PHYSICS FOR ARTS & SCIENCES

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PREFACE

This text is written for beginners. Its style is based on the assumption that its readers are beginners, but beginners capable of being interested in the way in which nature behaves.

Historical presentation is used only where it appears easier for the student and reasonably accurate for a modern perspective. For the most part modern explanations are given from the start, and experience with large numbers of students indicates that this method is usually both easier and more interesting for modern young people.

The presentation of material is based on the assumption that the physical world in which we live can be largely explained by mechanical and electrical concepts. Included in the general field of mechanics are the behavior of ordinary material objects, also the subjects of heat and temperature, the behavior of sound, and many molecular phenomena. In the broad field of electricity are included the ordinary behavior of electricity and also the subjects of magnetism and light. Light is shown to be electrical in nature, but the geometric optics associated with practical applications are also given. At the end of the text the general subject of nuclear physics is presented. The scope of the material covered in this text is that which is ordinarily included in a first year college course in physics.

The general pedagogical style is designed to encourage the student to think through particular types of problems by concrete reasoning rather than by an abstract mathematical approach. In some cases both methods are used but only in cases where a simple approach seems inadequate is the abstract analytical method used alone. The authors believe that the concrete reasoning approach develops the student's appreciation of the nature and methods of science and that it is desirable for both the general arts and sciences student and the prospective science major.

The material on electricity and associated subjects (Part II, Chapters 1 to 24, inclusive) was previously published under the title "Electronic Physics" (The Blakiston Company, 1943). Minor changes have been made in many of these chapters. A considerable amount of new material has been added to

Part II, Chapter 24 on nuclear changes and the energy available in these changes.

An overview is used as an introduction to each chapter. It gives the reader some idea of what to look for in the pages that follow. At the end of the chapter, the principal ideas developed are listed and a central thought summarizing the chapter is given.

Problems are arranged in graded groups. Those in the first group should be solved by all students. The more advanced students will find an outlet for their abilities in the second group. A third group is a list of suggestions for experimental work, some of which may be carried out in the home and some in the laboratory.

A feature of the text is the use of color in the line drawings. In general the material parts are represented in black, the special, pertinent features being depicted in red. For example, the iron of an electromagnet would appear in black lines, the electric currents in red. The beginner's attention is much more readily focused on the special features in these drawings than in ordinary all-black figures. The line drawings in Part II were prepared by Anne Scouten.

This text has been used with large numbers of students including regular college freshmen and sophomores, pharmacy classes, special courses in ground schools for pilots, and students in Signal Corps radio classes and a few high school classes. The present form of the text is based on this wide experience in the use of the material with a large number of teachers as well as with diversified groups of students.

The authors are indebted to Charles M. Fogel of the University of Buffalo for the critical reviewing of many chapters. The senior author is also indebted to Professor Sigmund W. Leifson, Department of Physics in the University of Nevada, for helpful suggestions on the presentation of material and for providing library and seminar room facilities for a part of the work. Considerable planning, supervision, and preparation of the illustrations have been contributed by the Art Editorial Department of The Blakiston Company, which the authors gratefully acknowledge.

THE AUTHORS

CONTENTS

PART I

MECHANICS, HEAT AND SOUND

THE FIELD OF SCIENCE

This chapter traces briefly the development of science from its beginnings in prehistoric times to the present day. Differences in the attitudes of the scientists as well as in their methods and achievements in different periods are pointed out.

It is interesting to notice that in the early periods little attempt was made to classify natural phenomena into all the various subdivisions that we now have. The branching out of specialized fields such as chemistry, biology, astronomy, geology, etc., represents the growth of knowledge in such specialties as time went on. In the early days the whole body of scientific knowledge was called Natural Philosophy. The part remaining after each specialty became sufficiently large and important to branch out for itself retained the name Natural Philosophy until relatively recent times. It is now known by the name physics.

This chapter attempts to show some of the relationships of the various sciences to one another and, in particular, indicates the way in which physics is basic to and intermingled with the other natural sciences.

1. Prehistoric Beginnings

The beginnings of science may be said to date back to the beginning of the human race as a more or less intelligent group of beings. And though today we like to think of science as being closely associated with man's desire to understand the world in which he lives, we probably will have to credit the first scientific achievements to the people who first used sticks for levers, who first tilled the soil, built fires, and otherwise adapted nature to practical needs.

To these forefathers of early civilization life was a curious mixture of the extremely practical and the extremely mysterious. Heat and light from the sun, the coming of rains or droughts, and other natural occurrences over which man had no control and little knowledge, seemed to call for worship and usually for fear. Mystic rites and sacrificial offerings were

used to propitiate the gods who were supposed to control these events.

In this manner early scientific thinking became intermingled with religious beliefs. Also the growth of science in later periods has often been influenced by the prevailing religious and philosophical attitudes.

2. The Scientific Method

To most of us the term science refers to a body of more or less well-organized knowledge that has been determined, collected and arranged in a purposeful manner. Comparatively, the early achievements of the cave man seem hit or miss in type. In fact, science means more than mere knowledge —it means also the method by which we attack problems.

The scientific method means careful observation of actual happenings, then honest thinking in an attempt to interpret the data and, finally, application of the results to new problems. More observations are then made on the basis of these predictions and so the cycle goes on through careful observation of the facts of nature, logical thinking and then application of the theory so developed. It is the method of good common sense, the method of intellectual keenness and honesty. It should be applied as far as possible to politics, business, religion, and all the affairs of everyday life as well as to the scientific events.

3. The Origin of Measurement

Such a method of attack necessitates accurate measurements and hence units of measurement. The first record of the recognition of these facts dates back to 2500 B.C. at which period rulers of Babylon legalized a number of units of length, for instance, the "finger" equal to two-thirds of our English inch, the "foot" equal to 20 fingers, the "cubit" equal to 30 fingers, and the "league" which was approximately equal to 6.65 of our miles. Units of weight, the grain, shekel, and talent, were also established in this period.

Considerable knowledge of arithmetic was common among the peoples of Chaldea, and both a decimal and duodecimal

system of counting were in vogue. The ownership of land led to elementary surveying and hence to some slight knowledge of geometry.

Somewhat similar knowledge of measurement, arithmetic, and geometry appeared in Egypt at about the same, or perhaps at a slightly later, period. At an even earlier period the computing of time and the establishment of a calendar came about in both these countries.

4. The First "Sciences"

A great deal of speculation existed as to the nature and importance of the earth with respect to the sun, stars and observable planets. On the basis of the limited knowledge then available the possible effects of the behavior of the heavenly bodies on the lives or welfare of people were unknown. Hence early attempts to note correlations between astronomical and terrestrial events were as scientific as other forms of observations and are not to be confused with what is now known as astrology.

Astronomy probably advanced further in Babylon than in Egypt, but on the other hand the Egyptians are credited with greater achievements in the art of medicine. Surgery in Egypt is recorded in carvings that date back as far as 2500 B.C. Egypt had men skilled in the setting of broken bones, the treatment of eye troubles, and the use of drugs. In fact, it is possible to trace medical knowledge from this early Egyptian era through the later Greek period and on through the Middle Ages to the present time.

Some developments in science, particularly in medicine, are recorded in India at a later time than those of Egypt (about 500 B.C.). Indians of this period were also skilled in arithmetical processes and our present system of numerals is reputed to have been derived from those used in India about 300 B.C.

5. Science and the Ancient Greeks

But it is the Greeks on whom modern civilization depended chiefly for its origin of science. It seems probable that Greek

knowledge and thought were influenced by Indian civilization and it is certain that much of the Greek learning was based on the achievements of Egypt. We are indebted chiefly to the Greeks for extensive written accounts of the early state of knowledge in these other centers of civilization.

It was in Greece, about 600 B.C., that honest and careful thinking began to play a role in scientific investigations. At this period some break in the tie between religion and science took place, with the result that both observation and thinking became more objective and less influenced by unfounded beliefs and mysticism.

The earliest Greek scientist of note was probably Thales of Miletus who achieved something of a reputation by observations on the food processes of animals and plants. He was also rated a good astronomer and is credited with organizing the geometry of Egypt into the form from which Euclid later developed it. His prediction of an eclipse in 585 B.C. shows the beginnings of the applied phase of the scientific method referred to in section 2.

The application of thinking to scientific problems was carried a step further by Pythagoras about a hundred years later in his emphasis on purely abstract concepts. But thinking abstractly instead of concretely also resulted in some tendency to get away from reality. It was in this period that pure numbers as such took on a special significance. Ten was considered the perfect number. The present fake science of numerology is related to this period somewhat as modern astrology is related to the serious astrology and astronomy of the early Egyptians, Babylonians and Greeks.

One of the important concepts of modern physics and chemistry is that matter is made up of atoms and that an atom is the smallest discrete particle which in any sense bears the characteristics of any particular element. This idea dates back to Democritus who was born in 460 B.C., and is reputed to have come to him through his teacher Leucippus. The concept of atoms at that time was in serious conflict with a prevailing early belief that matter was all different forms of a

single element, and it was also in conflict with a somewhat later notion that all matter was made up of fire, air, water, and earth. But all of these attempts to explain natural phenomena in simpler or better understood terms represent the addition of the second step of the scientific method to the earlier first step of making observations. In other words, they were attempts to develop reasonable explanations based on observations.

The third step in the scientific method, namely, the prediction of new effects and the carrying on of new experiments to check the theory, remained for our modern era of science to bring to its present high state. Its beginnings, however, may be seen in the work of Thales as noted above and in that of Aristotle and Archimedes as will soon be described. The limits of success in this third and important step in scientific procedure on the part of the Greeks was probably due only in part to failure to realize its importance. It must also have been largely due to inability to think of trial experiments with the meager amount of facts then known and the limited equipment with which to perform closely controlled experiments.

Failure of the early Greeks to make greater progress was also due to carrying to an extreme the very thing which constituted their greatest contribution to science, namely, thinking and philosophizing over their observations. The individual's conception of a thing became the real thing. If iron could be made to look like gold, then it was gold. Hence the early alchemist was not a faker, but rather a chemist whose knowledge permitted him to change the external appearance of metals.

Some success in the line of organized research however came with Aristotle (384 B.C.–322 B.C.). Much work was done by him in what is now called the field of biology, and some work also in physiology.

Further progress in predicting results and the verifying of theories dates from Archimedes (287–212 B.C.). His work in hydrostatics and his method for measuring the densities of materials have been immortalized in the story of his testing

a crown for King Hiero to ascertain whether or not the maker of the crown had used pure gold in its construction or had alloyed the gold with silver. The principle that a body immersed in a fluid has its weight reduced by an amount equal to the weight of the displaced fluid bears his name to this day. Not only did he do this experiment, but he also predicted by pure reasoning that it should be true.

The work of Aristotle in biological studies and to a still greater extent that of Archimedes in physics may perhaps be considered the nearest approach of ancient times to our now well-established scientific method of attack.

6. Science Among the Romans

In the period of the Romans just before and just after the birth of Christ we find some continued development in the fields of astronomy and medicine. But other branches of pure science received somewhat less attention than before although practical applications of scientific knowledge to architecture, road construction, water supplies for cities, public health, etc., went on at an unprecedented rate. The Romans were capable at doing things, but not greatly interested in any type of philosophic thought. Hence new knowledge did not increase appreciably during this era and the development of science ceased, although engineering, the practical application of science, flourished.

7. Science in the Dark Ages

In the first thousand years perier the birth of Christ the world seemed to remain in prs zy much of an intellectual morass. In part this condition was due to the subjective philosophy of the later Greeks who, following Pythagoras, believed that mental concepts were reality rather than the objects themselves. In part it was due to some of the teachings of the Christian religion then prevalent; in particular to a then common belief that only the world to come is of importance, and hence little attention should be paid to improving present life.

In marked contrast to this influence, learning was kept

alive in some medieval monasteries. In this manner the earlier knowledge of science was preserved and passed on to later periods even though progress in science was practically at a standstill.

8. Renaissance Science

At the end of the Dark Ages in Europe and somewhat earlier in Arabia a revival in learning began that finally resulted in the type of scientific procedure that we have today. Among the earlier names that stand out are: (1) Leonardo da Vinci (born 1452), who attacked scientific problems from the practical side but who symbolized the modern method of procedure by mathematical reasoning in arriving at experimental methods; (2) Copernicus (born 1473), whose fame resides chiefly in his exposition of the behavior of the earth and other planets in revolving about the sun; (3) Galileo (born 1564), who invented the telescope and made possible observations which verified the theories of Copernicus (for which Galileo was silenced by the church in 1616).

9. The Division of Science Into Specialties

The number of people engaged in scientific pursuits became rapidly larger and as knowledge increased the tendency to specialize in limited fields grew. The scientist no longer claimed to be an authority on all that was known of natural science. The study of living processes early became separated from other forms of nature study and has come to us under the name of biology. For convenience in specialization this subject has been further subdivided into botany, zoology, physiology, bacteriology, etc., some of the classifications being partially overlapping. As has been suggested in the above brief historical survey, medicine had early separated as a special field, and to some extent astronomy had done likewise.

10. Physics and Chemistry

The study of the properties of inanimate matter later divided under the arbitrary classification of physics and chemistry. The borderline between the two has never been

set with precision. The chemist is concerned chiefly with the abilities of elements to combine to form substances with physical properties not necessarily like those of the original substances. He is also concerned with the physical properties (hardness, density, elasticity, color, etc.) and he constantly uses purely physical processes such as weighing, measuring, etc.

Again, in electricity the processes of physics and chemistry are intermingled, so that elementary courses in both fields teach identical material on such subjects as electrolysis, electroplating, and the construction and operation of electric batteries.

In the study of the nature of atoms we find the two groups of scientists equally interested. The physicist, not being content with proof that gross matter is really made up of atoms as suggested by Democritus, has delved into the structure of atoms themselves and has produced evidence pointing to the atom as being electrical. He has also obtained some data throwing light on the arrangement of negative and positive electric particles in the atom.

But such knowledge, in addition to aiding the physicist to explain or predict physical properties of a substance, has also been invaluable to the chemist in predicting new chemical compounds, methods for producing them and the stability of the compounds when formed.

These cases are just a few examples of the overlapping of physics and chemistry. Some jealousy arises from time to time among the more ardent members of each group of scientists owing to the human tendency to classify in one's own field anything found to be useful in that field. More sober-minded scientists, however, waste no time in quarrels over questions of classification of this type.

In general it may be said that physics is a more basic science than any of the others since its methods and facts must be constantly used in all other sciences. It does not follow that chemistry, which is dependent on physics, and

biology, which is based on both physical and chemical processes, are any less essential in the whole scheme of life.

Physics as a separate science may be thought of as dating from the end of the Renaissance, roughly about 1600. In this period it was called Natural Philosophy. Discoveries and developments were made at an ever increasing rate so that by about 1800 the fund of knowledge was great enough to usher in the era of physics with which we are now familiar.

Many people like to consider the period from 1800–1900 as that of classical physics, and date modern physics from 1900. It is true that the nineteenth century saw the formal organization of much of the knowledge of physics, and in that sense it may be considered the classical period. However, the discoveries of this period were of great importance. Many of them led to important practical applications and others led to spectacular new discoveries in the period since 1900.

The developments in all the fields of science since the Middle Ages are too extensive to be covered here even in the brief manner in which the early history of science has been given. In the remainder of this book we shall confine our attention to physics and to those parts of chemistry, medicine, and biology where there is strong overlapping or where the applications of physics are particularly important or interesting. Moreover, we shall depart largely from the historical order and look at physics, both old and new, from the point of view of the present time.

Some Important Facts

1. In general, man's first conscious attempt to change his environment to suit his desires may be considered the beginning of science.

2. Science is an organized body of useful knowledge but, even more fundamentally, it is a systematic method of acquiring such knowledge.

3. Early in the evolution of the scientific method the need for measuring quantities accurately became apparent.

4. Astrology, medicine, geometry, arithmetic, etc., became more or less specialized in very ancient times.

5. The great Greek philosophers for the first time divorced science from theology. In fact, they anticipated many modern scientific ideas.

6. What kind of things were first to be measured?

7. Compare science in the Dark Ages and in the Renaissance.

8. Compare science in the Renaissance and in the present time.

9. What caused the subdivision of science into specialties?

What advantages and what disadvantages resulted from such subdivision?

Group B

1. Contrast the theories of the ancients with modern scientific theories.

2. Why is the science of measurement important?

3. How did the theoretical explanations of the Greeks differ from those of previous peoples?

4. Explain clearly what is meant by the scientific method.

5. From the point of view of long time progress what was wrong with the attitude of the Romans?

Experimental Problems

1. Trace the evolution of the theory of atoms back to Greek origins.

2. Write a biographical sketch of some Greek scientist.

3. Look up and write brief accounts on one or more of the following topics:

   (1) Motives and methods of alchemy.

   (2) The life story of Galileo.

   (3) An outline showing how to apply the scientific method to

      (*a*) An accomplished study such as Archimedes' work on flotation.

      (*b*) A new problem that you may want to tackle.

Chapter 2

THE SCOPE OF PHYSICS

This chapter gives in bold outline the major subjects included in physics and, in particular, it points out the chief fields of development in recent years.

Some reasons for a general belief in a mechanical view of the world are given, and the manner in which heat and sound appear as parts of mechanics is suggested.

The structure of atoms with their electrons, protons, and neutrons is given considerable space.

The subject of electricity is traced briefly in a historical manner and some of the recent trends, such as wireless, x-rays, and thermionic tubes, are outlined. Magnetism and optics are introduced as special phases of electricity.

An attempt is made to simplify the classification of all physics into mechanical and electrical phenomena. Since mechanical methods of measurement and mechanical ideas (for example: force, work, and power) are also used in electricity, it appears logical to begin a study of physics with elementary mechanics.

1. Becoming Science Conscious

Perhaps the first things to attract the attention of a small child in the world about him are mechanical objects either stationary or in motion. These early impressions the child accepts at face value and without prejudice of earlier conceptions of which he has none. Hence his first contact with the world is largely mechanical, and quite unwittingly he starts life with a mechanical philosophy of the world in which he finds himself.

Later in life, when something new comes under his observation he attempts to describe or explain it in terms of his already accepted mechanical knowledge. Perhaps it is safe to say that only a baby is truly open-minded, for as we grow older we insist more and more on having new things explained in terms of old ones, although often the new thing is more simple than the older ones which have been accepted earlier without question.

The whole human race, much like the individual newborn child, found itself in a world that appeared at first sight to be almost entirely mechanical. So mechanical developments led all others in the first days of scientific discovery, and thinking based on mechanical concepts became the basis of theories for every early scientist. Even now we find ourselves trying to explain optical and electrical effects in terms of mechanics.

It is possible that mechanics does form the actual basis of our world. On the other hand, perhaps it is energy, or perhaps it is electricity. We raise the question here, not because there is as yet any answer to it, but rather to show how mechanical concepts came to be the basis for scientific explanations and to indicate also that even so fundamental an issue is still an open question.

2. Our Mechanical World

We have already seen that simple devices such as levers, and the art of weighing and measuring, were among the early achievements along purely mechanical lines. On the other hand, the effects of forces in putting masses into motion, the kinetic energy and momentum of moving objects—ideas that are familiar to us in the use of automobiles, cannon, airplanes —were little understood until relatively modern times.

Heat, although originally considered a separate phenomenon of nature, is now explained on a mechanical basis since we believe that a hot body differs from a cold one chiefly in that the individual molecules of the hot body are in a more violent state of agitation than those of the cold one and hence may be said to have greater kinetic energy. Also, this energy can be taken from the hot body and turned into large scale mechanical energy. Steam and gasoline engines are familiar examples of devices that convert energy in the form of heat to that of motion.

The mechanics of fluids, both liquids and gases, obviously becomes a part of any complete study of mechanics. This field ranges all the way from simple problems concerning the

floating of objects on water or of balloons in air to the transmission of energy through liquids and gases by wave motion. The latter includes the familiar subject of sound, brought to new life in the last few years by the development of high-class radio sets and other sound-reproducing devices, by attempts to improve the acoustics of auditoriums, by an interest in antinoise campaigns, and by range and direction finders especially for use in times of war to locate guns, airplanes, and submarines.

In seeking for explanations in these subjects, as in the case of heat, one returns to the molecular nature of liquids and gases as the most likely hypothesis. Hence some study of the behavior of atoms and molecules is in order. Also, we find that many purely molecular phenomena, such as surface tension, osmosis, adhesion, cohesion and refrigeration, are of interest not merely for their own sake but also because of their applications to living processes and other everyday activities.

3. Early Knowledge of Atoms

In spite of the beauty of an atomic theory of matter for explaining so much of nature there has always existed a desire for still more direct proof of the existence of atoms and molecules. Practically no development of Democritus' concept of an atomic world was made until 1803 when John Dalton, an English chemist, announced experimental evidence for such a theory. This evidence was obtained from experiments on the chemical combining powers of various substances by weight. Unfortunately, other explanations of these experiments are also possible, and so Dalton's *proof* for the existence of atoms was not too good. However great progress in both physics and chemistry was made during the nineteenth century by assuming it to be at least an acceptable explanation.

During the past generation a number of more direct experiments have been performed. Of these, one of the best known is called Brownian movements. With the aid of a low power microscope the motions of small particles in tobacco smoke, or in a colloidal solution, can easily be seen. Such an experiment

can readily be set up by the teacher for observation. The motion of these particles is thought to be due to bombardment by the moving molecules in the surroundings.

The evidence for the existence of atoms and molecules is now considered so satisfactory that not much interest is left in the subject and physicists have turned their attention to the less well known and hence more exciting problem of the nature of atoms themselves.

4. Inside the Atom

Information on this subject started with experiments on electrical discharges in gases about 1890. Experiments with radioactivity (discovered in 1896) and studies of spectral lines of light (obtained from gaseous discharges similar to those now extensively used for advertising signs) contributed greatly to the knowledge. From these experiments there grew the belief that atoms are not little solid balls of material, but that they are made up of positive and negative electrical particles and neutral particles arranged something like a miniature solar system. The nucleus of the atom (corresponding to the sun) contains all of the positive and neutral particles while the negative particles revolve around the nucleus much as planets revolve around the sun.

Hence an atom is thought of as a sparsely occupied space the outer dimensions of which are roughly defined by the size of the largest orbit in which a negative particle of electricity is to be found. The positive particles are called protons, the negative ones electrons, and the neutral particles neutrons. The existence of protons and electrons was known long before the discovery of neutrons, and the early form of this picture of an atom included only the electrical particles. The early concept was named the Bohr-Rutherford theory after the two originators of the idea.

According to the above theory, atoms of all elements must be alike in the sense that they are built up of the same three primordial substances, and we are led to the conclusion that the only differences between atoms of different elements lie

are particles or waves, but where he simply states that under some conditions electrons behave as particles and under other conditions, as waves.

This search into the structure of atoms and the kind of units that go into their structure has been the high light of researches in pure science during recent years. Modifications and additions to the Bohr theory in an attempt to make it fit newly discovered facts, have also been numerous. Perhaps the most successful theory is one that assumes that electrons and protons may be considered as waves, as suggested by the experimental work mentioned above, and hence a so-called wave theory of atomic structure is now current. It has the advantage of predicting new facts better than the Bohr theory and the disadvantage of not being at all easy to picture in one's mind. A thorough study of this new theory will be left to such students as specialize in physics (by which time some other theory may be in vogue). Since the Bohr theory is responsible for much of our present advance in both physics and chemistry, it will be frequently used in this text to explain properties of matter and various electrical and optical effects.

6. Electricity and Magnetism—Early Knowledge

Theories of matter have held the spotlight of interest in physics for some years and hence have been given considerable space in this chapter. But we must also remember that the whole field of electricity comes within the domain of physics.

The earliest observed electrical effect was that amber acquired the property of attracting light objects after it had been rubbed. This experiment was first performed by Thales of Miletus in about 600 B.C. While this is now known as one of the simplest of electrical experiments, the real significance of it was not even guessed by Thales nor anyone following him for a period of 2000 years.

The attractive properties of magnetite (called lodestone) for pieces of iron or other pieces of magnetite were also known to Thales. This phenomenon is supposed to have been observed by the Chinese at an even earlier period (1000 B.C.)

and the use of lodestone to indicate directions (that is, as a compass) is credited to the Chinese of this early date.

No further progress along the lines of either electric or magnetic discovery was recorded until the time of William Gilbert (1540–1603) Court Physician to Queen Elizabeth and James I. Gilbert collected and published such meager facts as were known about these subjects, performed many experiments to measure magnetic and electric forces, and speculated (not very successfully) on the nature and causes of these effects.

It is interesting to notice that electric and magnetic effects were considered to be related, but nevertheless separate phenomena. We now attempt to explain magnetic behavior in terms of the known magnetic properties of electric currents. The new electrical theories of matter have been a great boon in these problems, especially with regard to explaining such things as permanent magnets.

In the 1600's and 1700's machines operating on the same principle as that of Thales' experiment were used to get charges of positive and negative electricity. These, as well as more improved types, were called static machines. They are used to this day to get sparks for demonstration purposes, but the amount of electricity that could be obtained each second in the early machines was too small to be very valuable. Extensive improvements in these devices in recent years have permitted the production of sufficient current at high voltages for much important experimental work.

7. Electricity and Magnetism—Present State

The development of batteries, the first devices to produce electric currents of appreciable magnitude, came in 1800, due to the work of the Italian scientists Galvani (1786) and Volta (1800).

Our modern electric power industry is based largely on discoveries made in 1831 by Michael Faraday, an English physicist. These experiments also furnished much information for the development of theories concerning the nature of electrical effects. Faraday himself was chiefly an experi-

menter, but his work was used by Clerk Maxwell to predict that electrical energy can be radiated. The discovery of wireless in 1888 by Heinrich Hertz was directly due to Maxwell's theories.

Maxwell's theory also suggested the explanation of ordinary visible light as a wave motion of an electromagnetic nature, and we now know that light waves and wireless waves are similar except for wave length, wireless waves being relatively long.

Experiments with electric discharges in gases led both to the discovery of x-rays (1895) (which are produced by bombarding a piece of dense material, such as a metal, with electrons) and also to experiments on spectral lines as already mentioned. These experiments, together with those made possible by radium (1898), led to all the electronic, as well as atomic structure, experiments of recent years. Many practical applications, such as the use of x-rays and radium in medical therapy, and the development of electron tubes (such as are in common use in radio receivers and transmitters), have accompanied these advances in pure science.

From the above brief account it must be evident that electric, magnetic, and optical effects are extremely closely related. In fact, the modern point of view seems to be to consider them all as electrical phenomena, and they are so presented in the latter half of this text.

We have thus simplified the classification of all the fields of physics into two large divisions, MECHANICS and ELECTRICITY. These fields overlap in the subject of atomic structure. We find also that such mechanical ideas as force, work, and power are used throughout the study of electricity.

These relations among the fields of physics seem to suggest some study of the elements of mechanics at the outset.

Some Important Facts

1. Mechanical concepts are among the first and most fundamental to demand our attention. Hence it is quite natural for us to try to explain new experiences in terms of old mechanical ideas.

2. Our mechanical view of the world has been extended to include not only force, motion, and energy but also such molecular phenomena as heat and sound.

3. From Democritus to Dalton, an "atom"—or indivisible unit of material structure—has seemed a reasonable speculation, but most of the experimental evidence as to the existence and characterization of atoms is rather recent.

4. We now believe that the atom is a minute solar system consisting of nuclear protons and neutrons and planetary electrons.

5. Another modern idea is that the electrons, protons, and neutrons of an atom may be treated as waves.

6. Prior to modern theories, electricity, magnetism and light were regarded as quite distinct. Now they are all explained in terms of electrical behavior.

Generalizations

In attempting to explain the world in which man finds himself he leans heavily on early impressions, most of which are in terms of the mechanics of his surroundings.

Mechanical explanations become strained in trying to cover some electrical phenomena. So we divide the realm of physics broadly into mechanics and electricity.

Questions and Problems

Group A

1. How did man first become interested in science?
2. How do you try to explain something to a person who is not familiar with what you are describing?
3. Why are so many of our explanations of a mechanical nature?
4. Draw a picture of an atom to show it as a solarlike system.
5. Describe the earliest discovery of electricity.

Group B

1. How do you extend mechanical explanations to the subject of heat?
2. How do you extend mechanical explanations to the subject of sound?
3. Compare the modern concept of the transmutation of elements with that of the Greeks.
4. How do we arrive at a picture of an atom?
5. Why was the discovery of electric batteries of great importance?
6. Give some reasons for considering light to be of an electrical nature.

Experimental Problems

1. Your baby sister is familiar with simple mechanical things about a house but has never seen a bow and arrow and you haven't got one to show her. Write up a description and explanation for her.

2. Think out an experiment to show some relation between ordinary mechanics and heat.

3. Think out an experiment to show some relation between ordinary mechanics and sound.

4. Think out an experiment to show some relation between electricity and magnetism.

SIMPLE MEASUREMENTS

Mass, length, and time are generally considered the three fundamental quantities in the physical world. It is also evident that these three properties are so common to our *everyday experience that in order to obtain quantitative* information it is desirable that we have generally accepted units by which to measure them.

Two sets of units for length and mass are in common use today. They are called the metric and the English units. They have been chosen in a very arbitrary manner.

The chapter opens with a discussion of the measurement of distance or length, then time, and then mass. Methods of measurement, especially for time and mass, are given as well as some description of the major properties of mass.

The chapter closes with a section on density and specific gravity.

1. The Measurement of Distance

For any exact work in science, and in fact for convenience in everyday life, it is necessary to have units by which things may be measured. We are familiar with expressing short distances in terms of inches or feet and longer distances in terms of miles. Other units to measure distances have been developed in various parts of the world at various periods. In the first chapter of this text we have credited the Babylonians with the first nationally recognized units of distance or length recorded in history.

At the present time there are two widely used sets of units of length, namely, the English, indicated above, and metric which uses centimeters for small distances and kilometers for large distances. It is obvious that the actual length of the table on which I write is just the same regardless of whether I measure it in terms of centimeters, meters, inches, feet, yards, or Egyptian cubits. But the number by which I specify the length will be different for each of these units since the units themselves differ. For example: if the table is 1 yard long it is also 3 feet, or 36 inches, or 91.44 centimeters long.

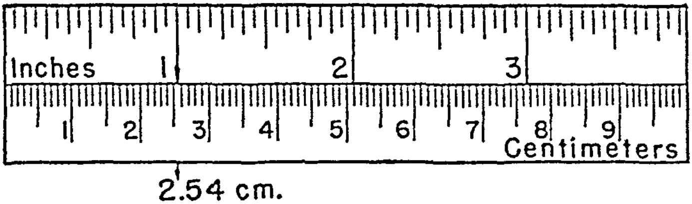


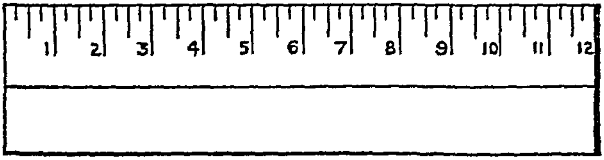
FIG. 1.—A comparison of inch and centimeter rules.

There is no particular virtue in one set of units of length as compared to another except that in the metric system there is progression by tens or powers of ten. For example: 10 millimeters make a centimeter, 100 centimeters make a meter, 1000 meters make a kilometer. In the United States we are already familiar with a progression by tens in our money system, for example, cent, dime, dollar.

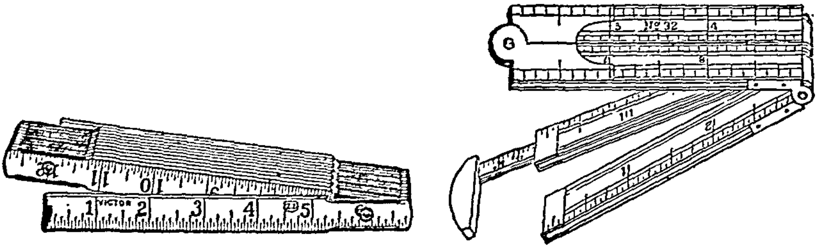
The actual length of a meter as well as the length of a foot or yard was a purely arbitrary choice. At the International Bureau of Weights and Measures in France is kept a bar made from an alloy of platinum and iridium. There are two marks on this bar and the distance between the marks (when the bar is at the temperature of melting ice) is, by international agreement, one meter. By law in the United States our yard (3 feet) is $^{3600}/_{3937}$ meter.

The convenience of decimal relations in the metric system has resulted in an increasing use of that system even in England, the United States and other English speaking countries. Common use of the system is to be noted on camera film whose cartons now state the size both in inches and centimeters, and again in the case of sports where there is a growing tendency to run distances of meters instead of yards. Below is a table showing the numerical relations between some of the units of the two systems.

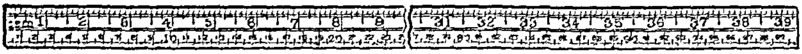
1 inch	=	2.540 centimeters (cm.)
1 cm.	=	0.3937 inches
1 ft.	=	30.48 cm.
1 yd.	=	0.9144 meter ($^{3600}/_{3937}$ meter)
1 meter	=	39.37 inches or 1.0937 yards



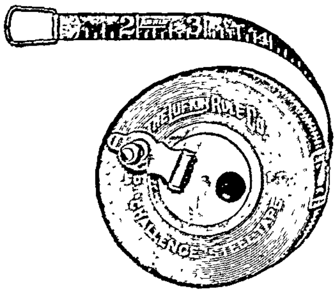
A. A foot rule



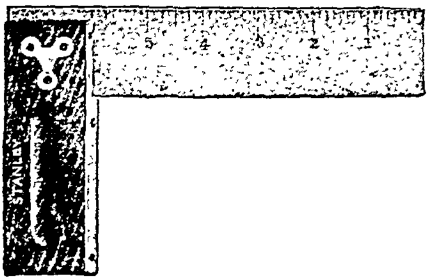
B and C. Folding rules



D. Meter stick



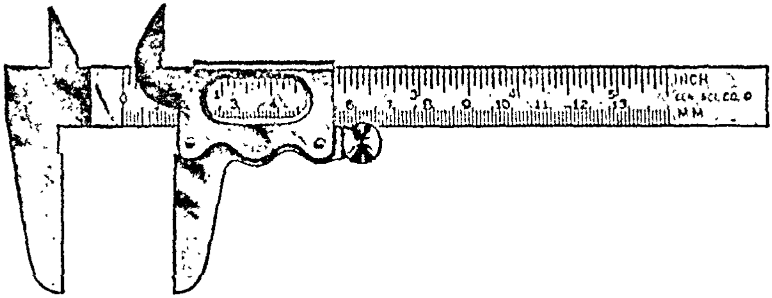
E. Tape measure



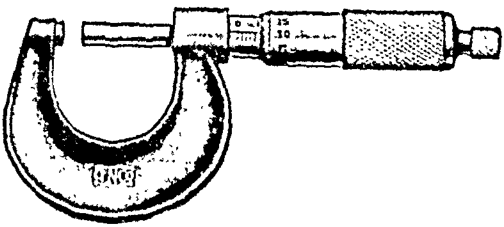
F. Steel square

G. Steel rule

Fig. 2(A–G).—Measuring instruments.



H. Vernier caliper



I. Micrometer

FIG. 2(H–I).—Measuring instruments (*continued*).

The table shown on page 23 should be used for reference rather than memorized unless a person has a great deal of occasion to convert distances in one set of units to the other. The first relation (that 1 inch is equal to 2.54 cm.) is probably used sufficiently by most people to justify remembering it, and the other relations can be easily derived from this one if the table is not at hand.

2. Instruments for Measuring Distances

A common instrument for measuring distance is the familiar foot rule or ruler shown in A in Figure 2. At B and C are seen folding forms of a ruler. They are easier to carry than the meter stick shown at D.

Another common type of measuring instrument is the tape measure, inexpensive models being made of cloth, and better ones of steel ribbon. (See E.) The tape measure will roll into small space and is easy to carry.

None of the above instruments are used for precise work. The steel square and rule, as shown in F and G, can be made with more accurate markings. For measuring short distances with even greater precision, special instruments such as the vernier caliper, shown at H, and the micrometer, shown at I, are used.

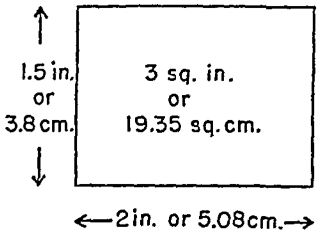


FIG. 3.—Area is the product of two linear dimensions.

3. Measuring Areas

Measuring areas is just an application or extension of measuring distances. For a simple case, suppose that the rectangle of Figure 3 has sides of 1.5 and 2 inches as marked. The area is then:

$$1.5 \text{ in.} \times 2 \text{ in.} = 3 \text{ square inches (sq. in.)}$$

If we use the table on page 23 we find that this rectangle can have its sides given in centimeters as follows:

$$1.5 \times 2.540 = 3.810 \text{ cm.}$$
$$2 \times 2.540 = 5.080 \text{ cm.}$$

The area must then be:

$$3.810 \times 5.080 = 19.35 \text{ sq. cm. approx.}$$

Areas of less simple shapes may offer greater difficulty in actual measurement, but the result can always be stated in terms of the square of the linear distance unit that is used.

4. Measuring Volumes

The measurement of volumes may also be thought of as an extension of measuring distances. This time we get an answer in terms of the cube of the linear distance unit that is used. For example, consider the volume of the object in Figure 4 with dimensions of 1.5, 2.0, and 1 inches as shown.
The volume is:

$$1.5 \text{ in.} \times 2.0 \text{ in.} \times 1 \text{ in.} = 3 \text{ cubic inches (cu. in.)}$$

The dimensions of this same object in centimeters is 3.810, 5.080, and 2.540. The volume is:

$$3.810 \text{ cm.} \times 5.080 \text{ cm.} \times 2.540 \text{ cm.} = 48.15 \text{ cubic centimeters (cc.) approx.}$$

It so happens that in the metric system there is also a unit of volume, called a liter, defined as the volume of 1000 g. of water at its temperature of greatest density (about 4° C.). This volume, called the liter, is approximately, but not quite, equal to 1000 cc. One one-thousandth part of the liter is called a milliliter and is approximately the same as one cubic centimeter.

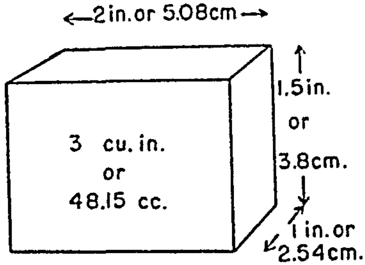


FIG. 4.—Volume is the product of three linear dimensions.

5. The Measurement of Time

The need for measuring intervals of time is almost as obvious as that for determining distances. It was apparent to the Egyptians, Babylonians, and Greeks; and from their work we have derived the concept of the year, month, week, day, hour, minute, and second. The exact length of the year in terms of the day was not accurately known to the ancients and some revision in calendars has been necessary from time to time to keep the seasons in the same part of the calendar year.

We now define the second as 1/86,400 of a mean solar day and this is both the national and international unit of time.

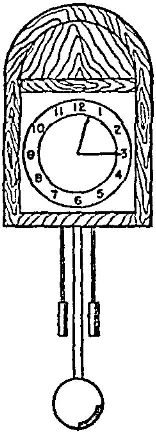
One of the more accurate devices in general use for indicating time are pendulum type clocks with some arrangement for compensating for any change in length of the pendulum when the temperature changes.

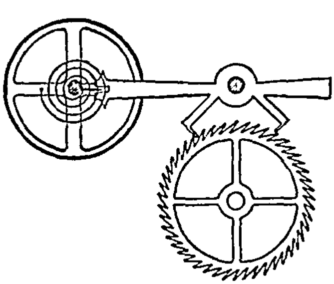
Recently, electric clocks have come into extensive use. They operate with tiny motors whose speed is controlled by the frequency of alternations in the electric power supply.

Where the power plant controls this frequency accurately, the electric clocks are almost perfect timekeepers.

Watches *employ an escapement system for regulating the* speed at which they run. This consists of a cogged wheel and ratchet where the latter is controlled by a wheel with a relatively heavy rim and with a coiled spring attached in such a manner that the wheel may rotate to and fro. Such devices can be made accurate enough for most practical purposes, but do not compare favorably with a high-grade pendulum type clock.

An accurate check on time can be made by daily observations on the transit of stars. The clocks of the U. S. Naval Observatory are checked in this manner. The approximately correct time is tolled off by seconds several times each day over government wireless stations at Arlington, Virginia. Minor corrections (usually not more than a few hundredths of a second) on these radio broadcast signals are mailed to laboratories throughout the country where experiments requiring accurate timing are carried out.





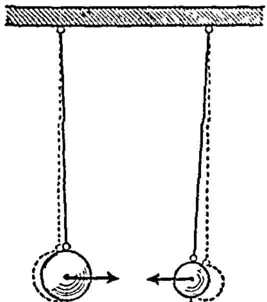


Fig. 5.—(*Left*) The to and fro motion of a pendulum is so regular that it makes an excellent control device for a clock.

Fig. 6.—(*Center*) The combination of a wheel which rotates to and fro with a ratchet and toothed wheel is used to control the rate of watches.

Fig. 7.—(*Right*) Gravitational attraction of matter.

6. Quantity of Matter—Mass and Weight

Distance, time, and mass are generally considered the three basic quantities involved in natural phenomena. All other quantities (force, work, power, etc.) can be expressed in terms of these three and so distance, time, and mass have received the name of "fundamental" units.

Mass may be considered as a measure of quantity of matter. Matter has two chief characteristics: first, inertia, or the ability to resist attempts to change its state of rest or motion, and second, mutual attraction with other quantities of matter.

Both of these effects are common in our daily experience. The engine of an automobile must perform work to get the car up to any specified speed from a state of rest. Effort is expended through the brakes to stop the car after it is moving. These are familiar examples of the inertia of matter of the car.

The attractive force (called gravitational force) between material objects is even better known, for this is the reason why an object falls when left free near the earth.

The gravitational force between two massive spheres may be measured by suspending them by long wires so that the spheres are close together. The attractive force can be measured from observations on the change in direction from the perpendicular of the supporting wires as indicated in the exaggerated drawing of Figure 7.

The experiment with two spheres is a laboratory job, but the attractive force between any object of ordinary size and the earth is so large that it is easy to measure. This attractive force with the earth we call the *weight* of the object.

Although the quality of matter called inertia may be equally important with the weight characteristic it turns out that the measurement of inertia is in most cases a laboratory job. Consequently we nearly always measure weight to determine quantity of matter quite without regard to whether our particular interest is in weight or inertia. We do, however, use two terms to indicate what our interest in the quantity of matter may be, *weight* for the gravitational attraction

characteristic, and *mass* for the inertia quality. So, for example, in the English system of units discussed in the next section we might speak of one pound mass or one pound weight and be talking about the same quantity of matter.

7. Units of Mass and Weight

The names given to commonly used units of weight and mass in the metric system are the gram and kilogram and in the English system the ounce, pound, and ton.

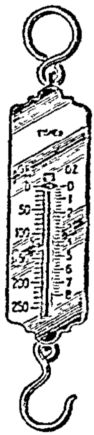


FIG. 8.—Spring balance. One of the simplest methods for measuring weight is by means of the stretch in a spring fastened at one end and with the object to be weighed fastened to the other.

An attempt to correlate the kilogram with the metric units of length was made by originally defining the kilogram as the mass of water in 1 liter (1000 cc.) at the temperature which gave water the greatest density. Actually, the kilogram is an arbitrary amount of mass. It is the mass of a one kilogram standard stored in the same bureau as that in which the international meter is kept.

The gram is $\frac{1}{1000}$ of a kilogram and is a commonly used unit for specifying the masses of small objects.

The pound is defined in the United States as 0.453593 kilogram. The following table gives the approximate values of some of the commonly used relations between these sets of units of mass and weight in the metric and English systems.

453.6 grams	≡ 1 pound
28.35 grams	≡ 1 ounce
1.0 kilogram	≡ 2.2 lbs. approx.

Note that the terms pound, kilogram, etc., are applied no matter whether we are interested in the inertia characteristic (mass) or the weight characteristic.

8. The Measurement of Weight

One of the simple methods for measuring weight is by means of the stretch in a spring fastened at one end and with

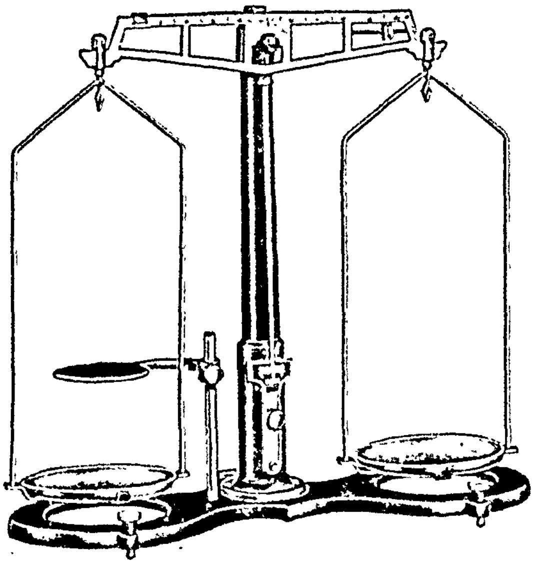


FIG. 9.—Equal arm beam balance. The object to be weighed is placed in one pan, and weights of known value are placed in the other until the beam assumes a horizontal position. The weight of the object is then equal to the value of the known weights.

the object to be weighed fastened to the other end. The stretch of the spring must first be calibrated by using weights of known value. Various modifications of this system, such as coiled springs and compression springs, are also in general use. Such a device is called a *spring balance.* It has the advantage of simplicity and cheapness of construction and the disadvantage of rather poor accuracy due in part to the limit of precision with which such a scale can be calibrated and read, and partly to the fact that springs do not keep their elasticity constant over long periods. Nevertheless spring balances are widely used.

A more accurate device for determining weights is called the beam balance. One is shown in Figure 9. If the distances on the arms are the same, the balance may be called an equal arm balance. The object to be weighed is placed in one pan, and weights of known value are placed in the other

until the beam assumes a horizontal position. The weight of the object is then equal to the value of the known weights.

When the object to be weighed is relatively heavy a balance with unequal arms is used. (See Figure 10.) The object is suspended from the shorter end and the known weights from the longer. The weight of the object, when the scale is balanced, is then given by the value of the known weights multiplied by the ratio of the length of the long arm to that of the short one. When scales of this latter type are pur-

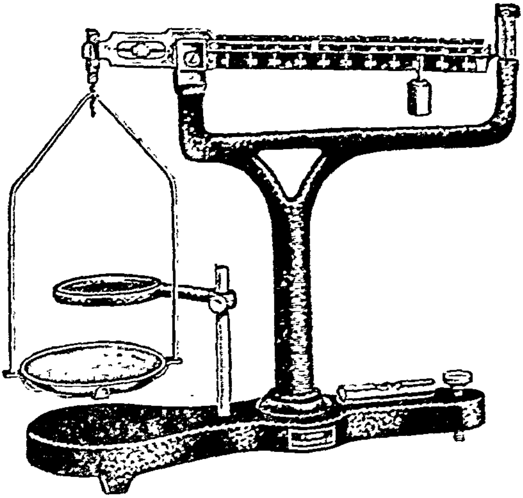


FIG. 10.—A balance with arms of unequal lengths.

chased, it is often possible to obtain standard weights marked in terms of what the corresponding weight of the object will be instead of with the actual value of the standard weight.

9. Density—Specific Gravity

The mass or the weight of bodies in proportion to the space that they occupy varies greatly from one kind of material to another and a measure of this property of matter is the mass in one unit of volume of the substance. This quantity is called density. It is usually given in grams per cubic centimeter or pounds per cubic foot.

Reference to the manner of choosing the unit of mass in the metric system as given earlier in this chapter shows that the density of water is exactly 1 g. per cc. at the temperature of greatest density. The density of water differs slightly from this value at other temperatures due to the expansion of water with change of temperature.

The unit of volume in the English system is the cubic foot and we find that a cubic foot of water weighs 62.4 lb. The density of water is therefore 1 g. per cc. or 62.4 lb. per cu. ft.

The density of some other familiar objects may be of interest. The density of air at normal temperature and pressure is 0.001293 g. per cubic centimeter or 0.0808 lb. per cubic foot. The density of iron is 7.85 g. per cc. or 490 lb. per cubic foot.

The densities of all substances (as well as water) vary with temperature and, especially in the case of gases, with pressure.

Since water is such a common substance it is sometimes convenient to state the *density of other materials relative to that of water.* This ratio of the density of any substance to that of water is called the *specific gravity* of that substance. In the metric system the specific gravity of any substance is numerically the same as the density, for the density of water, with which it is being compared, is unity.

The ratio of the densities of a substance to that of water in the English system is the same as in the case of the metric system. For example, the specific gravity of iron may be found by dividing 490 lb. per cubic foot, the density of iron, by 62.4 lb. per cubic foot, the density of water in the English system. The result is 7.85, the same as would be obtained in the metric system.

Specific gravity may also be defined as the ratio of the mass of any object to the mass of an equal volume of water. The value of specific gravity defined in this manner for any object will be the same as that obtained by comparing the densities.

It is often convenient to show the relations between mass, volume, and density by the following equation

$$\text{density} = \frac{\text{Mass}}{\text{Volume}}$$

$$d = \frac{M}{V}$$

where d is the density, M the mass of a specimen and V its volume. When any two of these quantities are known the third may be readily calculated.

Example. Find the weight of air in a room $12' \times 14' \times 8'$ if the density at the temperature of the room is 0.075 lb. per cu. ft.

The volume of the room is

$$V = 12 \times 14 \times 8 = 1344 \text{ cu. ft.}$$

The mass of air is

$$M = V \times d$$
$$M = 1344 \times 0.075 = 100.8 \text{ lb.}$$

Example. A brass cylinder is found to be 1.5 cm. in diameter and 13.0 cm. long. The weight of the cylinder is 195.5 g. Find the density of the brass.

The volume of the cylinder is $\pi r^2 L$ where r is the radius and L its length.

$$\pi r^2 L = \pi(0.75)^2 13 = 23.0 \text{ cc.}$$

The density is

$$d_{\text{brass}} = \frac{M}{V}$$

$$d_{\text{brass}} = \frac{195.5}{23.0} = 8.5 \text{ g. per cc.}$$

Example. Compare the spaces that will be occupied by 100 lb. of iron and 100 lb. of aluminum if the specific gravity of iron is 7.85 and that of aluminum 2.699.

From the specific gravities we can compute the densities in the English system.

$$\text{density} = \text{density of water} \times \text{specific gravity of substance}$$
$$d_{\text{iron}} = 62.4 \times 7.85 = 490 \text{ lb. per cu. ft.}$$
$$d_{\text{al}} = 62.4 \times 2.699 = 168.5 \text{ lb. per cu. ft.}$$

The space occupied by 100 lb. of iron and aluminum respectively will be

$$V = \frac{M}{d}$$

$$V_{\text{iron}} = \frac{100}{490} = 0.204 \text{ cu. ft.}$$

$$V_{\text{aluminum}} = \frac{100}{168.5} = 0.593 \text{ cu. ft.}$$

Some Important Facts

1. Time, distance, and mass are the three basic quantities of the mechanical world, and require distinct measuring systems.

2. From these three fundamental quantities several derived quantities and their measuring systems are built up, among which are: area, volume, weight, force, pressure, density, speed, velocity and acceleration.

3. Two systems of measuring length and mass are in common use—the English and metric. In the English system the foot is the unit of length and the pound of mass; in the metric system, the meter is the unit of length and the gram of mass. In the metric system, each unit is ten times the next smaller one, and one-tenth the next larger one.

4. Our second-minute-hour system of measuring time is universally used; it dates from early Babylonian times.

5. Mass and weight are measured in the same units—the gram in the metric system and the pound in the English system.

6. Matter has two fundamental characteristics: (1) inertia to changes in motion, called mass, and (2) mutual attraction to other matter. The important application of (2) in everyday life is the *weight* of matter near the earth.

7. The mass per unit volume of a body is its density, expressed in grams per cubic centimeter, or pounds per cubic foot. The relative density of a body, with water as a standard, is the specific gravity of the body.

Generalizations

There are three ideas or concepts in Mechanics—time, distance, and mass—which are so basic that we cannot break them down into anything more simple.

These three fundamental concepts must often be measured, sometimes with great precision, so we have invented units and instruments for measuring them.

Also, there are several other concepts derived from these three basic ones, and they too require measuring units and devices.

Questions and Problems

Group A

1. What are the three basic quantities which may be directly measured?

2. In what units are each of these basic quantities measured?

3. What quantities are directly derived from length alone? In what units is each measured?

4. What quantities are derived from mass and length? In what units is each measured?

5. What is density? In what units is it measured?

6. What is speed? In what units is it measured?

7. Why is measurement so important in physical science?

8. When and under what conditions was the metric system invented?

9. Tell what you know of the history of the English system of weights and measures.

10. Discuss the evolution of time measuring systems and devices.

11. Compare the English and metric systems as to convenience in use.

12. What is the approximate English equivalent of a 75 millimeter gun?

Group B

1. Explain the plan of construction of the metric system, illustrating your explanation by quoting from metric tables.

2. What is the weight of a liter of pure water at 4° C.: (*a*) in grams? (*b*) in kilograms? (*c*) in pounds? (*a*)1000 g.; (*b*) 1 kg.; (*c*) 2.2 lb.

3. A sprinter makes a record of 11 seconds flat for the 100 meters. At the same speed, what would his time be for 100 yards? 10.06 seconds.

4. Take the density of brass as 8 grams per cubic centimeter. What is its density in pounds per cubic foot? 500 lb./ft.^{3}

5. Find the metric dimensions of a photographic film $3\frac{1}{2} \times 5\frac{1}{2}$ inches. 8.89 cm. $\times$ 13.97 cm.

6. The distance from Buffalo to Rochester is approximately 70 miles. Find the distance in kilometers. 112.6 km.

7. Find the volume of a liter in terms of cubic feet. .0353 cu. ft.

8. Find the weight in grams of a 9 oz. baseball. 255.15 g.

9. If a man can run 100 yd. in 10.1 sec., how long will it take him to run 100 meters if his average speed does not change? 11.05 seconds.

10. A cylinder of steel has a diameter of 2.5 cm. and a length of 5 cm. It weighs 194.3 g. Find the density of steel. 7.92 g./cc.

11. A piece of cork $2 \times 3 \times 5$ inches is found to weigh 0.25 lb. Find the density in the English system. Find the specific gravity. 14.4 lb./ft.^{3}; .23

12. How much would a sphere of cork 5 feet in diameter weigh? Make a guess at the answer to this problem before you solve it.

13. The top of a table is 4×7 feet. It may be made of a wood 1.5 inches thick with a specific gravity of 0.95, a second type of wood 2.0 inches

thick with specific gravity 0.82, slate slab 1.25 inches thick with specific gravity of 2.6, or sheet steel 0.25 inches thick with specific gravity 7.85. Compute the weight of the table top in pounds for each of these cases.

207.5 lb.; 238.8 lb.; 473.2 lb.; 285.7 lb.

Experimental Problems

1. Select several small solid objects for volume determination. Compute the volume of each from their measured dimensions. Then determine the volume of each by measuring the volume of water it displaces.

The water may be measured by means of a graduated cylinder, or by catching it in an overflow can and taking its net weight in grams as numerically equal to its volume in cubic centimeters.

Which method, calculation from dimensions or water displacement, do you consider the more accurate? What are the limitations of each method?

2. Determine the densities of several common substances by dividing their weights by their respective volumes—determining their volumes by calculation from linear dimensions, or by water displacement, as you deem best. Compare your experimentally determined values of density with accepted densities of the substances used.

3. A vernier caliper may be calibrated to measure short lengths accurately to a tenth of a millimeter—a micrometer, to a hundredth of a millimeter. If you can get access to either or both of these instruments, study their method of operation and reading, and take several measurements. However, do not use a micrometer without expert advice, as you might damage the instrument.

THE MOTION OF OBJECTS

In the present motorized day, everyone is familiar with motion. The rate at which we travel, the time required to go from one place to another at various rates, and even the fact that we change from one rate of travel to another, are all included in our everyday working knowledge.

This chapter attempts to systematize this general knowledge, and since the ideas themselves are fairly simple, a great deal of the chapter is devoted to giving examples by way of illustration.

1. Simple Motion

Some understanding of the motion of ourselves and the motion of objects around us is a part of our early knowledge of the world. In fact, for most of us, the accepted use of trains, automobiles, and airplanes has made a fairly quantitative knowledge of velocity, distances covered, and change in velocity almost a necessary part of our everyday experience.

If the distance from Buffalo to Schenectady is 300 miles and an automobile used for the trip averages 40 miles per hour, it is apparent to everyone that Schenectady can be reached in 7.5 hours. This common knowledge relating time, distance, and average velocity may be written as an equation in which we say,

$$\text{distance} = \text{average velocity} \times \text{time}$$

In symbols we would write

$$d = \bar{v}t$$

where d is the distance covered, $\bar{v}$ is the average velocity, and t is the time involved.

As everyone knows, the road from Buffalo to Schenectady is not perfectly straight, and therefore the actual direction in which the automobile moves in going from one of these cities to the other is constantly changing. However, the direction

of travel in going from Buffalo to Schenectady is, on the average, towards the east.

In the above example we specified that the average rate at which the automobile traveled was 40 m.p.h. This 40 m.p.h. might be called the average *speed* at which the automobile traveled. If, however, we are also interested in the direction in which it traveled, we would say that 40 m.p.h. is to be called a *velocity*. So the difference in the use of the words speed and velocity is simply a matter of whether or not we are interested in the direction.

The indicating instrument on the panel of an automobile is called a speedometer. It tells us the rate at which the automobile is traveling, regardless of the direction in which it is going. In many cases, such, for instance, as driving an automobile in a race, the rate of travel is the only thing that is important. Obviously, however, if one wishes to travel from one point to another the direction is equally important with the rate of travel. In the next section we will consider still other cases where direction of travel is of importance as well as speed of travel.

2. Combined Motions

Calculating the time taken for an airplane to travel between two points is not quite so easy as is the case of the automobile, because with the airplane the possibility of wind affecting its motion must always be considered.

Suppose we have at our disposal a plane that can average 150 m.p.h. air speed and let us consider the effect of a 30 m.p.h. wind.

We can at once say that if it is a head wind, the plane's velocity with respect to the ground will be reduced to 120 m.p.h., while if it is a tail wind, the plane's velocity will be increased to 180 m.p.h.

If the wind is a cross wind, the plane will have a tendency to be carried off to one side of its course. In this case the pilot will probably head his ship slightly into the wind to counteract the effect of the wind in carrying him off course.

To study the effect of a cross wind, we may first suppose that the pilot's normal course is due east and that he is flying his plane by keeping it pointed in an easterly direction. We will assume that the wind is blowing from north to south. In the diagram of Figure 11, we can indicate the velocity of the plane by the horizontal arrow which is labeled 150. The shorter arrow drawn from north to south and marked 30 gives us the effect of the wind. The red line marked 153 gives us both the speed and the direction that the plane has followed with respect to the ground.

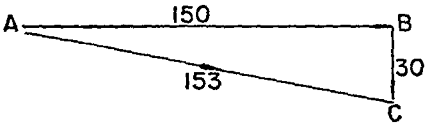


FIG. 11.—AB Steered course, 150 m.p.h. BC Wind vector, 30 m.p.h. AC Actual path, 153 m.p.h.

In the above simple case where the wind and the direction in which the plane was traveling are at right angles to one another, we can very easily find the combined velocity with respect to ground since this quantity is the hypotenuse of a simple right angled triangle. We square the value of each leg of the triangle, add the two, and extract the square root to get the length of the hypotenuse. We might also get the answer graphically by simply drawing the 150 and the 30 to scale, both as to size and direction. The length of the hypotenuse can then be obtained from the drawing and at the same time the direction will be indicated. This latter method is especially useful in case the wind and the direction in which the plane is pointed have some value other than 90°.

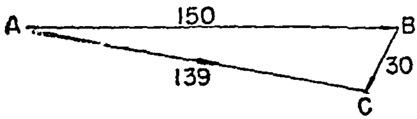


FIG. 12.—AB Steered course, 150 m.p.h. BC Wind vector, 30 m.p.h. AC Actual path, 139 m.p.h.

For example, consider the case illustrated in Figure 12. Here the wind is partly a cross and partly a head wind. Again, in Figure 13 we have the case of a wind which is partly a cross wind and partly a tail wind. In each of these cases, the actual direction of travel is indicated by the red line, and

the actual ground speed may be obtained from the red line, provided the figure has been drawn to scale.

In the example of cross winds which we have just discussed, it is evident that if the pilot wished to arrive at some point directly east, he would have been carried off course in all of these cases by the cross wind. If, in the case illustrated in

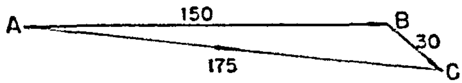


FIG. 13.—AB Steered course, 150 m.p.h. BC Wind vector, 30 m.p.h. AC Actual path, 175 m.p.h.

Figure 11, the pilot had actually wished to proceed in an easterly direction, it would have been necessary for him to point his plane slightly into the wind. This case is illustrated in Figure 14. The red line indicates the desired easterly direction of travel. The long arrow is the actual plane speed through the air, in this case 150 m.p.h. This arrow must be pointed sufficiently against the wind so that the plane will

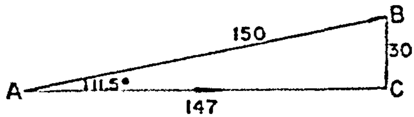


FIG. 14.—AB Steered course, 150 m.p.h. BC Wind vector, 30 m.p.h. AC Actual path, 147 m.p.h.

carry itself to the north an amount equal to that by which it is carried south by the wind. If Figure 14 is drawn to scale we see that the actual velocity in the desired direction of travel is 147 m.p.h., and if we now measure the angle with a protractor we discover that the plane must be pointed 11.5° north of east.

3. Vector Quantities

Any quantity in which direction as well as size must be considered is called a vector quantity. From this definition

it follows that speed is not a vector quantity. It is called a scalar quantity. On the other hand, velocity is a vector quantity.

Problems involving vector quantities can frequently be solved most easily by drawing the quantities to scale, as was suggested in the discussion of the airplane above. Persons who have a working knowledge of trigonometry will probably prefer to use that mathematical method for working such problems, but so far as the ideas are concerned, it is just as satisfactory to use the graphical method.

In this chapter we shall be concerned with velocity and rates of change of velocity (called accelerations), but in later chapters we will find other quantities in which direction as well as size is important. They will also be called vector quantities and will be treated in the same manner as velocities are being handled in this chapter.

4. The Parallelogram Method

In section 2 we added vector quantities by first drawing one vector quantity to scale, and then by drawing the next

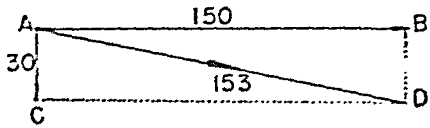


FIG. 15.—AB Steered course, 150 m.p.h. AC Wind vector, 30 m.p.h. AD Actual path, 153 m.p.h.

vector quantity beginning where the first one left off. The line connecting the beginning of the first vector line with the end of the second one then gave us the combined vector quantity.

Many people prefer to add two vectors by drawing both of them from a common point. For example, in Figure 15 we may draw a line representing 150 m.p.h. and another representing 30 m.p.h. from a common point instead of in the manner in which they are drawn in Figure 11. On these two lines

we may complete a parallelogram as is indicated by the dotted lines in Figure 15. The resultant of the two vector quantities will now be represented by a line drawn from the initial point to the diagonally opposite corner of this parallelogram. Of course, a little study of this figure and that of Figure 11 will show that the upper part of the drawing in Figure 15 is identical with the entire drawing of Figure 11.

We may take another example by considering the case of the cross wind shown in Figure 13. Here we again draw the plane's velocity and the wind's velocity both as to size and direction, from a common point. (See Figure 16.) We then complete a parallelogram as is shown by the dotted lines, and

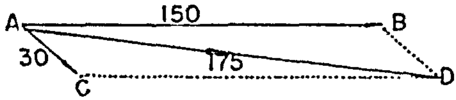


FIG. 16.—AB Steered course, 150 m.p.h. AC Wind vector, 30 m.p.h. AD Actual path, 175 m.p.h.

again a diagonal from the starting point represents the actual velocity in both size and direction.

There is very little difference between the parallelogram method in this section and the triangle method of section 2. The parallelogram method is usually preferred when there are only two vector quantities to be added. However, if there are more than two vector quantities to be added, the simple method of section 2 is considered by some people to be better.

5. Further Examples

1. A man travels at 30 m.p.h. for one hour and then 50 m.p.h. for one-half hour. How many miles does he cover, and what is his average speed? Here we use the equation

$$\text{distance} = \text{speed} \times \text{time}$$
$$\text{distance}_1 = 30 \times 1 = 30 \text{ miles}$$

and

$$\text{distance}_2 = 50 \times \frac{1}{2} = 25 \text{ miles}$$
$$\text{Total distance} = 30 + 25 = 55 \text{ miles}$$

The total elapsed time is 1.5 hours. We can rewrite the above equation to read

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

$$\text{speed} = \frac{55}{1.5} = 36.7 \text{ m.p.h. approx.}$$

2. Suppose that in example (1) the man had traveled for the first hour in a northerly direction, and for the next half-hour in an easterly direction. Where would he be at the end of the 1½ hours with respect to his starting point? Figure 17 shows an arrow drawn upward to indicate the distance covered in the first hour. The arrow drawn to the right indicates the distance traveled in the next half-hour. The red line joining the starting point and the finishing point gives, of course, the distance and the direction of the end point of the journey with respect to the starting point.

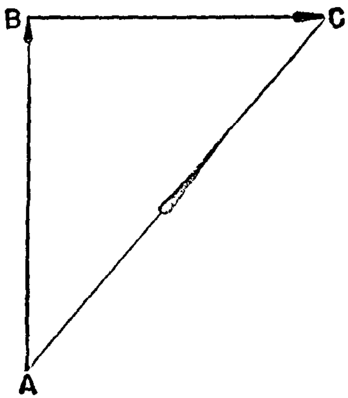


Fig. 17.—AB, 30 mi. in 1st hour. BC, 25 mi. in 3rd half-hour. AC, 39 mi. in 1.5 hours, equivalent straight course.

3. A man drives 50 miles at an average velocity of 25 m.p.h. He then continues in the same direction for three hours at an average velocity of 40 m.p.h. Find the total distance covered and his average velocity for the entire trip.

During the last three hours he travels

$$d = \bar{v}_2 t = 40 \times 3 = 120 \text{ miles}$$

Since he previously traveled 50 miles it is obvious that the total distance covered is 170 miles.

From the first statement in the problem he spent time

$$t = \frac{d}{\bar{v}_1} = \frac{50}{25} = 2 \text{ hours}$$

in the first part of the trip, and this combined with the three hours at the higher rate of driving gives an elapsed time of five hours.

The average rate for the entire trip is, therefore,

$$\bar{v} = \frac{d}{t} = \frac{170}{5} = 34 \text{ m.p.h.}$$

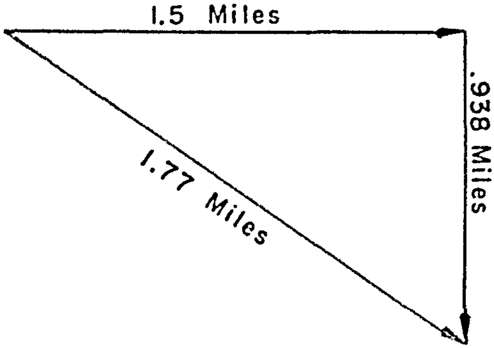


FIG. 18.—When a man rows cross stream the current carries him down so that he lands on the opposite bank at a lower point than that from which he started.

4. A man rows a boat at a velocity of 4 m.p.h. cross stream on a river whose current travels 2.5 m.p.h. If the river is 1.5 miles wide, how far downstream does he land?

Since he rows directly cross stream, he will cover the width of the stream in the time

$$\text{time} = \frac{\text{distance}}{\text{velocity}}$$

$$t = \frac{d}{v_1}$$

$$t = \frac{1.5}{4}$$

$$= .375 \text{ hours}$$

$$= 22.5 \text{ min.}$$

In this time the current will have carried him downstream

$$d = v \times t$$

$$= 2.5 \times .375 = .938 \text{ miles approx.}$$

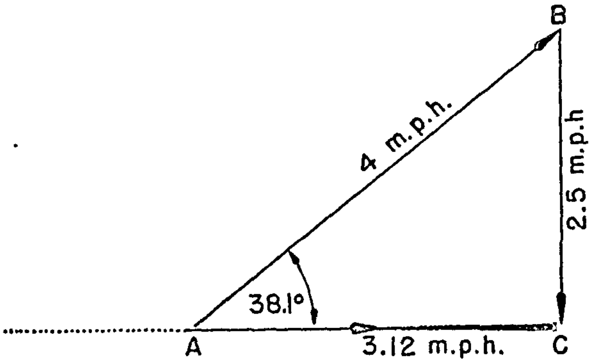


FIG. 19.—AB Steered course, 4 m.p.h. BC current, 2.5 m.p.h. AC Actual path, 3.12 m.p.h.

5. In what direction should the man of example 4 row in order that he move directly cross stream instead of being carried down? Also, how long will it take him to cross the stream? In Figure 19 the red line shows the desired cross-stream direction. The 4 m.p.h. line must be drawn at such an angle that it will just counteract the 2.5 m.p.h. of the current. If this figure is drawn to scale, the length of the red line between the start of the 4 m.p.h. line and the end of the 2.5 m.p.h. line shows us that the actual velocity cross stream is approximately 3.12 m.p.h. Hence the time required to cross the stream will be

$$t = \frac{d}{v}$$

$$t = \frac{1.5}{3.12} = .48 \text{ hours approx.}$$

$$= 28.8 \text{ min.}$$

With a protractor we can determine that the angle at which the boat is rowed with respect to the cross-stream direction is 38.1°.

6. Changing Velocities—Acceleration

In the above discussion care was taken to consider only constant velocities and average velocity, but when one drives an automobile at an average rate of 50 m.p.h. he keeps chang-

ing his actual rate from 0 m.p.h. at stop signs to perhaps 65 or 70 m.p.h. on the open road. When a velocity is changed, it is said to be accelerated. The rate at which it is changed is called acceleration.

In this chapter we will confine ourselves to cases where the acceleration is either in the direction of, or directly opposed to, the velocity. In other words, we shall be concerned with increasing or decreasing the amount of velocity, that is, speeding up or slowing down.

Since all of us are familiar with the operation of automobiles, we will again use them as examples. One of the things that we are always interested in is how fast will a car start out from a standing position? To make this question more definite, we may say, for example, how long will it take to get from a standing position up to 15 m.p.h.? Suppose that the answer were 5 seconds. We would then say that on the average we had increased our speed by 3 m.p.h. every second for 5 seconds. From this example we see that we can write a simple equation which says:

$$\text{final velocity} = \text{acceleration} \times \text{time}$$
$$v_f = at$$

where v_f is the final velocity starting from a standing position, a is the acceleration and t is the time required to get from the standing position to the final velocity. In the above example

$$15 = a \times 5$$
$$a = 3 \text{ m.p.h. per sec.}$$

Of course, we often change velocity from one value to another, without necessarily having the initial condition one of standing still. We might, for instance, be interested in the acceleration from 15 m.p.h. to 25 m.p.h. Suppose that our automobile in this case also took 5 seconds. Since the change from 15 m.p.h. to 25 m.p.h. is 10 m.p.h., we see at once that we have been changing our speed at the rate of 2 m.p.h. for each of the 5 seconds in this period. In this case we say that the acceleration was 2 m.p.h. per second,

as compared to an acceleration of 3 m.p.h. per second in the first case.

A similar example would be to consider slowing down from 60 m.p.h. to 40 m.p.h. Suppose that this change in velocity required 4 seconds. The change from 60 m.p.h. to 40 m.p.h. represents a loss in speed of 20 m.p.h. If this change requires 4 seconds, we see at once that we must have changed our speed downward by an amount of 5 m.p.h. every second for the 4 seconds. This 5 m.p.h. per second is called a negative acceleration, since it is opposed to the original velocity.

We can now define acceleration somewhat more broadly by saying that it is any change in velocity divided by the time required to make the change. In symbols we may write

$$a = \frac{v_2 - v_1}{t_2 - t_1},$$

If v_1 is 0 when t_1 is 0, we will have the simple case with which we started this section, namely, the case where we start from rest.

In the above case of the automobile we expressed all of our accelerations in terms of miles per hour per second. Of course, we can use many other combinations of units in which to express acceleration. Fundamentally, acceleration must have the unit of velocity divided by time, for it is a change in velocity divided by the time required to make the change. In the automobile cases, the unit of distance in determining the velocity was the mile, and the unit of time was the hour, hence miles per hour for velocity. However, the unit of time for determining the change in velocity was the second. Therefore we got the hybrid unit of miles per hour per second.

Another commonly used unit for velocity is feet per second. In this case, acceleration would be expressed in terms of feet per second per second. This is the most common set of units used in the English system. In the metric system, the corresponding set of units would be centimeters per second per second.

The problem of changing velocity from terms of miles per hour to feet per second occurs frequently both in simple velocity problems and in acceleration problems. If one substitutes the feet in one mile and the number of seconds in one hour in the miles per hour relationship, he will discover that a velocity of 1 m.p.h. is approximately 1.5 feet per second. If the interested student carries out the actual arithmetic for this case, he will find that the correct value is more nearly 1.47 feet per second. In most cases, however, the approximate value of 1.5 is sufficiently close for practical purposes, and this approximation will be used throughout this book.

7. Distances Covered While Velocities Are Changing

In the early part of this chapter we computed the distances covered very readily if we knew the average velocity and the time involved. In the problems which we studied we assumed that the velocity stayed at constant values. In such cases the actual velocity and the average velocity were equal to one another.

If a body is experiencing an acceleration, as described in section 6, we can still compute the distance covered in a given length of time if we can determine the average velocity. If the velocity changes at a steady rate, it is fairly easy to compute the average value. For example, suppose that we consider the case of the automobile starting from rest and changing to a speed of 15 m.p.h. in 5 seconds. Since the rate of change was steady, we can say that the average velocity is equal to the initial velocity v_0 plus the final velocity v_f divided by 2. In symbols we may write

$$\bar{v} = \frac{v_0 + v_f}{2}$$

In numbers this becomes

$$\bar{v} = \frac{0 + 15}{2} = 7.5 \text{ m.p.h.}$$

Since the distance covered in this time is small, and the time involved is small, it would be well to express this velocity in

terms of feet per second. If we use the approximation given above, we see that the average velocity is

$$\bar{v} = 7.5 \times 1.5 = 11.2 \text{ ft. per sec. approx.}$$

Since we now know the average velocity, it is easy to compute the distance covered, for we also know the time, which in this case was 5 seconds. Therefore we may write,

$$\text{distance} = \text{average velocity} \times \text{time}$$

Or in symbols $d = \bar{v} \times t$

Or in numbers $d = 11.2 \times 5 = 56.0$ ft.

Similarly, for the case where the automobile changed from 15 m.p.h. to 25 m.p.h. in 5 seconds, we may write

$$\bar{v} = \frac{15 + 25}{2} = 20 \text{ m.p.h.}$$

We may now convert this value to the velocity in terms of feet per second by writing

$$\bar{v} = 20 \times 1.5 = 30 \text{ ft. per sec.}$$

We may now easily find the distance covered by writing

$$d = \bar{v}t$$

Or in numbers $d = 20 \times 5 = 100$ ft.

Similarly, for the case of slowing down from 60 m.p.h. to 40 m.p.h., we may easily find that the average velocity was 50 m.p.h. Expressed in terms of feet per second, we find the approximate value to be 75 ft. per sec. Since we know that the time involved was 4 seconds, we may write

$$\text{distance} = \bar{v} \times 4$$

Or in numbers $d = 75 \times 4 = 300$ ft.

Evidently, simple problems involving velocity or acceleration or distances covered involve only the use of good common sense.

8. Falling Bodies

A familiar example of the case of accelerated motion is the behavior of a freely falling body. In this case, the gravita-

tional attraction between the body and the earth results in giving the body a constant downward acceleration. The value of this acceleration at or near the surface of the earth is approximately 32.2 feet per second per second, or 980 centimeters per second per second.

Example 1. Suppose we consider the case of an object dropping out of a window in the upper story of a building. It starts downward from a rest position, and at the end of one second it will have a velocity of 32.2 feet per second. We see at once that the average velocity was 16.1 feet per second. We can then write for the distance covered in this one second

$$d = \bar{v}t$$

Or in numbers $\quad d = 16.1 \times 1 = 16.1$ feet

Example 2. In the next second the velocity would pick up another 32.2 feet per second so that the velocity at the end of the second second would be 64.4 feet per second. The average velocity during this second second would be found by writing

$$\bar{v} = \frac{64.4 + 32.2}{2}$$
$$= 48.3 \text{ ft. per sec.}$$

The distance covered in this second second would be

$$d = \bar{v}t$$
$$d = 48.3 \times 1 = 48.3 \text{ ft.}$$

Example 3. At the end of 5 seconds from the time when the object was first dropped we could write

$$\text{final velocity} = \text{acceleration} \times \text{time}$$

Or $\quad v_f = at$

Or $\quad v_f = 32.2 \times 5 = 161.0$ ft. per sec.

For the average velocity we would write

$$\bar{v} = \frac{0 + 161.0}{2} = 80.5 \text{ ft. per sec.}$$

For the distance that the object had fallen we would write

$$d = \bar{v}t$$

Or

$$d = 80.5 \times 5 = 402.5 \text{ ft.}$$

Example 4. Sometimes the problem of a falling body is given in a somewhat different manner. Suppose, for example, that we know the distance between the window from which the object is dropped and the pavement below. We might then be required to find either the time that the object would take in falling or the velocity with which it would hit the pavement. Suppose, for example, that the distance involved is 100 ft. For the final velocity we may write

$$v_f = at = 32.2 \times t \text{ ft. per sec.} \tag{1}$$

when t is the time of descent. Since the object started from rest, we can write that the average velocity is equal to

$$\bar{v} = \frac{v_f}{2} = \frac{32.2 \times t}{2} = 16.1 \times t \text{ ft. per sec.} \tag{2}$$

Of course we already know that we may write

$$d = \bar{v}t \tag{3}$$

If we substitute the value for $\bar{v}$ from (3) in equation (2) we may write

$$d = 16.1 \times t \times t$$
$$d = (16.1)t^2 \tag{4}$$

We may put the value for distance in (4), in which case we write

$$100 = 16.1t^2$$

If we solve this equation for t we get

$$t^2 = \frac{100}{16.1}$$

Or

$$t = \sqrt{\frac{100}{16.1}} = 2.49 \text{ sec.}$$

We may now take this value of time t back into equation (1) for final velocity. In this case we have

$$v_f = at$$
$$v_f = 32.2 \times 2.49 = 80.2 \text{ ft. per sec. approx.}$$

Example 5. A slightly more difficult problem arises if a ball is thrown upward, since then the acceleration must first oppose the initial motion of the ball. Let us first consider the case of a ball thrown straight upward with a velocity of 50 feet per second. We then ask ourselves, how high will it rise?

It will rise until the downward velocity contributed by the gravitational acceleration equals the original upward velocity. This will occur in the period of time t, such that we may write

$$v = at \tag{1}$$

Or

$$50 = 32.2t$$

From this equation we see at once that

$$t = \frac{50}{32.2} = 1.55 \text{ sec.}$$

Since the velocity changed steadily from 50 feet per second to 0, at the top of the path, we see that the average velocity was 25 feet per second. Hence the height, d, obtained must have been

$$d = \bar{v}t \tag{2}$$
$$= 25 \times 1.55 = 38.8 \text{ ft.}$$

In this particular case we could have found the height, d, without determining the time, since we could have eliminated t between equations 1 and 2. The answer, of course, would have been the same as the one given here.

9. Effect of Air Resistance on Velocities

In the above example of falling objects we completely neglected the effect of the resistance of air on an object, as it moves through this common medium. The effect of this resistance is to slow down the object. The effect is more noticeable if the object is moving at a high velocity than at a

low one. Consequently, in the first second of fall an object probably would come close to attaining the estimated 32.2 feet per second velocity. However, as the velocity becomes higher the effect of the air resistance becomes greater.

This effect can be observed in spectacular form if we watch a man jump from an airplane at a great height. For perhaps as much as 1000 feet he gains downward velocity, but then he appears to come on down at about constant velocity. In other words, he arrives at a speed where the resistance encountered in the air is equal to the gravitational pull on him and then the velocity does not increase any further. This condition is known as terminal velocity. Of course, terminal velocity for a compact object like a person's body is very high.

However, if we can increase the effective area as, for example, by the use of a parachute, then terminal velocity is relatively low. In fact, it is low enough so that the man may be able to drop to the ground without hitting hard enough to do himself any serious damage.

10. Formulae for the Behavior of Objects in Motion

The use of formulae for the solution of all the examples given in this chapter has been purposely avoided. The simple aspects of velocities, accelerations, and distance covered are items familiar to all of us through experience, and most of the practical problems with which we meet can be thought through and answers obtained by the use of the same kind of good common sense that we apply to any other activity of life.

It is, of course, possible to work all of the types of problems given in this chapter using letters for quantities and developing algebraic equations for each type of problem. If one has many problems of a given type to do, the use of such equations is very time saving. On the other hand, it is possible to memorize such equations without thinking them through, insert numbers for letters and get answers without necessarily understanding the problem or the method of solving it. Formulae learned in this manner are also soon forgotten; whereas if the student thinks through some actual problems, he may

gain enough confidence to believe that he can think through the next one that he meets. For these reasons we have carried through each problem in this chapter with numbers.

For the student who likes to approach a problem from a more general point of view the development of formulae for a few of the more common cases is given in this section. From Section 1.

$$\text{distance covered} = \text{average velocity} \times \text{time}$$
$$d = \bar{v}t \tag{1}$$

In Section 6. For a body starting from a rest position

$$\text{final velocity} = \text{acceleration} \times \text{time}$$
$$v_f = at \tag{2}$$

For a body starting with initial velocity v_0

$$\text{final velocity} = \text{initial velocity} + \text{acceleration} \times \text{time}$$
$$v_f = v_0 + at \tag{3}$$

Acceleration is the rate at which a velocity changes. It may be determined from a change in velocity divided by the time in which the change took place. If velocity v_1 changes to velocity v_2 while time changes from t_1 to t_2 we may write for acceleration a

$$a = \frac{v_2 - v_1}{t_2 - t_1} \tag{4}$$

From Section 7.

$$\text{average velocity} = \frac{\text{initial velocity} + \text{final velocity}}{2}$$
$$\bar{v} = \frac{v_0 + v_f}{2} \tag{5}$$

In case the initial velocity is zero this equation becomes

$$\bar{v} = \frac{1}{2} v_f \tag{6}$$

The distance, d, covered in the time, t, for the case of starting from a rest position is, by using the value of $\bar{v}$ from

(6) in equation (1) and with the aid of equation (2)

$$d = \frac{1}{2} at^2 \tag{7}$$

and for the case of starting with initial velocity, we take the value of v from (5) and with the aid of equation (3) we obtain from equation (1)

$$d = v_0 t + \frac{1}{2} at^2 \tag{8}$$

For the case of a body starting from rest we can eliminate the time, t, from equations (7) and (2) to obtain an expression for the final velocity in terms of acceleration and distance.

$$v_f^2 = 2ad \tag{9}$$

This expression is often useful, especially in the case of falling bodies (see section 8).

Some Important Facts

1. Motion is the changing of position of a body with respect to its surroundings. Linear distance per unit of time is called speed, and speed in a definite direction, velocity. Speed is therefore a scalar quantity; velocity, a vector quantity.

2. When several forces act on a body to move it, they impart independent velocities. The combined effect of these several velocities may be found by vector addition. A convenient way to perform vector addition is by the graphic method, but greater accuracy may be secured by the use of trigonometry.

3. Distance covered is the product of average velocity and time. Distances, like velocities, may be handled by vector addition.

4. The rate at which a velocity changes is called acceleration.

5. In uniformly accelerated motion, the average velocity is one half the sum of the initial and final velocities.

6. Freely falling bodies are examples of uniformly accelerated motion. In such cases the acceleration due to gravity is called "g." The English value of g is about 32.2 feet per second per second; the metric value, about 980 centimeters per second per second.

Generalization

Much of our lives is concerned with the changing of objects from one location to another. We are interested in the rate at which these changes take place and the extent of these changes in position.

Questions and Problems

Group A

1. Distinguish between the terms "speed" and "velocity."

2. Define acceleration. Why does any expression of acceleration involve a double use of the time unit?

3. What is uniformly accelerated motion? Mention a few examples.

4. Distinguish positive from negative acceleration. Give examples.

5. A train travels at the rate of 30 miles per hour for ½ hour, 45 m.p.h. for ¼ hour, 60 m.p.h. for 2 hours, and 25 m.p.h. for ½ hour. Find the total distance traveled. Find the average velocity for the entire trip. 158.75 miles. 48.8 m.p.h.

6. On a 60 mile trip an automobile travels 30 m.p.h. for half of the distance. At what rate will the next part have to be traveled if an average of 40 m.p.h. for the whole trip must be made? 60 m.p.h.

7. A swimmer can average 2 m.p.h. in still water. He swims downstream in a current that is 1 m.p.h. for a distance of 5 miles and then swims back against the current. Find the time going down and also the time coming back to the starting point. 1 hr. 40 min. 5 hr.

8. If a body is uniformly accelerated from rest, how is the velocity attained at any instant related to the average velocity up to that instant? Why?

9. For uniformly accelerated motion of a body starting from rest, express the distance covered in terms of: (*a*) average velocity and time; (*b*) acceleration and time.

10. What is the meaning of the symbol "g"? What is its value in English units?

11. Assume that a car can pick up or accelerate at the rate of 5 miles per hour per second. How many seconds are required to attain a speed of 30 miles per hour? 6 sec.

12. A stone dropped from a bridge span is seen to hit the water just 3 seconds after being released. What is the vertical distance from the bridge to the water? 144.0 ft.

13. Why does the value of g vary with geographical location?

14. The brakes of a car can decelerate it at the rate of 15 feet per second per second. If the car is traveling at a speed of 40 miles per hour, how soon can it be stopped? In how many feet? 4 sec. 120 ft.

Group B

1. A boy can row a boat at a speed of 4 miles per hour in still water. He actually rows in a direction crosswise to a current of 3 miles per hour. At what speed and in what direction will he actually travel? 5 m.p.h.

2. A cross wind to the east of 50 m.p.h. carries an airplane that makes an airspeed of 125 miles per hour off its course. If the pilot was flying due

north by compass, find his location with respect to his starting point after 2 hours of flying. 269.2 mi.

3. In what direction should the pilot of problem 2 have flown to keep on his course? 23.5° W. of N.

4. A small private plane has gas for 6 hours at its normal cruising speed of 80 miles per hour. How far can it fly and be able to return to its home field:

(*a*) With no wind? 240 miles.
(*b*) With a 30 m.p.h. tail wind? 206 miles.
(*c*) With a 30 m.p.h. head wind? 206 miles.
(*d*) Directly across a 48 m.p.h. wind? 192 miles.

5. A man in an automobile increases his velocity uniformly from zero to 45 feet per second in 17 seconds. Find his acceleration and the distance traveled during this interval. 2.65 f.p.s.p.s. 382.5 ft.

6. A man increases the velocity of a car uniformly from zero to 60 m.p.h. while traveling a distance of 30 miles. (*a*) Find the average velocity. (*b*) Find the time required to cover the 30 miles. (*c*) Find his acceleration. (*d*) Find his location at the end of half the time. (*e*) At what time did he pass the 15 miles point? (*f*) The 20 miles point?

30 m.p.h. 1 hr. 60 m.p.h.p.h. 7.5 mi. 42.4 min. 49 min.

7. A man jumps off the bridge at Niagara Falls. If the bridge is 200 feet above the water, how long will the man have to think over his decision before he hits the water? (Use the approximate value for gravity, $g = 32$, for problems in this list.) 3.5 sec.

8. A man 150 feet above the ground drops a ball. A second ball he throws downward with a velocity of 40 feet per second. Compare the times of descent of the two balls. 3.06 sec. 2.06 sec.

9. A man traveling at the rate of 60 m.p.h. in a car applies his brakes so that he slows down uniformly and stops in a distance of 350 feet. (*a*) Find the time required to stop. (*b*) Find his negative acceleration. (Suggestion: First reduce the velocity to feet per second, next determine the average velocity while stopping.) 7.78 sec. 11.6 f.p.s.p.s.

10. An arrow is shot straight up at a velocity of 160 feet per second. How far from the starting position will it be at the end of 4 seconds? 5 seconds? 6 seconds? 10 seconds? 384 ft. 400 ft. 384 ft. 0 ft.

11. An arrow is shot upward with a velocity of 320 feet per second. How high does it rise? How many seconds does it rise? With what velocity does it reach the ground? How many seconds is it in the air?

1600 ft. 10 sec. 320 ft. per sec. 20 sec.

12. Suppose that an automobile traveling at 64 m.p.h. hits a rigid obstruction such as a large tree. From what height would the car have to drop as a freely falling body to hit a pavement with the same force?

144 ft.

Experimental Problems

1. Time the fall of a stone from a high bridge or cliff and calculate the distance of fall by the relationship, $d = \frac{1}{2} gt^2$. Check your calculation by comparison with other determinations of the same distance.

2. If friction is neglected it may be assumed that acceleration of an object on an inclined plane varies directly as the component of gravity parallel to the plane. The force on the object parallel to the plane then is to the weight of the object as the height of the plane is to its length.

With the aid of a stop watch time an object as it slides the length of the plane and from these data compute the acceleration. Repeat this experiment for several angles of inclination and check the relationship stated in the first paragraph.

Chapter 5

FORCE—WORK—FRICTION—POWER

In this chapter we begin a study of the nature of force and work. We are interested in the kind of things they are and also in methods for measuring them.

In many of the daily activities of life we encounter friction, and much energy is wasted through this medium. Consequently a brief study of frictional forces and the energy loss through friction is given here.

Not only are we interested in work but also we are interested in the rate at which work is done. The rate of doing work is called power, and the subject is introduced in this chapter.

1. Force

Some understanding of force is one of the first impressions gained by a small child. He is familiar with the sensation of pushing or pulling and also with the effects that may be produced on some object that is more or less free to move. Such a feeling for the meaning of force is so common that the term was used in the preceding chapter without any explanation.

In a later chapter (Chapter 11) we will delve further into the nature of force, but in the present chapter we shall depend largely on the simple understanding that everyone has of it.

It is possible to choose quite arbitrary units for measuring force, and such units are now in general use for most calculations.

The force with which the earth and a one pound mass attract each other (in other words, the weight of a one pound mass) is the practical unit of force in the English system. Hence when we say "one pound" we may mean a mass or we may be referring to a force. It is frequently advisable to specify which is meant; for example, "a mass of ten pounds," or "a force of ten pounds," or if the force is due to gravity you might say "ten pounds weight." The word "pound" by itself is a unit of either mass or force.

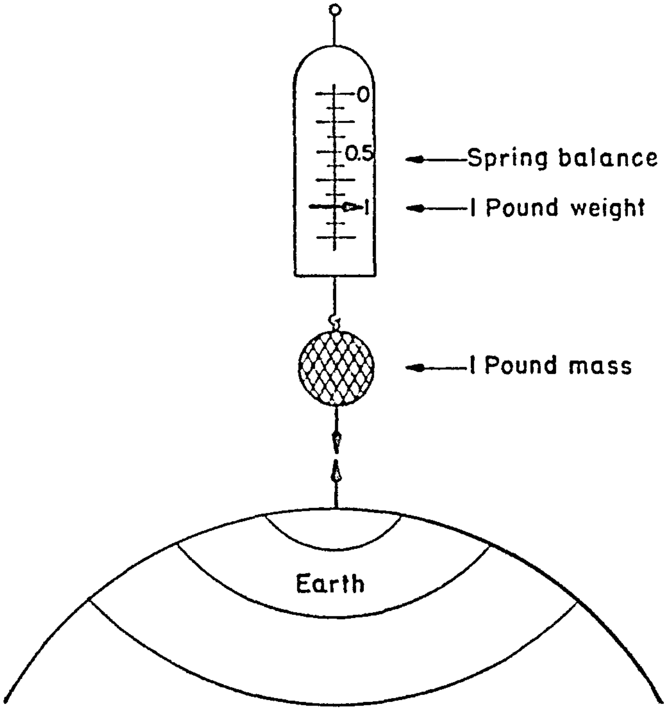


FIG. 20.—One pound of mass is attracted by the earth with a force of one pound weight.

The corresponding units of force in the metric system are the weight of one gram of mass or the weight of one kilogram of mass.

The pound and gram (or kilogram) when used as units for measuring force are often called *gravitational* units since they are the weights of 1 lb. or 1 g. or 1 kilogram of mass. In Chapter 11 we shall find other units of force in both the English and metric systems. These new units will be called absolute units.

2. Work

Most of us have a general understanding of the idea of work just as we have for force. The feeling of spending effort in lifting a suitcase, pushing a chair or table, or carrying one's

self up a flight of stairs is common to everyone. These impressions are quite subjective and we at once recognize that no two people would agree as to the amount of energy or effort they used to do any one of these things.

It seems wise to consider only what is accomplished and not how hard someone thinks he has worked. The whole idea of work can be made objective rather than subjective by defining work or energy as the product of a force by the distance through which the point of application of the force moves in the direction of the force.

This statement can be expressed in the equation

$$\text{Work} = \text{Force} \times \text{distance}$$
$$W = Fd$$

where W is the work done by the force F in moving its point of application through the distance d, where d is measured in the same direction as that in which the force is directed.

An examination of this definition shows that many forces exist that do not do work. For example: as I write this chapter I am pressing on a chair with a force equal to my weight. But the chair is standing up under the strain and consequently this force is doing no work. If the chair should crumble under me, my weight would do some work in crushing it.

Suppose, further, that a table stands close to a wall. To keep the problem simple, let us suppose that the table can slide along the wall without friction. If I push parallel to the wall I may be able to move the table along and so do work

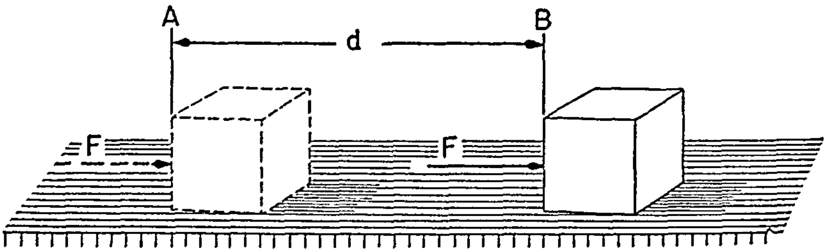


FIG. 21.—When the force F causes the box to slide through the distance d, work is done equal to F × d.

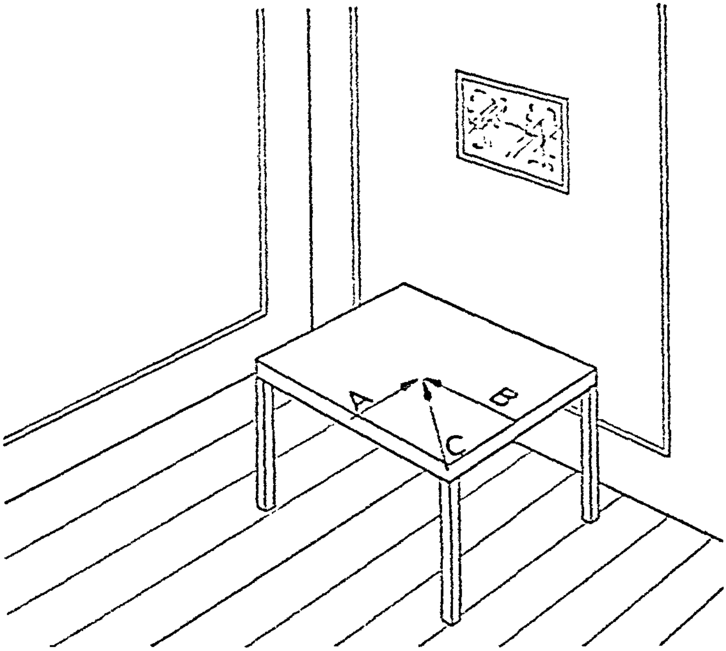


Fig. 22.—The force A does not move the table. All of the force B is available to move it. Only the component of C parallel to the wall is useful for moving the table.

against the friction between the table and the floor. On the other hand, if I push sidewise against the table, I merely press it harder against the wall, the table does not move, and I do not accomplish any work in the meaning of the above definition, in spite of the fact that I exert a force.

Let us now consider the intermediate case in the above example: that is, suppose that I push one corner in a direction pointed along the diagonal to the top of the table. In order to move the table I will have to push harder than in the case where the push was on the end of the table parallel to the wall, but the useful work accomplished will be no greater than in the former case, provided I push the table the same distance.

In the present chapter we shall confine our attention to cases where the motion is in the direction of the applied force, and the more involved cases such as the diagonal push on the table will be discussed in a later chapter.

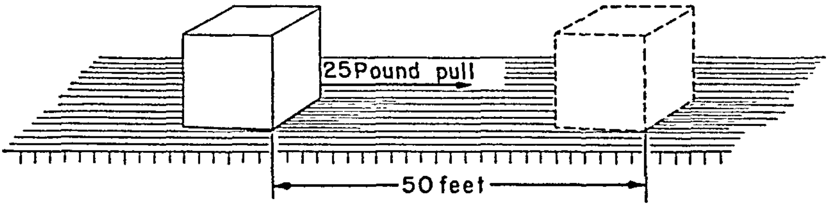


Fig. 23.—If a force of 25 lb. causes the motion of an object through 50 ft., the work done is $W = 25 \times 50 = 1250$ ft. lb.

3. Units of Work

We are now in a position to decide on units of work. It is neither necessary nor advisable to be entirely arbitrary in this choice for we see from the above discussion that work is to be expressed in terms of force and distance. So when we have settled on a unit of force and one of distance, a unit of work is automatically fixed.

If force is in pounds and distance in feet we get work in foot pounds. This is one of the more commonly used units in the English system, although it must be obvious that one could also speak of work in mile tons for large quantities or in inch ounces for small quantities if he wanted to be somewhat unconventional.

In the metric system we find the gram centimeter as a unit of work. The units kilogram centimeter and kilogram meter could also be employed.

If force is expressed in terms of the absolute units described in Chapter 11, other units of work will be obtained. These will be considered in later chapters.

4. Friction

Two of the more common reasons for using energy have been given in examples above: one is lifting objects, the other, moving them against friction.

It is well known that the difficulty encountered in sliding one object over another varies tremendously with the materials. For example: A block may be pulled very easily over a smooth tile floor but is moved with difficulty over rough concrete. Some measure of friction may be obtained by

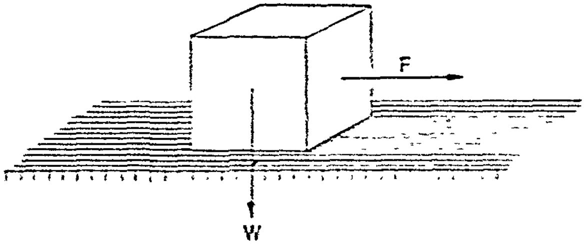


FIG. 24.—The block is pressed against the surface by a force equal to its weight, W. If the force F can just move the box, the coefficient of friction between the box and surface is $\frac{F}{W}$.

defining it as the ratio of two forces. The first of these is the force parallel to the surface required to move the object at a steady rate. The second force is that with which the object is pressed against the surface. If the surface is horizontal this latter force is the weight of the object. Hence, for such simple cases we may write

$$\text{Coefficient of Friction} = \frac{\text{Force}}{\text{Weight}}$$

$$n = \frac{F}{W}.$$

Example. A horizontal force of 100 lb. will cause a 550 lb. block to slide over a level floor. Find the coefficient of friction between the block and the floor.

$$n = \frac{100}{550} = 0.182$$

Example. The coefficient of friction between a 700 lb. sledge and snow is approximately 0.085. How much work will be done in pulling the sledge over a mile of level trail?

First we find the force required.

Since

$$n = \frac{F}{W}, \qquad F = nW$$

Hence

$$F = 0.085 \times 700 = 59.5 \text{ lb.}$$

The work done will be

$$\text{Work} = Fd = 59.5 \times 5280 \text{ (since there are 5280 ft. in a mile)}$$
$$= 314{,}000 \text{ ft. lb. (approximately)}$$

If one carries along all of the numbers in the above calculation he obtains 314,160 ft. lb. But to write this number as the exact answer would be to indicate that we know the exact weight of the sledge and the exact coefficient of friction much more accurately than the number of digits given in the problem. It is a more honest use of the numbers to round off the answer as indicated.

5. The Rate at Which Work Is Done—Power

Not only are we interested in the total amount of work done but we are also interested in the rate at which this work is done. The rate of doing work is called power.

Suppose that we consider a water tank at 100 ft. height above the source of water. Before we can decide how powerful a motor will be needed to pump water from the reservoir into this tank it will be necessary for us to decide how much water per hour or per minute or per second is required to be pumped. Obviously if we need a thousand gallons per hour instead of five hundred gallons per hour we will need twice as powerful a motor in the one case as in the other.

6. The Units in Which Power Is Measured

Since power is the rate at which work is done it can be expressed in the simple relation.

$$\text{Power} = \frac{\text{Work}}{\text{Time}}$$

$$P = \frac{W}{T}$$

We already have units of work in both the English and the metric systems, and consequently the unit of power is determined automatically as soon as we select the energy unit that is desired and the time unit. For making calculations in mechanics it is quite common to use the second as the time

unit in computing power. However, other units of time may also be used.

If we choose the foot pound as a unit of work and the second as a unit of time, the unit of power becomes the foot pound per second. This rate of work corresponds to raising a weight of one pound one foot each second. Five hundred fifty foot pounds per second is called one horse power. This unit of power does not represent the maximum rate at which a horse is capable of working for a short length of time. It is based roughly on the rate of work that can be obtained on the average from horses over full day periods.

For short periods of time a man can develop several tenths of a horse power. If a person has a good heart and is full of ambition he can determine his own maximum rate of working by timing, with the aid of a stop watch, the length of time it takes him to run up several flights of steps. The work which he does will be his own weight multiplied by the height of all the steps. If he divides this amount of work by the time in seconds and then by 550 he will have a measure of the horse power which he can develop.

Example 1. Suppose that a man can run up four flights of stairs of an average height of 12 ft. each in 20 seconds, and suppose further that the man weighs 150 lb. Then the work will be

$$\text{Work} = \text{Force} \times \text{distance}$$

$$W = 150 \times 4 \times 12 = 7200 \text{ foot pounds}$$

$$\frac{\text{Work}}{\text{Time}} = \text{Power}$$

$$\frac{7200}{20} = 360 \text{ foot pounds per second}$$

$$= \frac{360}{550} = .65 \text{ horse power approximately}$$

Example 2. Suppose that an elevator is designed to lift a load of 1000 lb. ten floors of a building averaging 12 ft. per floor in 15 seconds. Find the useful work done per second in terms of horsepower.

$$\begin{aligned}
\text{Work} &= \text{Force} \times \text{distance} \\
&= 1000 \times 12 \times 10 \\
&= 120{,}000 \text{ ft. lb.} \\
\text{Power} &= \frac{\text{Work}}{\text{Time}} \\
&= \frac{120{,}000}{15} \\
&= 8000 \text{ ft. lb. per sec.} \\
&\equiv \frac{8000}{550} = 14.5 \text{ horsepower approx.}
\end{aligned}$$

Example 3. An elevator car with its load weighs 2,500 lb. Find the maximum rate of travel upward if 20 horsepower is available for the work.

$$\begin{aligned}
\text{Power} &= \frac{\text{Work}}{\text{Time}} \\
&= \frac{\text{Force} \times \text{distance}}{\text{Time}}
\end{aligned}$$

Since 20 horsepower is equivalent to 11,000 foot pounds per second we may write

$$11{,}000 = \frac{2500 \times \text{distance}}{\text{Time}}$$

If time is one second this equation gives

$$\text{Distance} = \frac{11{,}000}{2500} = 4.4 \text{ ft.}$$

Hence the upward velocity is 4.4 ft. per second.

Some Important Facts

1. From early experiences we interpret a force as a push or a pull. A unit of force in common use in the English system is the pound; in the metric system, the gram.

2. When a force succeeds in moving matter, work is done. The amount of work depends on the size of the force and the distance through which the force moves and so may be measured in foot-pounds or gram-centimeters, etc.

$$W = F \times d$$

3. Energy, that is work-doing ability, is sometimes used in lifting objects against gravity and it is also often used in overcoming friction.

4. The coefficient of friction between two surfaces is the ratio of the force necessary to move one on the other to the force pushing them together, commonly expressed as

$$n = \frac{F}{W}$$

5. The rate at which work is done is called power. In symbols

$$P = \frac{W}{T}$$

Generalization

Force and work are always involved in moving an object, and in many cases part or all of the work is dissipated as a result of friction.

Problems

Group A

1. A horizontal force of 50 lb. will just keep an automobile moving over a level road. Find the work done in moving the car ⅛ mile.
33,000 ft. lb.

2. A man carries a hod full of brick to the floor of a building which is 50 feet above his starting point. Find the work done if the hod and brick together weigh 125 lb. 6250 ft. lb.

3. A horizontal force of 32 lb. will cause a box to slide over the floor. If the box and its contents weigh 175 lb., find the coefficient of friction between box and floor. .183.

4. Find the horizontal force required to slide the box of problem 3 if a 75 lb. child sits on top of it. 46 lb. approx.

5. Discuss the variance between physiological sensation and the doing of work in the case of a man who carries a bag of grain for 100 feet over a level plain.

6. A man carries 100 lb. of cement from the ground to the roof of a building that is 60 ft. high. (*a*) How much work does he do on the cement? (*b*) At what rate does he do the work if the time required to carry the cement is 40 seconds? 6,000 ft. lb. 0.273 H.P.

7. The man in problem 6 must carry his own weight in addition to that of the cement. If he weighs 150 lb., find the total power he develops in carrying himself and the cement to the top of the building. 0.682 H.P.

Group B

1. An elevator on a construction job hauls 500 lb. of concrete between floors that are 142 feet apart. It makes an average of 15 trips per hour. Find the total amount of work done in an eight-hour day.
8,520,000 ft. lb.

2. The engine of a train applies a tension of 750 lb. in its coupling to the train as it travels at a speed of 30 m.p.h. Find the total work that the engine does on the train in one hour. In one minute. In one second.
118,800,000 ft. lb./hr. 1,980,000 ft. lb./min. 33,000 ft. lb./sec.

3. The coefficient of friction between a sled and ice is known to be approximately 0.085. A spring scale in a rope indicates a force of 8 lb. when a horizontal pull on the rope just keeps the sled moving on a level surface. Find the weight of the sled. Find the work done in moving the sled a distance of 150 ft. 94 lb. 1200 ft. lb.

4. A box is placed on the sled of problem 3 and the force pulling the rope has to be increased until the scales read 17 lb. How much does the box weigh? 106 lb.

5. The coefficient of friction between iron and a concrete floor is approximately 0.30. How large a horizontal force will be required to slide a block of iron over such a surface if the block is $1 \times 1.5 \times 2$ ft. (See density of iron in Chapter 3.) 441 lb.

6. How much work is done on the block of problem 5 if it is moved over 15 feet of floor and then raised by a hoist to a floor 20 feet above its starting point? $6{,}615 + 29{,}400 = 36{,}015$ ft. lb.

7. A tractor pulls a gang of plows at the rate of 12 ft. per second. If the tension in the coupling between the tractor and plows is 800 lb. find the useful power developed. 17.5 H.P.

8. A tension of 5000 grams is applied through a rope to drag a box over a floor. Find the useful work done and the power if the box is moved 3000 cm. in 20 seconds. 15,000,000 g. cm. 750,000 g. cm. per sec.

9. An engine applies a tension of 750 kilograms to the coupling to a train as it pulls it at the rate of 15 meters per sec. Find the power in kilogram-meters per second. 11,250 kg. meters per sec.

10. A tug boat tows a large boat by means of a cable at the rate of 6 m.p.h. If the tug boat has 75 horsepower available to move the big boat, what is the tension in the cable? (Suggestion: First reduce the velocity to feet per second. Then reduce the power to foot pounds per second and equate to the work done on the big boat in one second by the unknown tension.) 4,583 lb.

11. A road up a hill rises 1 foot in every 8 ft. of road. If a 2600 lb. car goes up the hill at the rate of 36 ft. per sec. what horsepower is used in lifting the car? 21.3 H.P.

12. A 3300 lb. car has 30 horsepower available for hill climbing. How fast can it go up a hill that rises 1 ft. in every 12 ft.? (Suggestion: First find the energy available in one second in terms of foot pounds. Then find the vertical height through which this amount of energy can raise the car.) 40 m.p.h.

Experimental Problems

1. Walk leisurely up one or more flights of stairs of known or measurable vertical elevation. Compute the physical work done in elevating your own weight.

Then run briskly up the same course and compute the physical work done. Which seemed to be the harder work? Did you work in walking back down?

2. Attach the hook of a heavy spring balance—one reading up to about 25 lb.—to one end of a wooden packing case of moderate size so that the case can be pulled uniformly along the floor by means of the horizontal balance. Read the balance while so used and divide this reading by the actual weight of the box to find the coefficient of friction.

Repeat this procedure with about 25 lb. added weight in the box, then with 50 lb., etc.

Also repeat all trials with the box on its side, if a side area differs much from the bottom area.

What effect has the change of weight on the coefficient of friction? The change of area? How could you change the force of friction very noticeably?

Chapter 6

MORE ABOUT FORCES

In the previous chapter we learned about forces and their ability to do work. Under these conditions the forces always move.

There are many cases where objects stand still under the influence of forces. In other words, two or more forces may act on an object in such a manner as to balance each other and hence to leave the object unmoved.

In this chapter we learn how to add forces in such a manner *as to find the resultant effect of several forces acting on* an object at one time.

There are also many cases where the application of a force results in the motion of an object in some direction other than that of the force with which we are most concerned. A typical example is the case of pushing along the handle of a lawn mower whereas the mower actually moves horizontally over the ground if the latter is level. Consequently we must also study how to divide a single force into components in various directions.

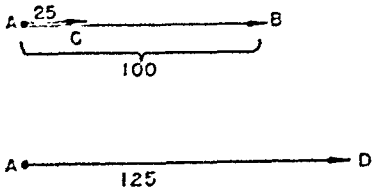
Most of the material in this chapter is presented by means of giving actual examples.

1. Addition of Forces

In Chapter 5 we began the study of forces and learned something about their ability to do work. In that chapter we considered only one force at a time.

In many cases two or more forces act at the same point on an object at one time. For example, in a simple case we might have two ropes attached to the same point on an object. For the sake of argument we may assume that a large force is pulling on one of these ropes, while a rather small force is pulling on the other one. If both ropes pull in the same direction, we can say at once that the combined effect is equal to the sum of the two. On the other hand, if one force pulls forward and the other pulls backward, then the net result is the difference between the big force and the little force.

To make this problem more pointed, we might consider a particular case where we assign values to the two forces.



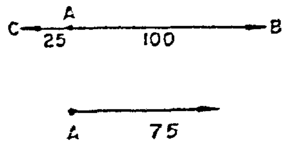


FIG. 25.—(*Left*) Two ropes are attached to a post at A. They pull in the same direction, one with a force of 100 lb. and the other with a force of 25 lb. The combined effort is a force of 125 lb.

FIG. 26.—(*Right*) The two forces of Fig. 25 pull in opposite directions here. The net result is a force of 75 lb.

Let us say that the larger of the two forces is 100 pounds, and the smaller of the two, 25 pounds. When the two pull together, we know at once that we will have a resulting force of 125 lb. Similarly, if they pull against one another, the resulting force on the object is only 75 lb.

Now we will consider the possibility that the 25 lb. force is pulling neither with nor against the 100 lb., but is pulling at some angle to the direction of the 100 lb. force, an angle

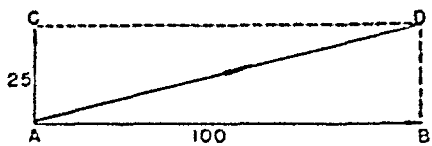


FIG. 27.—The two forces of Fig. 25 pull on A at right angles to one another. The combined effort of the two forces is shown graphically by AD.

other than 0 or 180 degrees. This condition is illustrated in Figure 27, for an angle of 90° and in Figure 28 for an angle less than 90°.

Forces are the kind of things that have both size and direction. They are therefore vector quantities, and they can be added by the methods described for vector additions in Chapter 4. So, in Figures 27 and 28, we can complete the parallelogram as is indicated by the red lines. Then the diagonal drawn from the origin to the opposite corner will

represent the combined result of the two forces, both as to the amount of the force, and as to its effective direction. This force is called the resultant of the two original forces.

In Figure 27, we use the rather simplified case of having the 25 lb. pull at right angles to the direction of the 100 lb. force. However, the same method would apply for any other angle. Such a case is illustrated in Figure 28. Here again we use the parallelogram method. The two forces are drawn from a common point and the parallelogram is completed as is indicated by the red line. Again the diagonal tells us both the size and the direction of the resultant force.

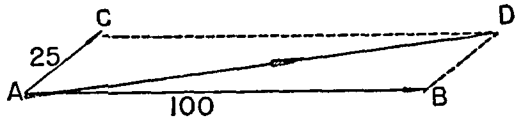


FIG. 28.—The two forces of Fig. 25 pull on A at an acute angle to one another. The diagonal, AD, of the parallelogram ABDC shows the resultant of the two forces.

Of course, just as in the velocity examples of Chapter 4, one can solve such a problem by means of trigonometry. However, the problem can also be solved simply by drawing the graph to scale and then by measuring the resultant to determine the magnitude of the force. By measuring with a protractor, one can also determine the angle that this resultant makes with either of the two original forces.

If a force equal to the resultant is attached to the two original forces in the opposite direction to that in which the resultant lies, we find that the two original forces are just balanced by this new force. This force is called the *equilibrant.*

A simple experiment can be set up to prove the truth of this discussion concerning addition of forces. Three ordinary spring balances may be hooked together as shown in Figure 29. Suppose, for example, that we work with the data shown in Figure 27. Here we have a force of 100 lb. and a force of 25 lb. at right angles to one another. This figure shows that the

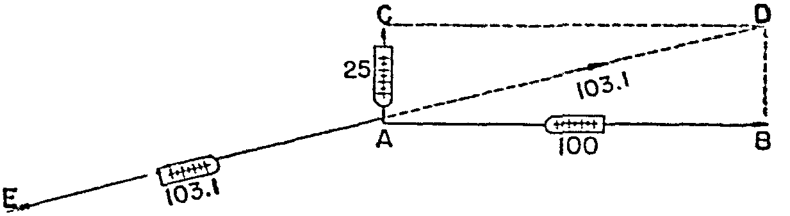


FIG. 29.—Two forces AB and AC can be balanced by an equilibrant AE which is a third force equal to the resultant of the first two but oppositely directed.

value of the resultant is 103.1 lb. approximately, and that this resultant makes a direction of approximately 14° with the 100 lb. force. The third scale is in line with the resultant of these two forces, but pointed in the opposite direction. We discover that the 100 lb. force and the 25 lb. force are just balanced by an equilibrant force of 103.1 lb. Other angles and other values of forces and their equilibrants can easily be tried with this simple apparatus, and the result can be compared with the calculated values.

There are many practical applications of the use of two or more forces which balance one another. Let us consider a 100 lb. load being pulled upward by a rope. A second rope attached to the first is pulled horizontally with a force of 50 lb. This situation is illustrated in Figure 30. It might be of interest for us to find out the value of the tension in the rope above the point where the 50 lb. force is applied. In Figure 30 we draw to scale and in the proper direction the 100 lb. force

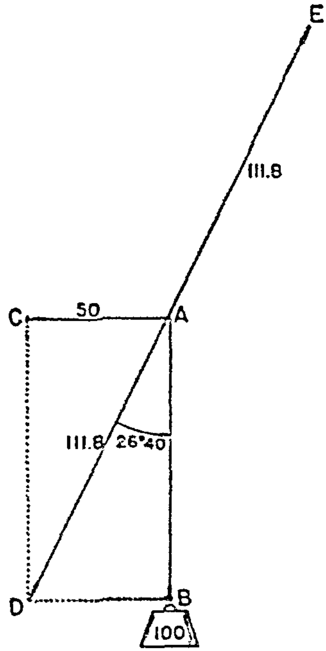


FIG. 30.—AB, Vertical force of gravity, 100 lb. AC, Horizontal force, 50 lb. Resultant, 111.8 lb., or AD. Equilibrant, tension on rope, 111.8 lb. or AE.

and the 50 lb. force. The parallelogram is completed with dotted lines, and the diagonal drawn from the initial point tells us that the value of the resultant force is approximately

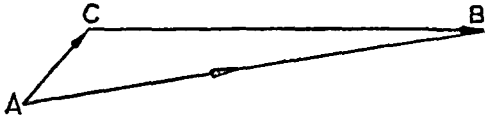


FIG. 31.—The resultant will be the line connecting the beginning of the first vector with the end of the second. This triangle corresponds to half the parallelogram of Fig. 28.

111.8 lb. This resultant force makes an angle of approximately 26°40′ with the 100 lb. force. The rope above the 50 lb. point of application must then have a tension in it equal to the resultant and opposite in direction. Hence the magnitude of this tension is 111.8 lb.

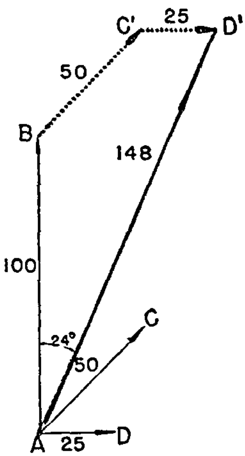


FIG. 32.—AD′, 148 lb., is the resultant of AB, 100 lb. North; AC, 50 lb. Northeast and AD, 25 lb. East.

In Chapter 4 we worked many vector problems by drawing first one vector and then the second one. The second vector is started where the first one ends, and we find the resultant by connecting the first end of the first vector with the final end of the second vector. The problems worked so far in this chapter can be solved in the same manner. Suppose, for example, that we take the problem of Figure 28 and instead of connecting the two vectors to the initial point, we connect the second one to the end of the first vector. (See Figure 31.) Then the resultant will be the line connecting the beginning of the first vector with the end of the second. We can see at once that this triangle corresponds to the upper half of the parallelogram of Figure 28.

This method of adding forces is sometimes more convenient where more than two forces are to be added. For example,

suppose that we consider three ropes attached to a common point. A pull of 100 lb. is exerted through the rope to the north. A pull of 50 lb. to the northeast, and a pull of 25 lb. to the east.

In Figure 32 we see lines representing these forces connected one to the other in the manner which we have described. If the quantities are drawn carefully to scale, we discover that the resulting force has a value of approximately 148 lb. and that it makes a direction of 24° east of north.

2. Resolution of Forces

In the first part of this chapter we were concerned with finding a single force, called a resultant, which would be equivalent to two or more other forces. The new single force could be used to replace the original forces.

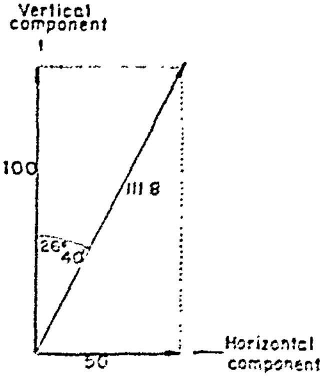


FIG. 33.—Resolution of forces. In Fig. 30 we added two initial forces to obtain the resultant. In Fig. 33, we begin with the larger force and resolve it into two components at right angles to one another.

It might occur to us that this process could be worked backwards. In other words, if we have a single force it might be possible to find two other forces which would completely replace the one force. It is customary to choose the two new forces so that they are at right angles to one another. These forces are called the components of the original force. When a single force is replaced by two other forces, usually at right angles, we say that the first force has been *resolved* into its *components*.

A simple example is found by taking the diagram of Figure 30 where two forces at right angles to one another were added vectorially by the parallelogram method. Both the resultant and the equilibrant are shown. The latter is the tension in the rope which supports the weight and also counteracts the side pull. In Figure 33 we show the equilibrant with the black

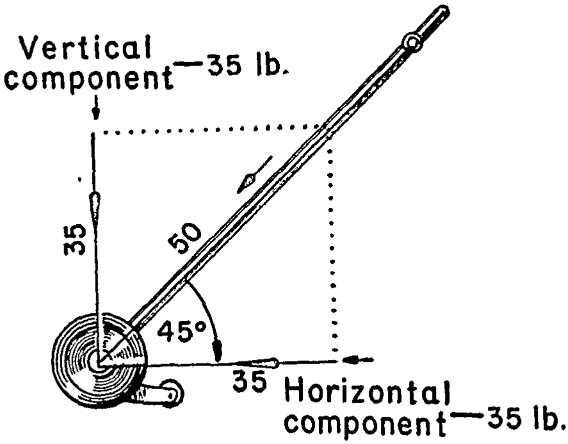


FIG. 34.—A simple application of the resolution of forces is involved in pushing a lawnmower. This is done by drawing a horizontal line from one end of the 50 lb. force line and a vertical line through the other end.

arrow, 111.8. From each end of this vector we draw horizontal and vertical lines and determine their lengths by their intersections. The value of the vertical line is found to be 100 lb. and that of the horizontal line, 50 lb. These forces are called the vertical and horizontal components, respectively, of the 111.8 lb. tension in the rope. The upward component balances the weight of 100 lb. in Figure 30 and the horizontal component balances the 50 lb. side pull of Figure 30.

Example 1. A simple application of the resolution of forces is involved in pushing a lawnmower. Suppose that the handle of the lawnmower makes an angle of 45° with the ground, as is illustrated in Figure 34. Suppose also that a person pushes along the handle in this direction with a force of 50 lb. We can now resolve this force into two forces, one perpendicular to the ground, and one parallel to the ground. This is done by drawing a horizontal line from one end of the 50 lb. force line and a vertical line through the other end. The intersection of these two lines cuts them off at the proper lengths to represent the vertical force and the horizontal force. In this particular case where the handle makes an angle of 45° with the ground, we find that the two components

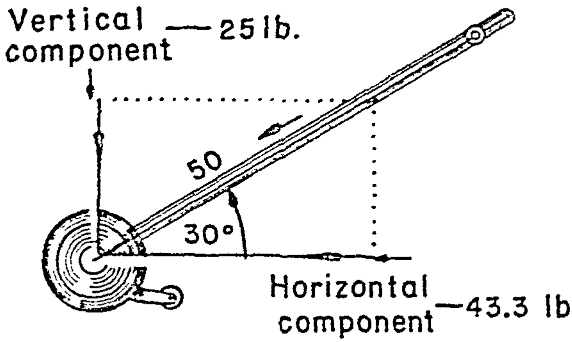


FIG. 35.—Where the handle of the lawnmower makes an angle of 30° with the ground, the lawnmower is pushed forward with a force of approximately 43.3 lb. while it is pushed against the ground with a force of 25 lb. in addition to its own weight.

are equal. The force pushing the lawnmower forward is approximately 35 lb. and so also is the force pushing the lawnmower harder against the ground.

Example 2. We may repeat this problem for the case where the handle of the lawnmower makes an angle of 30° with the ground. This condition is illustrated in Figure 35. The vertical and horizontal lines are drawn in the same manner as in the preceding case. From this graph we can easily find that the lawnmower is pushed forward with a force of approximately 43.3 lb. while it is pushed against the ground with a force of 25 lb. in addition to its own weight.

Example 3. It often happens that a railroad switch engine is used to push a car on a track parallel with the one on which

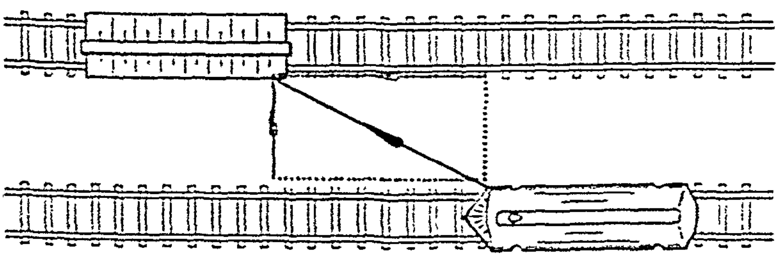


FIG. 36.—It often happens that a railroad switch engine is used to push a car on a track parallel to the one on which the engine is operating. A short wooden pole is used as shown.

the engine is operating. A short wooden pole is used, as shown in Figure 36. Suppose that the push along this pole amounts to 500 lb., and that the pole makes an angle of 30° with the track. In Figure 36 we represent this 500 lb. force at an angle of 30° to the track. A line drawn through one end parallel to the tracks is shown, and a second line through the other end of the 500 lb. line is drawn perpendicular to the first line. These two mutually right-angled lines intersect each other in a manner such that their respective lengths show the force available for pushing the car along the track

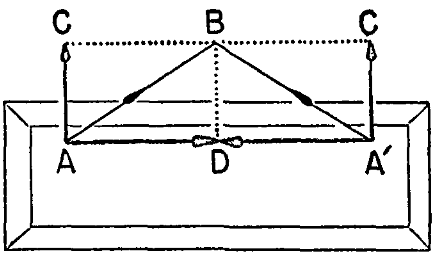


FIG. 37.—A common problem of the type discussed in this chapter is the case of hanging a picture by means of two cords, BA and BA′.

and also the force with which the car is pushed sidewise against the rail.

Example 4. A common problem of the type which we have discussed in this chapter is the case of hanging a picture or any other object by means of a rope as shown in Figure 37. In the simplest case, the rope is hung over a peg at B so that the rope sections BA and BA' are of equal length.

Let us consider the tension in the rope BA. If we choose the proper scale for drawing, we may let the length of rope BA represent the force in this section of rope. Then, as in the problem shown in Figure 33, we may draw vertical and horizontal lines through both ends of the line BA, and their mutual intersections will give us the vertical and horizontal components of the tension in BA. The vertical component AC is available to help support the weight of the picture. The

horizontal component AD is useless and will need to be counteracted by an equal and oppositely directed horizontal component in the section of the rope, BA'.

In this simple case where the two sections of rope, BA and BA', make the same size of angle with the vertical, the tensions in the two sections of rope will be equal, and their upward components will be equal, so that each will support one half the weight of the picture.

Example 5. In Figure 38 we see two beams standing with their lower ends twenty feet apart on the horizontal ground. Each beam is 15 ft. long. From the top of the two is hung a weight of 500 lb. The problem is to find out what thrust there is in each of the beams. To solve this problem we draw a vertical line whose length represents the 500 lb. From each end of this line we draw lines which are parallel one each to the two beams. These two lines intersect, and the section of the line from either end of the 500 lb. line to the intersection represents the thrust in the beam. In this case we see that the two thrusts are equal. The student should measure the

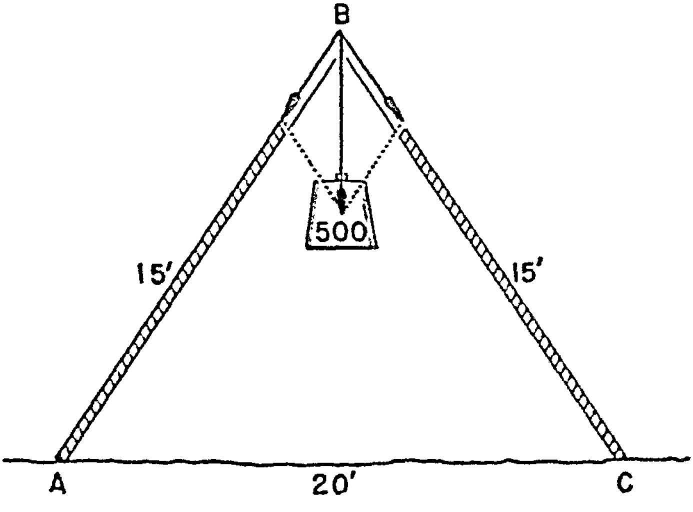


FIG. 38.—Two beams, AB and CB, stand with their lower ends 20 ft. apart on the horizontal ground, AC. Each beam is 15 ft. long. From the top of the two is hung a weight of 500 lb. The problem is to find what thrust there is in each of the beams.

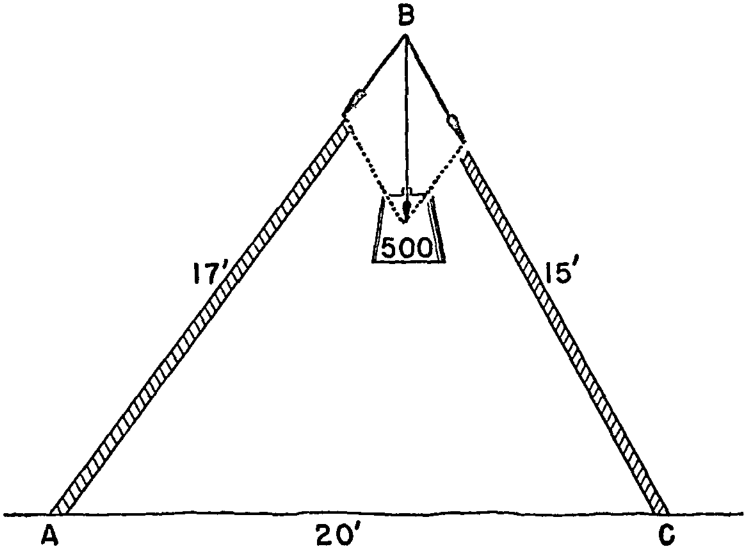


FIG. 39.—In this figure, one of the beams, AB, has a length of 17 ft., the other, CB, a length of 15 ft., thereby increasing the thrust along CB but decreasing the thrust along AB.

line in this drawing to determine the size of the thrust. In Figure 39 this problem is repeated where one of the beams has a length of 17 ft. and the other 15 ft. This problem also is solved by drawing a vertical line representing the 500 lb. and by drawing from the end of this line two lines which are respectively parallel to the two beams. Again the intersected lengths of these lines will give us the amount of the thrust in each beam.

Notice in this case that the shorter beam carries the greatest thrust.

This result you should have anticipated by just considering the problem.

Some Important Facts

1. When two forces act together in the same direction, that is at a 0° angle, the net result, or resultant force, is their sum. When they directly oppose each other, that is act at a 180° angle, the resultant is their difference. In no case can the resultant of two forces be more than their sum nor less than their difference.

2. When two forces act together at any angle other than 0° or 180°, their resultant may be found as follows:

Let two adjacent sides of a parallelogram be the vectors of the two forces, then the diagonal to their common vertex is the vector of their resultant.

3. The process of finding the resultant of two or more forces is called composition of forces. The reverse process—finding one or both forces from their resultant—is called resolution.

The parallelogram vector method is equally effective for composition or resolution of forces.

Generalizations

A single force, called the resultant, can be used to replace any number of forces acting at a common point on an object. Similarly a single force can be replaced by two or more forces acting at the same point.

Questions and Problems

Group A

1. What are concurrent forces? Give examples.
2. Define the terms "resultant" and "equilibrant."
3. Distinguish between "composition" and "resolution" of forces.
4. What is the meaning of the term "component"?
5. How do you find the resultant of two forces acting along the same straight line and in the same direction—that is, at a 0° angle with each other?
6. How do you find the resultant of two forces acting along the same straight line but in opposite directions—that is, at a 180° angle with each other?
7. If two concurrent forces act at an angle of more than 0°, how is their resultant related to their sum and difference? How is their resultant found?
8. State the Parallelogram Law. What is meant by the "graphic" method of applying it?
9. One tug is pulling a freighter north with a force of 900 pounds, and another is pulling it east with a force of 1200 lb. With what force and approximately in what direction would a single tug have to pull to produce the same effect? 1500 lb. E. N. E.
10. A guy wire makes an angle of 30° with a telegraph pole. The tension in the wire is 500 lb. What component of this 500 lb. acts horizontally to keep the pole upright? What component tends to push the pole into the ground? 250 lb.; 433 lb.
11. Why is it impossible to stretch telephone, telegraph and power line wires sufficiently tight to be horizontal?
12. Each of two ropes of a hammock makes an angle of 60° with the vertical. The tension on each rope is 125 lb. A person is sitting in the

hammock. Ignore the weight of the hammock. What does the person weigh? 125 lb.

Group B

1. A canal barge is towed from the bank of the canal by a rope which makes an angle of 30° with the direction of travel. The effective propelling force is 866 lb. What is the tension on the rope? 1000 lb.

2. A 100 lb. girl sits in a swing. A boy pulls her back so that the swing ropes make a 30° angle with the vertical. With what horizontal force does he pull? What is the tension on the swing ropes? 58 lb.; 116 lb.

3. Two ropes are attached to a common point on a pole. The ropes make an angle of 90° to one another. Find the resultant force on the pole in magnitude and direction when

a. The tension in rope 1 is 200 lb. and the tension in rope 2 is also 200 lb. 282.8 lb.

b. The tension in rope 1 is 200 lb. and the tension in rope 2 is 100 lb. 223.6 lb.

4. Repeat problem (3) when the angle between the two ropes is 30°.

a. 386 lb.
b. 291 lb.

5. Find the size and direction of a single force which will just balance two forces of value 300 lb. and 800 lb. respectively when there is an angle of 40° between these two forces. 1048 lb.

6. As a beam weighing 1000 lb. is being raised by a single rope it is pulled sideways by a force of 200 lb. Find the tension in the first rope. 1020 lb.

7. In the above problem (6) the pulley through which the first rope is being pulled is 25 ft. above the point at which the sideways pull is applied. Find the distance sideways that the 200 lb. force moves the beam. 5 ft.

8. A man weighing 150 lb. sits on a swing with ropes 50 ft. long. What horizontal force will be required to pull him 10 ft. away from the original perpendicular position in which the ropes hang? What will then be the tension in the ropes of the swing? 30.6 lb. 153 lb.

9. A man pushing a hand car leans over so that the force with which he pushes the car is at an angle of 40° to the horizontal. What fraction of his push is useful in moving the car forward? ¾ or 75% approx.

10. A picture weighing 50 lb. is supported at the center of a rope 10 ft. long whose ends are attached to pegs 8 ft. apart. Find the tension in each section of the rope. 41⅔ lb.

11. Repeat (10) when the picture is attached to the rope at a point such that one section is 4 ft. long and the other 6 ft. 45 lb. 35 lb.

12. A beam stands at an angle of 20° with the vertical. A rope from the top supports a weight of 1000 lb. Find the thrust in the beam. How could a beam be held in this position? 1064 lb.

Experimental Problems

1. By means of a strong thread attach the hooks of three spring balances to each other on a horizontal plane, so that the pull of each balance serves as the equilibrant of the other two. Slip a sheet of paper under the three threads and indicate their three directions by three straight lines. Record the three balance readings, and choosing a convenient scale, lay off the three vectors. Using each two vectors as adjacent sides, complete the three parallelograms and draw the diagonal of each. How does each diagonal compare with the vector of the corresponding equilibrant in length? In direction? What general conclusion is justified?

2. Hang a known weight, say 500 g., near the middle of a string, the ends of which are tied to the hooks of two spring balances. By moving one or both balances, vary the angle made by the two string segments from 0° to as near 180° as possible, noting the balance readings for each angle. Formulate a general conclusion.

PARALLEL FORCES—LEVERS

The tendencies of forces to produce turning motions is common to our everyday experience. The use of levers, turning the steering wheel of a car, pedaling a bicycle, cranking an automobile engine, are familiar examples.

The tendency of a force to produce rotation depends on the magnitude of the force and also on the length of the lever arm. The product of the two is called a torque or a moment of force.

If the clockwise torques about any arbitrarily chosen fulcrum are equal in magnitude to the counterclockwise torques, the object remains in equilibrium so far as motion of rotation is concerned. Many problems involving parallel forces can be solved by applying this principle.

Further application of the principle justifies the idea of center of gravity and still other extensions of the principle explain the action of the beam balance.

1. Parallel Forces

In our study of forces up to this chapter, all of the forces that acted on any one body either actually were attached to the body at a common point or behaved as if they were applied at the same point. But in many common examples of forces acting on bodies this condition is not true. For example, let us look at the case of a simple lever.

Figure 40 shows a rod or beam AB used as a lever. It is supported at C, while two forces press down at A and B respectively as shown in the diagram. Our experience tells

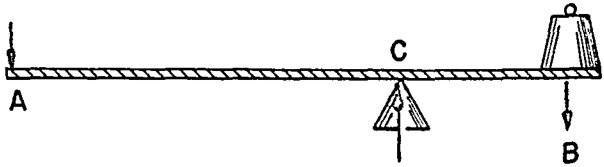


FIG. 40.—A small force at A can balance the lever against a larger force at B, since the lever arm AC is longer than the lever arm BC. The point about which the lever balances, C, is called the fulcrum. The upward thrust at C must be just equal to the combined forces at A and B.

us that a small force at A can balance the lever against a larger force at B, for in the case shown in this diagram the lever arm AC is longer than the lever arm BC. The point about which the lever balances, C, is called the fulcrum. Common sense tells us that the upward thrust of the support at C must be just equal to the combined forces at A and B.

In this case the three forces are parallel to one another. They do not meet at a point. Since the upward forces are equal to the downward forces, there is no tendency for the lever as a whole to move up or down.

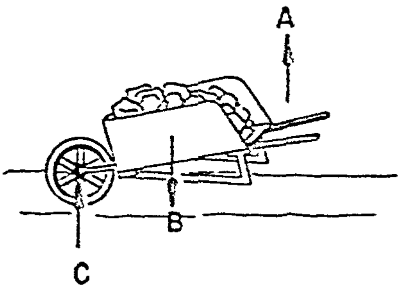


Fig. 41.—Parallel forces applied to different points on a wheelbarrow.

The force at A tries to make the lever revolve about C in a counter-clockwise direction and the force at B tries to make the lever revolve in the opposite direction. These tendencies to cause a revolving motion of the lever neutralize one another.

Figure 41 shows another familiar example of the behavior of parallel forces applied to different points on an object. In this case we have a wheelbarrow. There is an upward thrust on the handle at A and another upward thrust, this time by the road against the wheel or the wheel against the axle, at C. The downward force at B is caused by the weight of the load. The fixed point about which the upward force at A tries to produce rotation is at the axle of the wheel, C, which is then called the fulcrum. The force at B tries to produce rotation in the opposite direction around this same point.

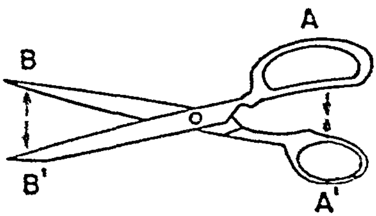


Fig. 42.—A pair of scissors is another example of parallel forces.

The combined lifts at A and C must equal the weight indicated at B.

A simple pair of scissors as shown in Figure 42 is another example of parallel forces. Here equal and opposite forces occur at corresponding points on the two halves of the scissors; for example at A and A' and also at B and B'.

2. Torques

An inspection of these problems shows that any force acting at some point other than the fulcrum tends to rotate the object. The tendency of the force to produce rotation depends on the amount of the force and on the effective length of the lever arm.

In the simple case illustrated in Figure 40 the lever arm is the distance measured along the lever from the fulcrum to the point where the force is applied. However, if the lever were placed in some direction other than the horizontal while the forces remain vertical, the effective lever arm would be decreased.

The truth of this statement will be more apparent from a study of the simple process of pedaling a bicycle. Five positions of the pedals are indicated in Figure 43. Experience tells us that the most effective position is shown in (*c*), that no tendency to produce rotation is obtained in positions (*a*) and (*e*), and that results between nothing and the maximum are obtained for positions (*b*) and (*d*).

The true lever arm for these cases is shown by h, the perpendicular distance between the fulcrum and the line of direc-

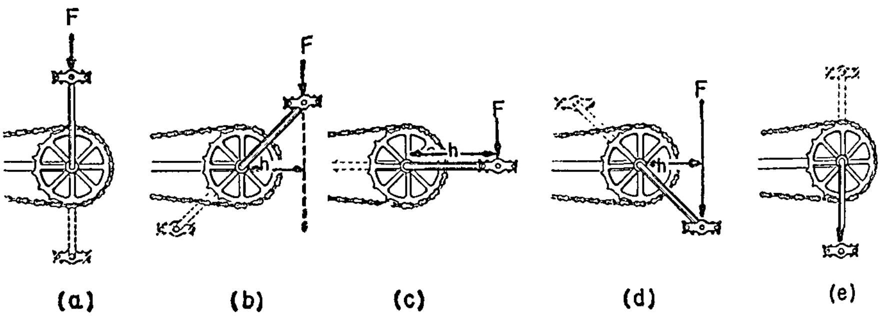


FIG. 43.—Torques on bicycle pedals. The most effective position is shown in (c). No tendency to produce rotation is obtained in positions (a) and (e). Results between nothing and the maximum are obtained for positions (b) and (d).

tion of the force. The largest value of h is found for case (*c*), while it has zero value in cases (*a*) and (*e*).

The tendency of a force to produce rotation is called a *torque*. Some writers call it a *moment of force*. Torque is technically defined by saying that the torque of a force about a point is the product of the force and the perpendicular distance from the point to the line of action of the force. So we may write

$$\text{Torque} = \text{Force} \times \text{Lever Arm}$$

Two equal but oppositely directed parallel forces produce torques that are called a *couple*. Two hands on the opposite sides of a steering wheel of a car may exert a couple on the wheel.

Some objects are arranged for extensive turning about a point, as for example the wheels of an automobile, or to a more limited extent the steering wheel of an automobile. Other objects such as the levers and scissors of Figures 40 and 42 are arranged for very limited turning about some one point. Still other objects, although acted on by parallel forces are not expected to turn at all about any point. See the bridge of Figure 44 carrying its own weight and that of two cars.

3. The Principle of Moments

If the sum of all the forces acting on an object in one direction is equal to the sum of those in the opposite direction, the object will be in equilibrium as far as the linear motion parallel to these forces is concerned. However, we have to examine the torque effect of all the forces about some point to learn

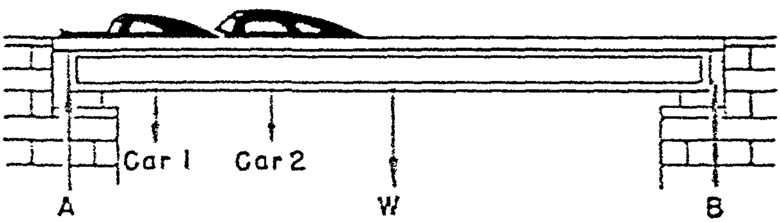


Fig. 44.—Some objects, although acted on by parallel forces, are not expected to turn at all about any point as the bridge carrying its own weight and that of two cars.

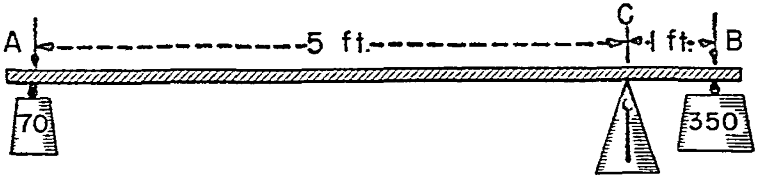


FIG. 45.—If the clockwise and counter-clockwise torques are equal, the object does not tend to rotate. This statement is sometimes called the Principle of Moments.

whether or not the object will start to rotate or remain stationary under the influence of the forces.

With the aid of the definition of a torque this matter can easily be decided. If the location of the fulcrum is not known, any point on the object may arbitrarily be chosen as a fulcrum. One then computes all the torques that tend to rotate the object in a clockwise direction about that point. Similarly, the counter-clockwise torques are found about the same point. If the clockwise and counter-clockwise torques are equal, obviously the object does not tend to rotate. This statement is sometimes called the *principle of moments*.

Example. In Figure 45 the distance from A to C is 5 feet measured along the lever. The distance from C to B is 1 foot. If the force at B is 350 pounds, what force at A will just balance it? The clockwise torque about C due to the force at B is

$$350 \times 1 = 350 \text{ lb.} \times \text{ft.}$$

And the counter-clockwise torque about C due to the force at A is

$$A \times 5$$

These torques must be equal in magnitude, hence

$$\begin{aligned} 5A &= 350 \\ A &= 70 \text{ lb.} \end{aligned}$$

The total force at C must be equal and opposite to the sum of A and B; that is

$$C = 350 + 70 = 420 \text{ lb.}$$

The principle of moments can be applied very effectively to all problems where there are parallel forces. Consider for instance the case of a simple bridge, which of course is not supposed to rotate about any point under any circumstances. See Figure 44 and the numerical example below.

Example. In Figure 44 the weight of the bridge, 20,000 pounds, is treated as though concentrated at the center of the structure. Car 1 is 10 feet from A, and car 2 is 15 feet from A. Car 1 weighs 3100 pounds and car 2 weighs 2900 pounds. The bridge is 40 feet long. Find the weight supported by the piers A and B.

It is obvious that the two piers together must push up with a force equal in magnitude to the entire weight supported; that is

$$20{,}000 + 3100 + 2900 = 26{,}000 \text{ lb.}$$

No fulcrum is indicated and we may arbitrarily choose any point, say A.

There are three clockwise torques due, respectively, to the weight of each car and the weight of the bridge. These are

$$\begin{array}{lrl} \text{Car 1} & 3100 \times 10 = & 31{,}000 \text{ lb.} \times \text{ft.} \\ \text{Car 2} & 2900 \times 15 = & 43{,}500 \text{ lb.} \times \text{ft.} \\ \text{Bridge} & 20{,}000 \times 20 = & \underline{400{,}000} \text{ lb.} \times \text{ft.} \\ & & 474{,}500 \text{ lb.} \times \text{ft.} \end{array}$$

The only counter-clockwise torque is that due to the thrust B of the pier, and since the bridge is 40 feet long this torque is

$$40B \text{ lb.} \times \text{ft.}$$

By equating the clockwise and counter-clockwise torques we obtain

$$\begin{aligned} 40B &= 474{,}500 \text{ lb.} \times \text{ft.} \\ B &= 11{,}862.5 \text{ lb.} \end{aligned}$$

It follows that the rest of the total weight must be supported at A. Hence

$$A = 26{,}000 - 11{,}862.5 = 14{,}137.5 \text{ lb.}$$

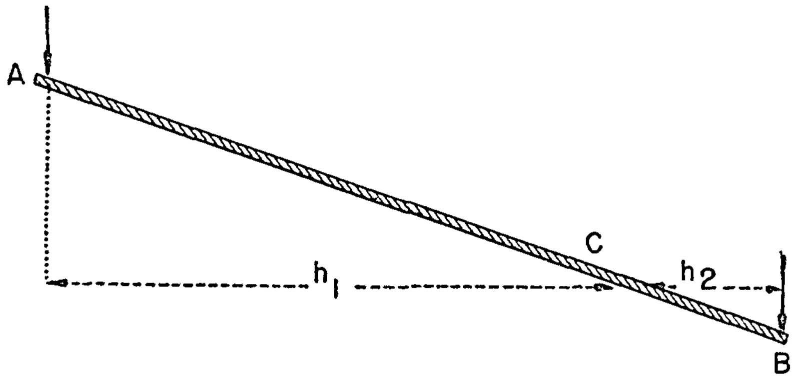


FIG. 46.—A lever in an inclined position. The lengths of the true lever arms are indicated by h_1 and h_2.

The student should now choose some other point for a fulcrum, say *B*, and solve this problem again. He will find that no matter what point is arbitrarily chosen for a fulcrum the same results will be obtained as above.

Example. Figure 46 shows a lever in an inclined position. Suppose that the distance along the lever from *A* to *C* is 8 feet and that from *C* to *B* is 2 feet. The lengths of the true lever arms are indicated by h_1 and h_2. In a practical problem these distances may be measured. Or they can be obtained from the drawing of the problem, if the drawing is made to scale. Suppose that h_1 is 6 feet and h_2 is 1.5 feet. Then the clockwise torque is

$$1.5B$$

and the counter-clockwise torque is

$$6A$$

For equilibrium

$$6A = 1.5B$$

and if either *A* or *B* is known the other can be determined.

4. Center of Gravity

In a number of the above examples, the weight of an object was assumed to be acting at a single point. Justification for this assumption may be found in this manner.

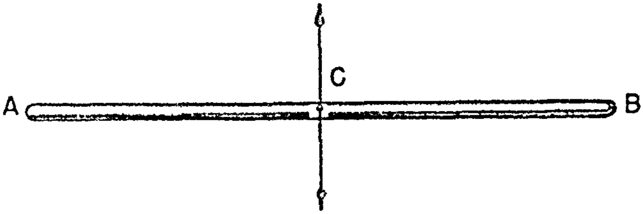


FIG. 47.—AB is a uniform rod. If picked up at the exact center, C, it will remain in a horizontal position.

In Figure 47, *AB* is a uniform rod. If it is picked up at the exact center, *C*, it will remain in a horizontal position; or if it is turned into some other position as indicated in Figure 48 it will remain as placed.

In Figure 49, *AB* is a rod which is not uniform. Some point, *C*, however can be found on this rod such that the rod

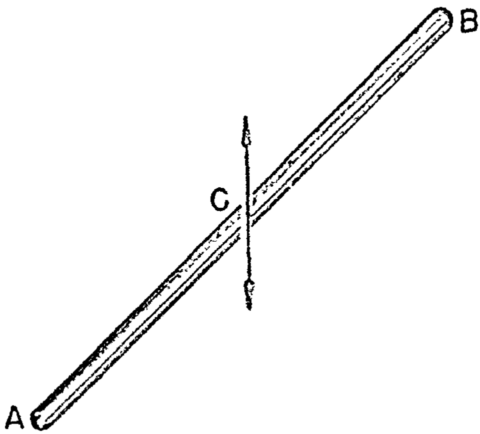


FIG. 48.—If the uniform rod, AB, of Fig. 47, is turned into some other position as indicated, it will remain as placed.

will not start to rotate when picked up at this point. We can say that the torques about this point due to the distributed weight of the object must be the same in the clockwise as in the counter-clockwise direction. Such a point is called the *center of gravity*. In the case of an irregularly shaped body, as for example a chair, this point may not even lie in the object.

The point has the particular property that as far as gravitational forces are concerned, all the mass of the object may be considered as concentrated at this point. For example, the bar of Figure 50 is pivoted on a fulcrum at P and balanced by means of a weight W. The entire mass of the rod behaves as though concentrated at its center of gravity to the right of the fulcrum. The rod can therefore be bal-

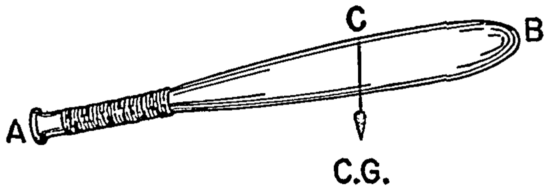


FIG. 49.—A point, C, can be found on AB, which is not a uniform rod, such that AB will not start to rotate when picked up at this point.

anced by the additional weight attached as shown to the left of the fulcrum.

The equation of moments is

$$WL_2 = W_bL_1$$

where W_b is the total weight of the bar, L_1 is the distance of the center of gravity from the fulcrum, and L_2 is the corresponding distance of W, the weight attached.

5. Center of Mass

The point called the center of gravity also marks the approximate position of the *center of mass.* The latter is defined as a point about which no rotation of the body will start if a single force is applied in any direction at that point on the object.

Although the definitions of center of gravity and center of mass are different, the points determined are almost identical. The slight discrepancy is due to the fact that lines representing gravitational force are not quite parallel but actually are radial toward the center of the earth. This effect is too small to be of practical significance in most problems.

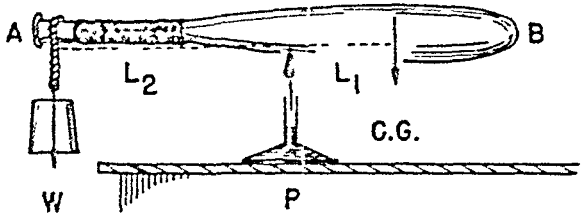


FIG. 50.—Rod, AB, is pivoted on fulcrum at P and balanced by means of weight, W.

6. Beam Balance

In Chapter 3, page 31, a balance for the measurement of weight is described. This device depends for its action on the principle of moments for a body in equilibrium as described in the present chapter. The balance of Figure 9, page 31,

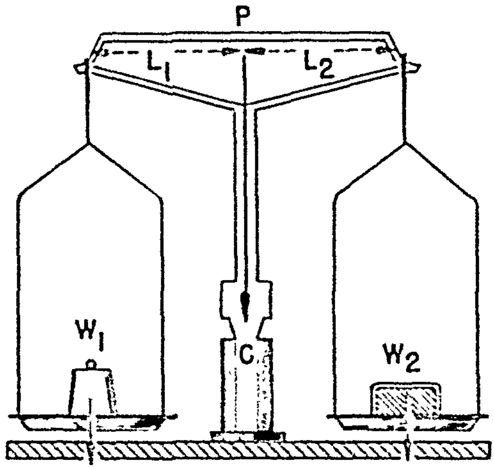


FIG. 51.—Directly beneath the pivot is the point C, the center of gravity. This point is directly beneath the point of support when the beam is balanced and hence does not produce any torque about the pivot.

is shown schematically in Figure 51 with an object to be weighed and weights added. The fulcrum (called a pivot in this case) is marked *P*. Directly beneath the pivot is the point *C* which is the center of gravity for the beam and scale-

pan system. Notice that this point is directly beneath the point of support when the beam is balanced and hence does not produce any torque about the pivot.

The equation of moments is

$$W_1L_1 = W_2L_2$$

where, for example, W_1 may be the weight of some object to be determined, and W_2 that of standard weights of known

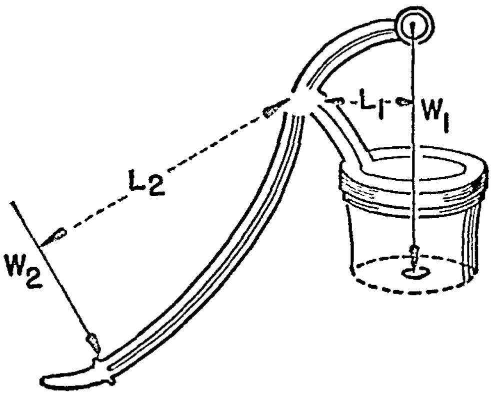


FIG. 52.—The lever in the shape shown may still be in rotational equilibrium if the forces and lever arms are such that the clockwise torques and counter-clockwise torques are equal in magnitude.

value. The weight of the beam does not come into the equation because, with the center of gravity directly beneath the point of support, it does not produce a torque about this point.

If the device is an equal arm balance L_1 equals L_2 and the equation becomes

$$W_1 = W_2$$

Otherwise we must write

$$W_1 = W_2 \frac{L_2}{L_1}$$

7. Torques Due to Nonparallel Forces

Suppose that the lever of Figure 40 is bent into the shape shown in Figure 52. It may still be in rotational equilibrium

if the forces and lever arms are such that the clockwise torques and counter-clockwise torques are equal in magnitude.

An interesting application of the principle of moments for this type of problem occurs in the case of a ladder leaning against a wall. (See Figure 53.)

The weight of the ladder itself, W_L, acting at a lever arm, h_1, produces a counter-clockwise torque about A, the foot of

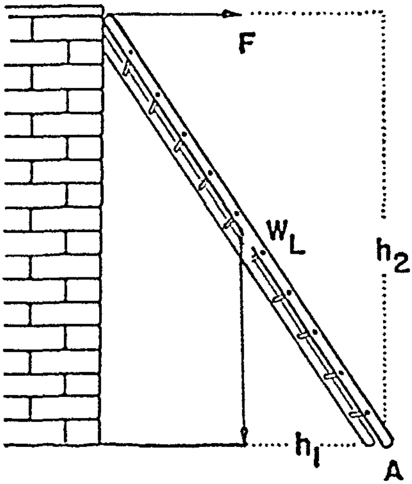


FIG. 53.—The weight of the ladder, W_L, acting at a lever arm, h_1, produces a counter-clockwise torque about A, the foot of the ladder. The wall pushes against the top of the ladder with a force F that gives a clockwise torque about A.

the ladder. The wall pushes against the top of the ladder with a force F that gives a clockwise torque.

The clockwise torque is

$$F \times h_2$$

and the counter-clockwise torque is

$$W_L \times h_1$$

These torques must be equal and so, for example, if the weight of the ladder and the distances h_1 and h_2 are known,

the force with which a building must push against the top of the ladder can be found.

Example. The ladder in Figure 53 is 20 feet long, weighs 75 lb. and is of uniform construction. The bottom end of the ladder is 6 feet from a vertical wall. Find the force with which the wall must push back against the ladder.

The vertical height, h_2 may be found from plane geometry since the hypotenuse and one leg of the triangle are known.

$$h = \sqrt{20^2 - 6^2} = 19.1 \text{ ft.}$$

The clockwise torque is then

$$19.1F$$

and the counter-clockwise torque is

$$3W_L = 3 \times 75 = 225$$

Hence $$19.1F = 225$$

and $$F = 11.8 \text{ lb.}$$

Some Important Facts

1. Forces which are not concurrent (for example, parallel forces) may be harnessed together by a rigid bar called a lever.

The lever arm of a force is the perpendicular distance from that force to a center of rotation, or fulcrum.

2. The torque or moment of a force is the product of the force times its lever arm.

$$\text{Torque} = \text{Force} \times \text{Lever Arm}$$

Two equal but oppositely directed parallel forces produce torques that are called a couple.

3. When a system of parallel forces is in equilibrium, the sum of the clockwise torques equals the sum of the counter-clockwise torques, and the sum of the forces in any direction equals the sum of the forces in the opposite direction.

4. The center of gravity and for practical purposes the center of mass of a body is the point about which torques due to the weight of the various parts of the body are in equilibrium.

5. The principle of moments is utilized in the beam balance,

$$W_1L_1 = W_2L_2$$

For the equal arm balance, therefore,

$$W_1 = W_2$$

6. In an equilibrium system of nonparallel forces, rotational equilibrium exists when the clockwise torques are equal to the counter-clockwise torques.

Generalization

When any object is in equilibrium, the sum of all clockwise torques about any point on that object is equal to the sum of all the counter-clockwise torques about the same point, and the sum of all forces in any direction is equal to the sum of all forces in the opposite direction.

Questions and Problems

Group A

1. If forces are parallel, or nearly so, how can they act on each other?
2. What is a lever? A lever arm? A fulcrum?
3. What is meant by the moment of force?
4. What is the center of gravity of a body, and how might it be determined?
5. Distinguish translatory and rotational motion. Mention examples of bodies having each and both.
6. What is equilibrium? Define stable, unstable, and neutral equilibrium. Distinguish translatory and rotational equilibrium.
7. State the principle of moments.
8. A 75-pound girl sits 10 feet from the fulcrum of a see-saw. Where must a 125-pound boy sit to balance her? 6 ft.
9. A hunter and his son can carry home a deer hung on a 14-foot pole. The center of gravity of the deer is 10 feet from the boy and 4 feet from the man who supports 125 lb. How much does the boy support? How much does the deer weigh? 50 lb.; 175 lb.
10. Why may a force applied at the fulcrum be ignored in a moment equation?
11. What determines the selection of the fulcrum on which a moment equation is to be based?
12. A bridge is 30 feet long and weighs 20 tons. A 10-ton truck is 12 feet from one end. Find the total upward force at each end of the bridge. 14 T. 16 T.
13. A and B carry a 200-lb. weight on a 15-foot pole. Where must it be hung so that A supports 40 per cent of the load? 9 ft. from A.
14. In a wheelbarrow, a 150-lb. load is centered one foot from the axle. The effort is 50 lb. How long are the handles? What is the upward thrust at the axle? 3 ft.; 100 lb.

15. A 10-foot pole has its center of gravity 4 feet from one end. When a 10-lb. weight is hung one foot from the lighter end, the pole balances at the middle. What is the weight of the pole? 40 lb.

16. Forty pounds of force can be applied on the handles of a pair of wire cutting pliers at a distance of 8 inches from the pivot or fulcrum. If a force of 640 lb. is required to cut the wire, how close to the pivot must it be placed? 0.5 in.

Group B

(Draw diagrams to represent each of the following situations.)

1. Two children are balanced on a teeter-totter. One weighing 75 lb. sits 8 ft. from the fulcrum. The other sits 10 feet from the fulcrum. What does the second child weigh? 60 lb.

2. Two children weighing 85 and 50 lb. respectively are balanced on a teeter-totter. If the 50-lb. child sits 10 ft. from the point of support, where must the other child sit? 5.88 ft.

3. A plank is used as a lever to pick up one end of an automobile. One end of the plank is placed under the rear axle and the plank rests across a small log which acts as a fulcrum. The horizontal distance from the log to the rear axle is 1.5 feet and the corresponding distance from the log to the free end of the plank is 10 feet. Neglect the weight of the plank and find the effect on the automobile that can be produced by a man weighing 150 lb. located at the free end of the plank. 1000 lb.

4. The plank of problem 3 may be used to lift the car by slipping the end under the rear axle so that the end rests on the ground or a block of some kind. Find the force on the automobile if the car axle is 1.5 feet from the end of the plank and the total length of the plank is the same as in problem 3. Find the force on the automobile if the man picks up with a force of 150 pounds. 1150 lb.

5. Find the force exerted by the ground or block against the end of the plank in problem 4. 1000 lb.

6. A force of 40 lb. is used to squeeze a pair of pliers. If the force is applied at an average distance of 5 inches from the pivot, what force is present on an object in the jaws of the pliers located 1 inch from the pivot? 200 lb.

7. A uniform bar 10 feet long and weighing 50 lb. is picked up at a point 4 feet from one end. Compute the amount of weight that could be hung on the shorter end to make the bar just balance. 12.5 lb.

8. An irregularly shaped bar 9 feet long is found by trial to have its center of mass 4 feet from one end. It is now mounted on a fulcrum at its midpoint and the bar is balanced by placing a 10-lb. weight on the lighter end. Find the weight of the bar. 90 lb.

9. A simple bridge is supported by piers at either end. An automobile weighing 3000 lb. is located 10 feet from one end. The bridge weighs

10,000 lb. and is 28 feet long. Find the load supported by each pier. *a*. 6928.6 lb. *b*. 6071.4 lb.

10. A man weighing 70 kilograms sits on the end of a 10-meter beam of uniform construction. If the beam weighs 400 kilograms, where shall a cable be fastened so that the beam will remain horizontal when picked up? 74.5 cm. from center of beam toward end with man.

11. An irregularly shaped bar weighs 100 kilograms and is 7 meters long. It is picked up at the center and balanced by placing a 10-kilogram weight on the lighter end. Find the center of gravity of the original beam. 35 cm. C. to C. G.

12. Many balances are made so that a standard weight may be moved along one end of the beam to balance the load. Make a diagram of such a balance where the shorter end of the beam is 1.5 inches. The sliding weight weighs ½ lb. Indicate positions of this weight by pounds from 1 to 10. 3 in. to 30 in.

13. A beam balance is made with unequal arms so that weights marked in ounces will actually indicate pounds. If the shorter arm of the balance is 0.2 inch long, find the length of the arm from which the standard weights are to be hung, for 1 to 10 lb. 3.2 in. to 32 in.

14. Repeat problem 12 for a metric balance where the shorter end of the beam is 5 cm. and where the sliding weight weighs 100 g. Determine positions of the slider for weighing 100-g. variations up to 1000 g. 5 cm. to 50 cm.

15. A ladder 25 feet long and weighing 100 lb. leans against a vertical wall. The ladder is uniform in construction. Its bottom end is 6 feet from the wall. Find the force with which the wall must resist the top of the ladder. 12.34 lb.

16. A ladder 25 feet long and of unknown weight has its upper end resting against a window which will break if required to resist a force of more than 20 lb. The bottom of the ladder is 6 feet from the wall. Find the limiting weight of the ladder if it is of uniform construction. 162 lb.

17. A man climbs up the ladder of problem 15. Find the force of resistance of wall against ladder when the man is half way up the ladder, if he weighs 125 lb. 15.43 lb. more.

18. Repeat problem 17 for the case when the man is two-thirds of the distance up the ladder. (Suggestion: Determine the lever arm to the line of action of the weight of the man by using similar triangles.) 20.58 lb. more.

Experimental Problems

1. Suspend a meter stick from its center of gravity and hang weights on the stick so as to produce equilibrium. Place the sum of the clockwise moments equal to the sum of the counter-clockwise ones. Does the

equation reduce to an identity? If not, what are some probable sources of error?

2. Suspend a meter stick from some point other than the center of gravity, and hang weights so as to produce equilibrium. Using "X" as the unknown weight of the stick, calculate the moment of the stick and add it to the proper side. Check by weighing the stick.

Chapter 8

MACHINES—MECHANICAL ADVANTAGE

It is often more convenient to apply a force at some point other than where it is wanted, and it also often happens that the needed force is larger or smaller than the one which is available. Devices which enable us to make these manipulations with forces are called machines. The ratio of the delivered to the applied force is called the mechanical advantage of the machine.

A machine may be thought of as an amplifier of force, but of course the energy expended through the delivered force cannot be greater than that put into the system by the applied force. This means that if the application of a small force results in developing a large force, the smaller force must move through a large distance in comparison to that through which the larger force may act. The mechanical advantage of a machine can often be determined most easily by equating the energy put into the machine to that taken out of it and then computing the relative motions of the points of application of the applied and the delivered forces.

In any practical machine, the mechanical advantage is less than the ideal value computed because of the work that must be done against friction.

---

1. Mechanical Advantage of Levers

Many practical situations arise where it is desirable to exert a large force when only a small force is available. Levers are often used for such purposes and the development of the preceding chapter and many of the problems of that chapter dealt with such cases.

A typical use of a lever is to lift a heavy object when only a relatively small force is at hand. In the drawing of Figure 54 a heavy weight W rests on the lever at B. C is the fulcrum and a force F can be applied at A. The respective lever arms of A and B from C are L_1 and L_2.

From the principle of moments as developed in the preceding chapter we write

$$FL_1 = WL_2 \tag{1}$$

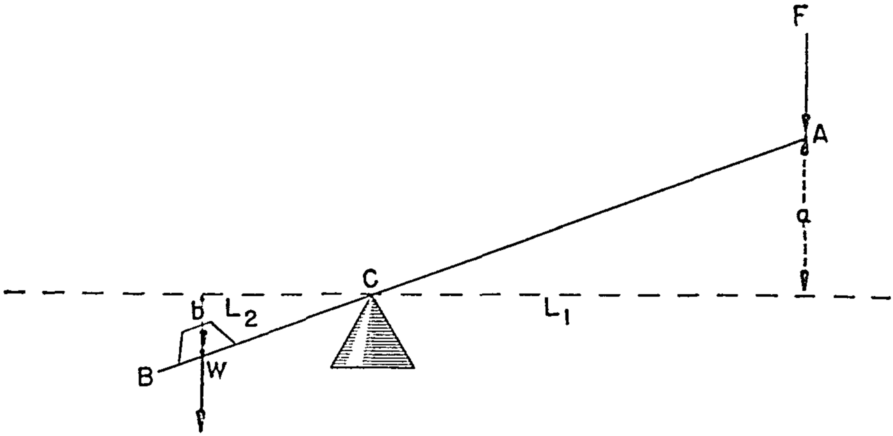


FIG. 54.—The work done by force A in moving through the distance a is equal in magnitude to that done by force B moving through the distance b.

From which

$$\frac{L_1}{L_2} = \frac{W}{F} \qquad (2)$$

The ratio of the force obtained (in this case W) to that applied (F) is called the *mechanical advantage* of the lever, and a lever used in this manner is called a *machine.*

We know from experience, and it can easily be seen from the geometry of the drawing, that if the force F moves with the lever, the weight W will be moved through a smaller distance. These distances are indicated by a and b in Figure 54.

The amount of work done by the force F in moving through the distance a is

$$Fa$$

and that done on W is

$$Wb$$

If we assume that no energy is wasted in the process we may write

$$Fa = Wb \qquad (3)$$

From which

$$\frac{a}{b} = \frac{W}{F} \qquad (4)$$

From a comparison of equations (2) and (4) it is evident that

$$\frac{a}{b} = \frac{L_1}{L_2}$$

That this statement is true may also be seen by noting the similar triangles in Figure 54.

The fact that the work put into a machine is, in the ideal case, equal to the work gotten out of it, furnishes a simple method for computing the mechanical advantage of many machines that look more complicated than a simple lever.

2. The Jackscrew

Figure 55 shows schematically a simple jackscrew such as is used for raising heavy objects. A force F is applied horizontally near the end of the bar as indicated. In one revolution of the bar the head of the jackscrew will go up or down the distance of one thread to another.

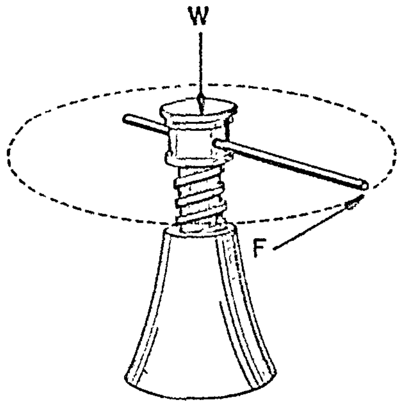


FIG. 55.—A jackscrew. A force F is applied horizontally near the end of the bar. In one revolution of the bar the head of the jackscrew will go up or down the distance of one thread to another.

Example. The end of the bar of the jackscrew in Figure 55 is 24 inches from the center. The screw moves ¼ inch for one revolution. Find the mechanical advantage and find the upward force for a horizontal force on the handle of 30 lb.

For one revolution the end of the handle will move

$$d = 2\pi 24 = 151 \text{ in. approx.}$$

in comparison to ¼ inch for the vertical motion of the screw. Any motion of the end of the bar is therefore 604 times that of the up or down motion of the screw. But in the ideal case of no losses by friction the work put into moving the end of the bar is equal to the work that can be done by the end

of the screw. Since the motion of the end of the bar is 604 times that of the screw the forces must have the inverse relation; that is, the force the screw can exert must be 604 times that exerted on the handle.

Hence the mechanical advantage of this jackscrew is 604 and if a force of 30 pounds acts on the lever, the force that the screw can exert must be $30 \times 604 = 18{,}120$ pounds.

3. Pulleys

Machines are not only used to obtain forces that are different from those applied to the machine, but are also employed

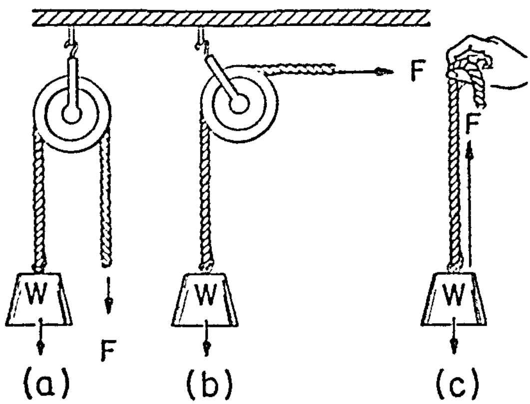


FIG. 56.—A single pulley used to change the direction of a force. The mechanical advantage of one fixed pulley is one and the force is just equal in magnitude to the weight, W.

to change the direction of application of a force. A rope and simple pulley is a common example of this arrangement.

Figure 56(*a*) shows a pulley used to change the direction of a force along a rope so that a downward pull on the end of the rope will raise the weight W. The mechanical advantage of this system is one and the force F is just equal in magnitude to the weight W.

Another familiar arrangement of rope and pulley is shown in Figure 56(*b*) where a horizontal pull is used to raise a weight vertically. Neglecting friction the force F in both cases (*a*)

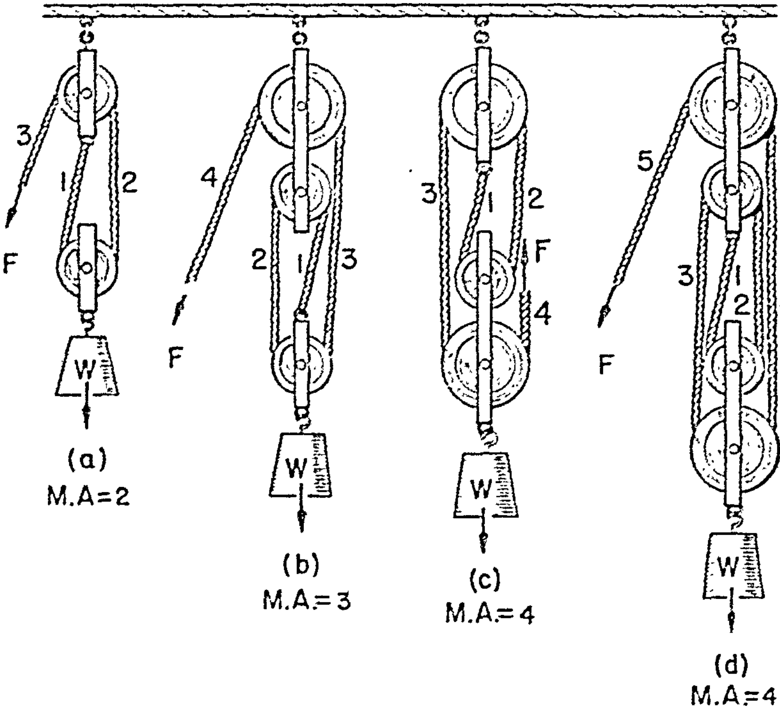


FIG. 57.—A few "blocks and tackle" or pulley systems.

and (*b*) is just the same as that required for a straight upward pull as indicated in Figure 56(*c*).

Mechanical advantage greater than one can be obtained by using more involved arrangements of pulleys and ropes. These are often called "blocks and tackle." Figure 57(*a*) shows a system with a mechanical advantage of 2. An inspection of this figure shows that ropes one and two grow shorter as the weight is raised by the force F. In other words, twice as much rope must be pulled in on rope three as the distance through which the weight W is raised. Hence the force F need be only half as great as the weight W in order that the work done by the force F will just equal that done on W.

From this analysis, or by an examination of more involved pulley systems we come to the conclusion that the ideal

mechanical advantage of a block and tackle arrangement is numerically equal to the number of ropes that grow shorter. (See Figure 57(*b*) for a mechanical advantage of 3, and 57(*c*) and (*d*) for a mechanical advantage of 4.) Pulley wheels of different sizes have been used for convenience of drawing in these figures. In actual practice the wheels in one block are usually of the same size and mounted beside one another on a common axle.

4. Pulleys of Different Size

A commonly used arrangement in engines and motors is a pulley and belt or geared wheels for permitting the device

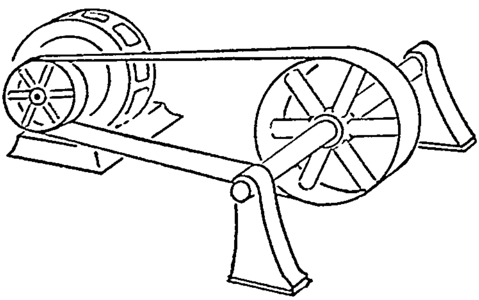


FIG. 58.—The radius of one pulley is twice that of the other. If the larger one is connected to the device that is to be turned and the smaller one is connected to the motor, the device will run at half the speed of the motor but the torque applied to the device will be twice that delivered by the motor.

driven to run at a different speed from that of the engine or motor.

Figure 58 shows two pulleys where the radius of one is twice that of the other. If the larger one is connected to the device that is to be turned and the smaller one is connected to the motor, the device will run at half the speed of the motor but the torque applied to the device will be twice that delivered by the motor.

On the other hand if the motor is connected to the larger pulley and the device to be turned connected to the smaller

pulley, the latter will run at twice the speed of the motor, but will have only ½ the torque applied to it that the motor can deliver.

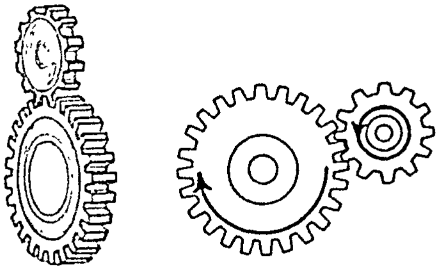


Fig. 59.—An arrangement of geared wheels.

Figure 59 shows an arrangement of geared wheels where similar results are obtained.

Figure 60 shows two pulleys attached to the same shaft and rigidly fastened to one another. The belt A travels faster than belt B but the amount of work delivered through

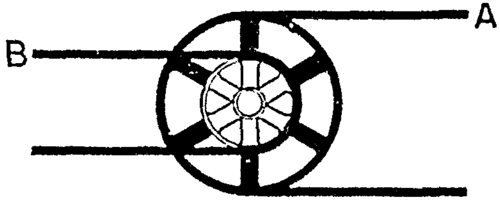


Fig. 60.—Two pulleys attached to the same shaft and rigidly fastened to one another. Belt A travels faster than belt B and hence the greater tension will exist in belt B if the amount of work delivered through one belt is to be equal to that supplied over the other.

one belt is equal to that supplied over the other. If the radius of one pulley is twice that of the other, the tension in the belt on the smaller pulley will be twice that of the other.

Differential pulleys such as those of Figure 60 are common in machine shops. Geared wheels are also often used in connection with motor driven lathes and other machine tools. Geared wheels of more complicated design are used in the

transmission box of an automobile. Here a choice of the gear arrangement to be used can be made by the driver by moving the gear shift lever. Other gears are to be found in the differential at the junction of the drive shaft and rear axle of a car. These gears change the direction of rotation transmitted by the drive shaft to that of the rear wheels and also permit one rear wheel to turn independently of the other. The latter arrangement is necessary for the equalizing of driving power on the rear wheels when a car travels in a curve.

5. Inclined Plane

Inclined planes also are often used to raise objects when the available force is less than that required for a straight lift. Figure 61 shows such an arrangement. The block has weight W and the useful work required to lift it to the height of the top of the plane is

$$\text{Work} = Wh \text{ lb.}$$

However, a force F, smaller than W, may be used to drag the block up the plane. The work done by this force will be FL and if friction between block and plane is negligible these two expressions for work must be equal; that is

$$Wh = FL$$

From which

$$\frac{W}{F} = \frac{L}{h}$$

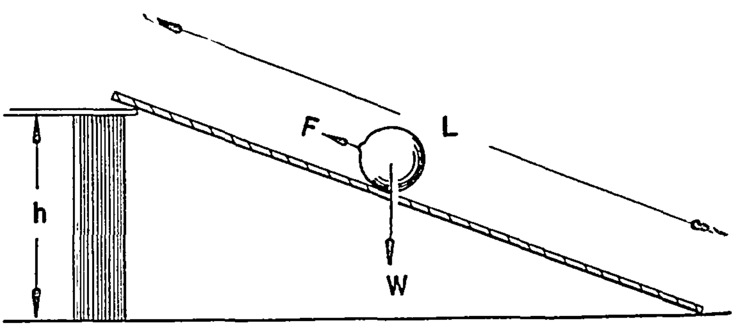


Fig. 61.—Mechanical advantage with an inclined plane.

We see that the ideal mechanical advantage of an inclined plane is equal to the ratio of the length to the height of the plane.

6. Efficiency of Machines

Just as the force required to operate a machine is greater in any practical case than the ideal force computed from the mechanical advantage, so is the work put into the machine greater than that delivered by it. The efficiency of the machine may be defined as the ratio of the work which it delivers to what is put into it.

$$\text{Efficiency} = \frac{\text{Work delivered}}{\text{Work put in}}$$

The efficiency of a machine may also be defined as the ratio of the experimentally found mechanical advantage to the ideal mechanical advantage. This definition gives the same result as the commonly given definition in the paragraph above. It also suggests an experimental method for finding the efficiencies of levers, jacks, inclined planes, pulley systems, and other simple machines which may be found around the laboratory. It is usually possible to determine the ideal mechanical advantage of such machines from measurement, and the actual mechanical advantage by experiment.

Some Important Facts

1. When a lever is used in doing work, the work done by the forces producing the clockwise and counter-clockwise torques are equal. Therefore, the resistance overcome, (W), and the necessary effort, (F), vary inversely as the respective distances moved by these forces.

Either of these ratios, W/F, or D_F/D_W, is the mechanical advantage of the system, ignoring friction.

2. In the jackscrew, the effort distance, D_F, is 2π times its radius of rotation, while D_W is the pitch of the screw, so the mechanical advantage may be expressed:

$$\text{M.A.} = \frac{D_F}{D_W} = \frac{2\pi r}{P}$$

3. The ideal mechanical advantage of a system of pulleys is numerically the number of rope strands which shorten when the resistance is moved, that is, the number pulling directly on the resistance.

4. Pulleys of different sizes may be connected by belts, gears, or by being mounted on the same shaft. In any case, the ideal mechanical advantage is the total travel distance of the effort force divided by the distance the resistance is moved in the same time.

Pulleys driving one another by gears, or a crossed belt, rotate in opposite directions.

5. The ideal mechanical advantage of an inclined plane is the length of the plane divided by its height, that is

$$\frac{W}{F} = \frac{L}{H}$$

6. The efficiency of any machine is the ratio of actual work delivered to total work put in.

$$\text{Efficiency} = \frac{\text{Work Delivered}}{\text{Work Put In}}$$

This ratio is also equal to actual mechanical advantage over ideal mechanical advantage.

$$\text{Efficiency} = \frac{\text{W.D.}}{\text{W.I.}} = \frac{\text{M.A.}_a}{\text{M.A.}_i}$$

Generalizations

Mechanical systems may be used so that the force delivered by the system may be equal to, greater than, or less than that applied to the system. If the forces move, the work put into the system must equal that delivered by the system including that done against friction. The ratio of the force delivered by the system to that applied to it is called mechanical advantage.

Questions and Problems

Group A

1. What is a machine?
2. What is the basic simple machine for dealing with parallel forces? What are some of its variations?
3. What is the basic simple machine for dealing with concurrent forces? What are some of its variations?
4. What is meant by mechanical advantage?
5. Distinguish real or actual mechanical advantage from ideal mechanical advantage.
6. What is mechanical friction? Distinguish sliding from rolling friction.
7. Give examples of useful friction, and of friction that is not useful.
8. What is meant by the efficiency of a machine?

9. Express efficiency as a work ratio and as a mechanical advantage ratio.

10. State the General Law of Machines.

11. How would you determine the ideal mechanical advantage of a lever? A wheel and axle? A block and tackle? A chain of gears?

12. How would you determine the ideal mechanical advantage of an inclined plane? A jackscrew?

13. Find the mechanical advantage of a lever 10 feet long with a fulcrum 2 feet from one end. 4.

14. Find the mechanical advantage of a lever 10 feet long with a fulcrum at one end and the weight to be lifted placed 2 feet from that end. 5.

15. A 12-foot plank, used as a lever to raise the end of an automobile, has one end 1.5 feet under the rear axle, which is near the ground. Find the vertical lift required on the other end of the plank if the end of the automobile weighs 1300 lb. 162.5 lb.

16. A 12-foot plank is used to roll a 300-lb. barrel up on to a 3-foot platform. Neglecting friction, what effort is required? 75 lb.

17. Find the mechanical advantage of a jackscrew with a bar 1.5 feet long and a screw of 0.125 inch pitch. 905.

Group B

1. The crank of a windlass is 2½ feet long and the drum is 10 inches in diameter. Neglecting friction, what effort is needed to raise 360 lb.? What effort is needed if the machine is 60% efficient? 60 lb.; 100 lb.

2. The pitch of a jackscrew is ¼ inch and the bar is 28 inches long. If the efficiency is 30 per cent, what effort is needed to raise 7040 lb.?

M.A., 704; *E*, 33⅓ lb.

3. A steel beam weighing 3000 lb. is supported at the ends on jackscrews. Each jackscrew has a 2-foot bar and the screw is ¼ inch pitch. Find the force required to turn the bar on either screw. 2.5 lb.

4. In an actual trial of the conditions of problem 3 the force required is found to be 5.0 lb. What is the efficiency of the machine?

50 per cent.

5. In a block and tackle similar to that of Figure 57(*b*) the lower block weighs 5 lb. and the weight to be lifted weighs 409 lb. Find the force necessary to apply to the rope, if friction is neglected. 138 lb.

6. Allow for the fact that the block itself is not a useful part of the load and compute the efficiency of the above (problem 5) system, still neglecting friction. 98.7 per cent.

7. A block and tackle of 3 fixed and 2 movable pulleys is used to raise a 500-lb. weight. Neglecting friction, what effort—acting downward—is needed? What effort is needed if the machine is 80 per cent efficient?

100 lb.; 125 lb.

8. A plank 10 feet long is used as an inclined plane to raise objects 1.5 feet. What is the mechanical advantage? Neglect friction and find the force along the plank required to move a 450-lb. object.

M.A. $6\frac{2}{3}$; *E*, 67.5 lb.

9. Neglect friction on an inclined plane and find the length of plane required to raise a 500-lb. object 10 ft. if the available force is 75 lb.

$66\frac{2}{3}$ ft.

10. If the efficiencies of the planes in problems 8 and 9 are 60 per cent as a result of friction find the actual values asked for.

112.5 lb.; 111.1 ft.

11. What is the mechanical advantage of a carpenter's claw hammer when used to pull a nail if the handle is 12 inches as measured from the fulcrum when the hammer is used and the point where the nail is caught is 1.4 inches from this point? $8\frac{4}{7}$.

12. A pail of water weighing 40 lb. is attached by a light rope to a horizontal axle with an attached handle for turning. If the axle has a diameter of 6 inches and the handle is 14 inches as measured from the center of the axle, find the force required to raise the pail of water if friction may be neglected. $8\frac{4}{7}$ lb.

Experimental Problems

1. Set up blocks and tackle having ideal mechanical advantages of 2, 3, 4, 5, and 6. In each case choose a convenient resistance and hang the ideal effort on the free end of the cord.

Can the ideal effort support the resistance? Pull it up? Explain.

In each case add sufficient effort to slowly and uniformly pull the resistance and determine the actual mechanical advantage.

Measure the effort distance necessary to raise each resistance weight a definite distance, such as 20 cm., and compute the work input and output.

Finally, calculate the efficiency of each block and tackle. Do the efficiencies vary? Does this seem reasonable?

2. Adjust an inclined plane so that it makes an angle of 30° with the horizontal. Measure the length and height and determine the ideal mechanical advantage.

Choose a roller weight of convenient size, and apply the effort by either pulling directly on the roller with a spring balance or by hanging weights on the vertical segment of a cord attached to the roller and passing over a pulley at the upper end of the plane.

After several trials, decide on the effort which pulls the roller up at a uniform rate; then determine the actual mechanical advantage and the efficiency.

Repeat the above procedure for two other angles of inclination of the plane.

Chapter 9

BEHAVIOR OF LIQUIDS

The term fluids is used to refer to both liquids and gases. The complete lack of rigidity as compared to solids gives liquids and gases many special qualities of importance; but at the same time many of the ideas of mechanics that we have learned in preceding chapters can also be applied to them.

For convenience in studying the subject, it is customary to study first the properties of fluids that do not depend on motion of the fluids, and to study next the properties that do depend on their motion. In this chapter and the next we will be chiefly interested in the static properties. This part of the subject is called hydrostatics—a somewhat misleading name, for hydro refers to water and actually we are interested in any and all fluids. We will first study liquids, and then in the next chapter gases.

This chapter opens with a discussion of the nature of the three common states of matter—solid, liquid, and gaseous—and then it proceeds to a study of pressure in liquids. We first consider pressure on the bottom of a vessel, then at any position in the liquid. Pressure in a liquid is the same in any direction at any one point and is independent of the size and the shape of the container. On the other hand, it is dependent on the density of the liquid and on the height of the top level of the liquid above the point in question.

In a closed vessel any additional pressure introduced is transmitted undiminished to all parts of the liquid. Applications of the use of this fact are found in hydraulic presses, in city water supplies and many other devices.

Whenever an object is immersed in a fluid it is buoyed up by a force equal to the weight of the displaced fluid. If the body is less dense than a liquid in which it is placed, it will float with part of its volume above the surface of the liquid.

1. States of Matter

The three common states in which matter is to be found are the solid, liquid, and gaseous. Under the more commonly experienced conditions of temperature and pressure some substances are always found in a single state. For example, iron is a solid, air a gas, and oil a liquid. Other substances, such for example as water, may exist in any one of the three forms at moderate temperatures (e.g., steam, water, ice).

Iron and many other substances, although solid at moderate temperatures, may be changed to the liquid, or even the gaseous state at temperatures more or less readily attainable in furnaces. On the other hand, air and other gases may be liquified or even frozen if their temperatures are reduced far enough.

The gaseous state of matter is distinguished from other states by the tendency of the gas to diffuse without limit. That is, it is not confined to any one volume, but will spread out and occupy all the volume that is available. It is also easily compressible.

Liquids, in contrast to gases, occupy definite volume but they are not limited to any definite shape as would be the case with a solid. They flow and make themselves conform to the shape of any container. They are only slightly compressible —a great force is required to make an appreciable change in the volume of a liquid.

2. Pressure in a Liquid

If a liquid is placed in a simple container with vertical walls as shown in Figure 62, it is easy to see that the total

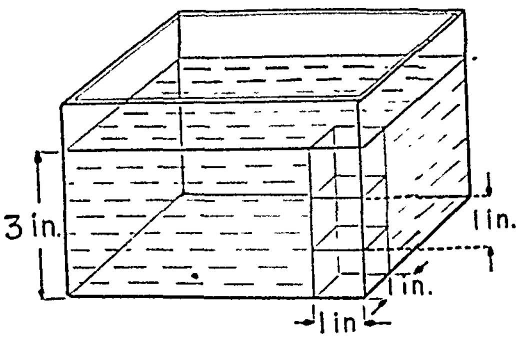


FIG. 62.—Liquid pressure at the bottom of a vessel.

weight of the liquid must be supported by the bottom of the container. The weight supported by each square inch of the bottom is equal to the weight of a column of liquid one square inch in cross section and as high as the top of the water in the container. Similarly, in the metric system, the weight sup-

ported by one square centimeter of the bottom of the container is equal to the weight of a column of liquid one square centimeter in cross section and as high as the top of the liquid.

The force per unit area exerted by the liquid on the bottom of the container is called the pressure of the liquid at that level. It is commonly measured in pounds per square inch or grams per square centimeter.

If we insert a plane (*a*) at some other level in the liquid as indicated in Figure 63, the pressure P on top of this plane may be found in the manner suggested for the bottom of the pail. However, experience shows us that such a plane, unless made of material more dense than the liquid, does not tend to sink in spite of this pressure. This leads to the conclusion that there must be an equal pressure upward on the bottom of the plane.

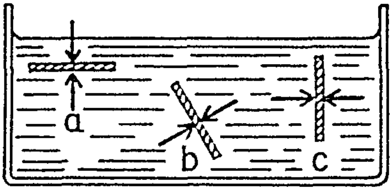


Fig. 63.—Liquid pressure at any point in the liquid.

Similar experiments with planes in other positions, as shown at (*b*) and (*c*) in the same figure, lead to the conclusion that pressures must be the same in both directions in these cases also.

Finally, we conclude that pressure as measured at any one position in a liquid is the same in any direction. A rigorous proof of this fact will be left to more advanced texts on the subject.

When a vessel of irregular shape is used as a container, it is less easy to arrive at the above conclusions by a superficial examination, but actual experiment will show them to hold. In Figure 64 is shown a container with a large reservoir and a small side tube. In order that the pressure at the entrance of the side tube into the reservoir may be the same in both directions, it is necessary that the vertical height of the liquid in each part of the container be the same.

In a container with sloping sides as shown in Figure 65 (*a*) and (*b*), the pressure on the bottom is determined by the

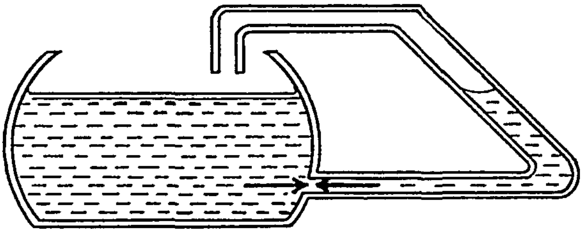


FIG. 64.—In order that the pressure at the entrance of the side tube into the reservoir may be the same in both directions, it is necessary that the vertical height of the liquid in each part of the container be the same.

weight of a column of unit area in cross section and of height as measured to the level of the top of the liquid just as in the previous cases. Hence, since the areas of the bottoms are different, the total force on the bottoms of the containers is different for cases (*a*) and (*b*) even though the amount of liquid in the two cases and the depth of the liquid should be the same.

The total force on the bottom of the container (*a*) is less than the total weight of the liquid because part of the weight is supported by the sloping sides. In the case (*b*) the total force on the bottom of the container is greater than the total weight of the liquid because a component of the pressure of the liquid against the sides is upward.

An extension of this latter idea may be seen in the case of Figure 66. Here the pressure at the bottom of the container is determined by the height of a liquid column as measured from the bottom of the container to the top of the liquid in the tube and by the density of the liquid.

At the level of the top of the main part of the vessel, the pressure is determined by the height of the liquid in the tube alone and the density of the liquid. This value of pressure must be the same at all points at the same level and since it is the same in any direction, it follows that it presses upward against the inside of the top of the container. In the case of the regularly shaped container in this figure the difference in the total force downward on the bottom of the container

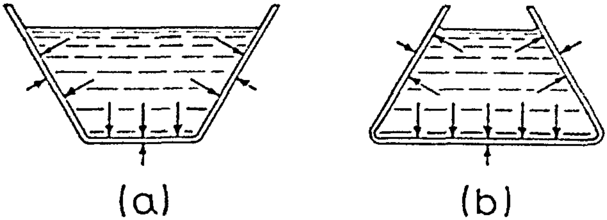


FIG. 65.—Liquid pressure on side walls of vessels. The total force on the bottom of the container is: in (a) less than the total weight of the liquid; in (b) greater than the total weight of the liquid.

and the total force upward on the inside of the top will be equal to the total weight of the liquid in the vessel and tube.

Pressure in a liquid in an open container is directly proportional to the height of the top surface of the liquid as measured vertically upward from the point where the pressure is to be determined. The horizontal dimensions, and the shape of the vessel have no effect on the pressure.

Example. In a vessel such as is shown in Figure 66, let the bottom be square with side b, and let the height be a. The height of the auxiliary tube is h.

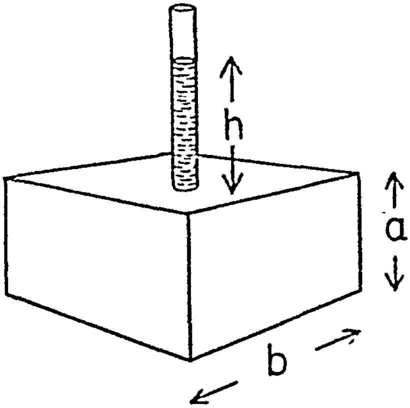


FIG. 66.—The pressure at the bottom, b, of the container is determined by the height of a liquid column as measured from the bottom of the container to the top of the liquid in the tube. $P = (h + a)d$.

(a) Find the pressure and the total force on the bottom of the vessel when the main part is full of water but the tube is empty, if b is 20 cm. and a is 10 cm.

The pressure P is determined by the height a and the density d and is

$$P = ad \text{ grams per sq. cm.}$$

For water, in this problem we have

$$P = ad = 10 \times 1 = 10 \text{ g. per sq. cm.}$$

The total force on the bottom is equal to the pressure (force per unit area) multiplied by the total area.

$$F = P \times \text{area} = 10 \times (20 \times 20) = 4000 \text{ g.}$$

(*b*) Find the pressure and total force on the bottom of the vessel when the tube h is 15 cm. long and is also full of water.

The pressure at the bottom of the vessel may be found either by determining the height of the liquid from the bottom of the vessel to the height of the water in the tube or by determining the amount of pressure due to the water in the tube alone and then adding it to the part found in section (*a*) of this problem above.

From the first of these methods we write for the pressure P'

$$P' = (a + h)d = (10 + 15) \times 1 = 25 \text{ g. per sq. cm.}$$

The total force F on the bottom would be

$$F = P \times \text{Area} = 25 \times 20 \times 20 = 10{,}000 \text{ g.}$$

For a liquid with density other than that of water, the same general method may be used if the density of the liquid is known. Similarly, it may be applied with slight change in the English system of units.

In the English system the most common units for specifying pressure are pounds per square inch as indicated in the first paragraph of this section. The operation of this unit may be seen by examining Figure 67 where a cubic vessel of 1 foot edge is shown. If this vessel is filled with water, the total weight of the water is 62.4 lb. (since the density of water in the English system is 62.4 lb. per cu. ft.).

In this case the area of the bottom of the vessel contains 144 sq. in., and hence the pressure on the bottom is

$$P = \frac{62.4}{144} = 0.433 \text{ lb. per sq. in.}$$

Obviously this is the weight of a tube of water of one square inch cross section and one foot in height.

For liquids of other densities the value of the weight for a similar tube may be found by multiplying this value by the specific gravity of the liquid. (From Chapter 3, p. 32, we know that specific gravity is the relative density of a substance as compared with that of water.)

Example. A container shaped as in Figure 66 has the main part of the vessel with bottom 8 × 8 inches and it is 6 inches high. The tube is 15 inches in height. Find the pressure and the total force on the bottom of the vessel when the entire vessel and tube are full of water.

The total height of the liquid in the vessel including that in the tube is

$$\text{Height} = 6 + 15 = 21 \text{ inches} = \frac{21}{12} \text{ ft.} = 1.75 \text{ ft.}$$

Hence the pressure P on the bottom of the vessel is

$$P = \text{height} \times d'$$

where d' is the weight of a tube of liquid of one square inch cross section and one foot in height. Hence

$$P = 1.75 \times 0.433 = 0.758 \text{ lb. per sq. in.}$$

The total force on the bottom of the vessel due to the liquid is

$$F = P \times \text{area} = 0.758 \times 8 \times 8 = 48.5 \text{ lb.}$$

When experiments such as those discussed above are performed under ordinary conditions in open vessels there is additional pressure in the liquid due to pressure from the earth's atmosphere. The above examples and discussions, as well as those that follow in this chapter, consider the pressures due to the weight of the liquid alone. Cases due to both atmospheric and liquid pressures will be considered in a later chapter.

3. Pressure on Side Walls

Since the pressure at any point in a liquid has the same value in all directions, we can compute the pressure at any point on a side wall of a vessel in the same manner as would

be used to determine the pressure on a horizontal plane at the same level. Obviously, the value of pressure on a side wall will vary from point to point as we consider higher or lower positions. Since it varies directly with the height of liquid above the point it is permissible to take the average value of the pressure if we wish to compute the total force on any given section of such a wall.

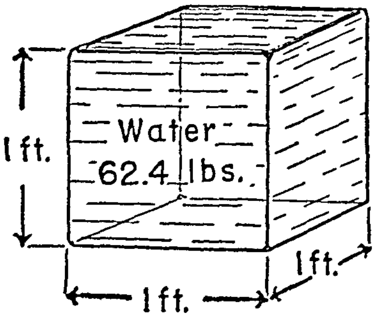


FIG. 67.—The total gravity force of one cubic foot of water is its weight, 62.4 lb.; and its pressure, therefore, 62.4 lb. per square foot.

Example. In Figure 67, if the vessel is full of water, the pressure at the bottom edge of any side wall due to the water is the same as that on the bottom of the vessel. For a water depth of 1 foot

$$P_b = 0.433 \text{ lb. per sq. in.}$$

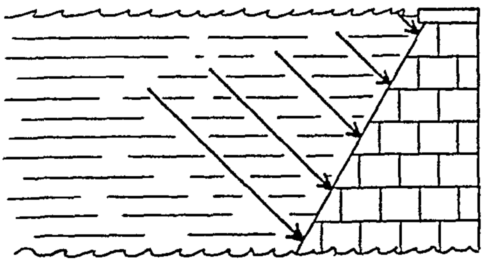


FIG. 68.—The total force on the wall equals the the area times the average pressure.

The pressure along the top edge of a side wall is zero. Hence, the average pressure, P, is

$$P = \frac{0.433 + 0}{2} = 0.2165 \text{ lb. per sq. in.}$$

The total force on a side wall is

$$F = P \times \text{area} = 0.2165 \times 12 \times 12 = 31.2 \text{ lb.}$$

The fact that a side wall may be other than vertical—see Figure 68—would not make any difference in the method of determining the average pressure.

4. Transmission of Pressure in a Liquid

In the case of the container with a tube as shown in Figure 66, the lower part of the container may be thought of as a closed vessel. There we saw that additional pressure was added to that already existing at any point in the main part of the vessel by the additional liquid in the tube. This experiment shows that in a closed system additional pressure introduced at any point is distributed to all other points in the liquid without loss. This conclusion was first stated by Pascal and is often referred to as *Pascal's principle.*

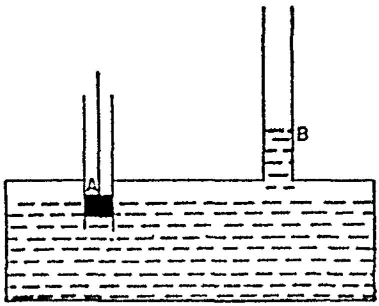


Fig. 69.—In a closed vessel, a force applied on the plunger A will cause a rise of fluid in tube B.

One application of Pascal's principle is in the special case illustrated in Figure 69. A force applied to the plunger *A* will result in a rise in the liquid in the tube *B*.

Experiments with different size plungers will show that for a given *total* force applied to the plunger, the water will rise higher for a small plunger than for a large one. This result is due to the fact that for a given *total* force, there will be a larger force *per unit area* in the case of the small plunger than with the larger plunger, and it is the force per unit area that determines pressure.

5. Force Pumps

The principle just described is used for the operation of a force pump for pumping water or other liquids. Such a pump is shown schematically in Figure 70. At the bottom of the cylinder is a valve A which closes when the piston is pushed downward. Water is forced through the check valve *B* and

into the vertical pipe or other system to which it may be attached. When the motion of the piston is reversed, the check valve *B* closes to prevent the return of the water and the valve in *A* opens to permit liquid to move into the cylinder of the pump.

Such pumps are used to fill reservoirs at levels high in comparison to the level of the source of water or other liquid. In particular, the system is useful for either a city water supply

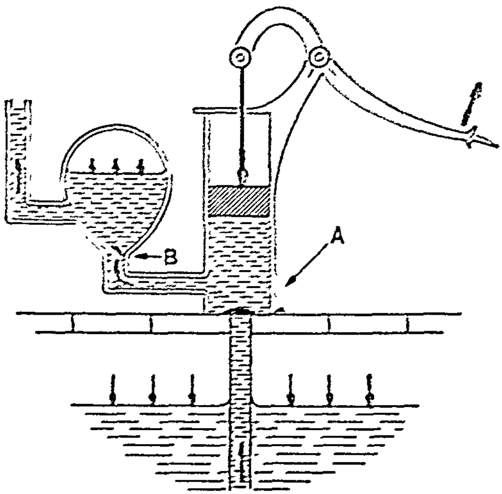


FIG. 70.—A force pump.

or the water supply for an individual building. Pipe lines are run from the high reservoir to the various outlets where it is desired to use the water.

Additional diagrams of arrangements of this type are to be found in texts on general science.

6. The Hydraulic Press

The hydraulic press (see Figure 71) is another adaptation of the fact that additional pressures at one point in a liquid in a closed vessel are distributed to all points in the liquid with undiminished magnitude.

The arrangement of the pump part of the device is similar to that of the force pump just described. The piston is

usually small in area so that a small total force will result in a relatively large pressure. The piston *A* is moved up and down, thereby pumping oil from the reservoir *C* through the check valve *B* into the cylinder *D*.

The additional pressure at *A* is distributed undiminished to the bottom surface of the piston *E*. The total force on *E* is much greater than that on *A* since the area of the surface *E* is greater than that at *A*, and of course the total force on *E* is equal to the product of the pressure and the area.

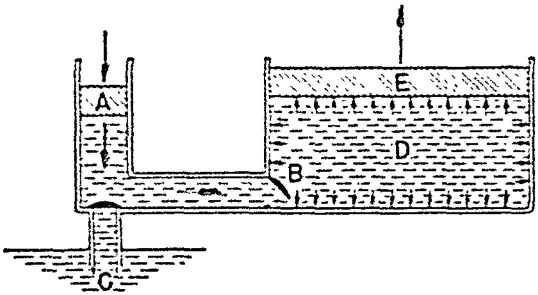


FIG. 71.—The hydraulic press may be considered as a machine whose mechanical advantage is equal to the ratio of the areas of the respective pistons.

The hydraulic press may be considered as a machine whose mechanical advantage is equal to the ratio of the areas of the respective pistons.

A common use to which such an arrangement is put is that of a barber's chair. The small piston is attached to a lever so that the whole device is really a compound machine in which mechanical advantage is obtained both from the lever and from the hydraulic press.

The hydraulic press is also used as a jack for lifting extremely heavy objects and is also employed where tremendous forces are required for compressing materials.

7. Buoyant Effect of a Liquid

When a body is immersed in a liquid it appears to lose weight. This phenomenon is a matter of common experience, and may be noted by anyone in such a simple case as swim-

ming. In fact, most people lose so much weight in water that they float. This loss of weight in a liquid is called buoyancy. It is more noticeable in heavy liquids and less noticeable in less dense liquids.

If a body of regular shape, so that it can be easily measured, is weighed in air and also when suspended in water or other liquids, it will be found that the loss in weight is always equal to the weight of a volume of liquid just equal to the volume of the body. Of course there is nothing unique about using an object of regular shape except to make the measurements easy; so we arrive at the conclusion that the buoyancy effect on any object immersed in any liquid is just equal to the weight of the displaced liquid.

The statement of this experimental fact was first given by Archimedes who also presented theoretical reasons as to why it should be true. It was used by him for the determination of the density of a crown belonging to Hiero, and is still a commonly used method for determining the densities of irregularly shaped objects.

8. Density Determinations by the Method of Archimedes

In Figure 72 is indicated the weighing of an object when immersed in a liquid of density d_1. The true weight of the object is indicated by W_t and the upward force due to the buoyancy effect by B. The observed weight W must be the difference between these two; that is

$$W = W_t - B \tag{1}$$

W_t may easily be found by weighing the object in air and so B can be determined by solving equation (1) for B

$$B = W_t - W \tag{2}$$

Since B is the weight of the displaced liquid and hence numerically equal to the mass of the displaced liquid we can write

$$B = Vd_1 \tag{3}$$

where V is the volume of the displaced liquid, and, of course, it is also the volume of the immersed object.

If the density d_1 of the liquid is known we can at once determine V, since

$$V = \frac{B}{d_1} \tag{4}$$

The density of the material d_m of the object immersed can now be determined since both the mass (equal numerically to W_t) and volume are known; for

$$d_m = \frac{W_t}{V}$$

The student should notice that it is the volume of the displaced fluid and the density of the fluid that determine the

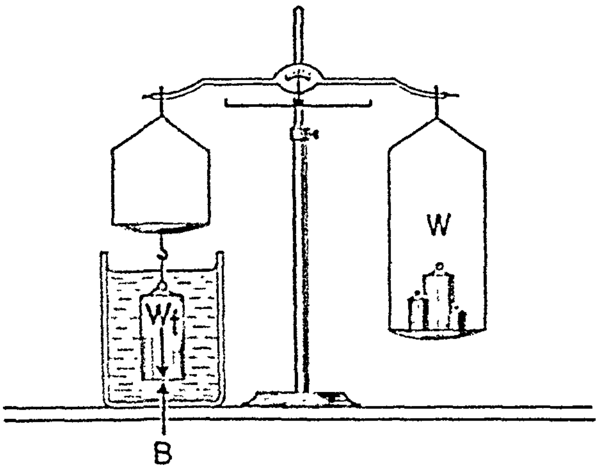


FIG. 72.—Buoyancy on object immersed in a liquid. The true weight of the object is indicated by W_t and the upward force due to the buoyancy effect by B. The observed weight W must be the difference between these two.

buoyancy—not the density of the immersed object. If a piece of iron and a piece of wood of the same size are both submerged in the same liquid the buoyancy effect will be the same on the iron as it is on the wood. Of course, the buoyancy lift may be great enough to float the wood while it may not come near to counteracting the weight of the iron.

Example. An irregularly shaped piece of brass is found to weigh 85 g. in air and 75 g. in water. What is the density of the brass?

From the above development we have that the volume of the displaced liquid and hence of the brass is,

$$V = \frac{B}{d_1} = \frac{(85 - 75)}{1} = 10 \text{ cc.}$$

Hence the density of the brass is

$$d = \frac{W_t}{V} = \frac{85}{10} = 8.5 \text{ g. per cc.}$$

Example. The brass object in the previous example is found to weigh 76 g. when suspended in oil. Find the density of the oil.

Since the density, volume and mass of any substance are related by the equation

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

$$d = \frac{m}{v}$$

and since the volume of the displaced oil is 10 cm. and the mass of displaced oil is numerically equal to the loss in weight of the brass when weighed in oil as compared to air we may write

$$d_{\text{oil}} = \frac{(85 - 76)}{10} = 0.9 \text{ g. per cc.}$$

9. Flotation

If an object is less dense than the liquid in which it is immersed the buoyant effect will be greater than the weight of the object, and if a force used to push the object beneath the surface of the liquid is released, the object will rise so that part of it is above the level of the surface. A sufficient part of the object will remain below this level so that there will be displaced a weight of the liquid just equal to the total weight of the object.

Example. If the density of cork is 0.25 g. per cc. find the fraction of the volume of a cork block that will float above the surface of the water.

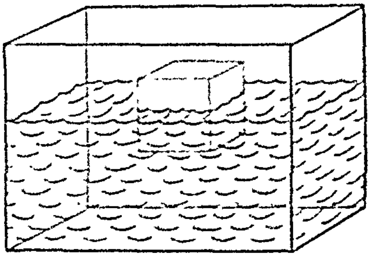


FIG. 73.—An object less dense than a liquid will float with just enough of itself below the liquid surface to displace a weight of liquid equal to the total weight of the object.

If the volume of the cork is V, its weight, W_c, will be

$$W_c = Vd_c = V \times 0.25$$

The weight of the water displaced must have this same value in order that the cork may float, and if v is the volume of water displaced and d the density of the water we have

$$V \times 0.25 = vd$$

Hence the volume of water displaced (and that of the cork which is submerged) is

$$v = \frac{V \times 0.25}{d}$$

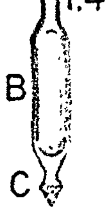


FIG. 74.—Hydrometer for measuring the density or specific gravity of liquids.

And since the density of water is approximately unity

$$v = 0.25V$$

The part of the cork floating above the water must be

$$V - v = V - 0.25V = 0.75V$$

10. Hydrometers

The principle of flotation may be applied to a device for the measurement of the density of liquids by calibrating the amount of the device that will float above the surface for liquids of various densities. Such a device is called a hydrometer and is indicated in the diagram of Figure 74.

The tube is usually divided into three sections. A is a small tube containing a scale reading densities directly, the

point of reading being the place on the scale that is just at the surface of the liquid. Section *B* is relatively large and since it is filled with nothing but air, it is light. Section *C* is usually weighted to give greater stability to the instrument.

The weight and volume of the entire object must be such that it will just float in the lightest liquid whose density is to be determined and such that it will float with practically all of the tube *A* above the surface in the liquid of greatest density to be measured.

Some Important Facts

1. Matter may exist in one of three states—solid, liquid, gaseous. Solids have definite volume and shape; liquids, definite volume only; gases have neither.

2. Pressure—force per unit area—below the surface of a liquid, due to the liquid, depends on the depth and density of the liquid, regardless of the size or shape of the container.

$$P = hd$$

The total force on any area is the product of area and pressure.

$$F = ap = ahd$$

The total force exerted by a liquid on the bottom of an open container equals its weight only in case the container has vertical sides. If the sides flare out at the top, the total force on the bottom of the container is less than the weight; if they flare in at the top, more than the weight.

3. The average liquid pressure on side walls of an open container is the average depth times the density of the liquid.

The total force on a side wall is its area times the average pressure.

4. Liquids transmit gravity or outside pressure freely, without loss and normal to all surfaces contacted, provided the motion of the liquid is small enough to be neglected.

5. In force pumps, the pressure transmitted by liquids is supplied by a moving piston, valves preventing a back flow of liquid on the piston's recovery stroke.

6. The hydraulic press is essentially a force pump in which pressure exerted by a small piston is transmitted to a piston of relatively very large area. Since force equals area times pressure, the force on the large piston is greater than on the small piston, the two forces having the same ratios as the piston areas.

7. When a body is immersed in a fluid it experiences an apparent loss of weight equal to the weight of fluid that it displaces.

8. The volume of a submerged body equals its buoyancy, divided by the density of the liquid.

$$V = \frac{B}{d_1}$$

from which its density may be readily found by the relationship,

$$dm = \frac{W_1}{V}$$

9. When a body is less dense than a liquid, it sinks only to the point where it displaces its own weight of liquid. Therefore, the submerged volume of the floating body and the volume of displaced liquid vary inversely as the density of the liquid.

10. Since the submerged part of the volume of a floating body varies inversely as the density of the liquid, bodies designed to float in a vertical position may be calibrated to read liquid densities.

Generalizations

Pressure in a liquid is developed as a result of the weight of the liquid and is proportional to the density of the liquid and the depth below the surface. Pressure may also exist in a liquid due to the application of external forces such as the weight of the atmosphere on an open liquid surface, or force exerted by a piston on liquid in a closed container. These added pressures are transmitted to all parts of the liquid. Pressures at any point in a liquid due to any cause act perpendicularly to any surface at the location.

Questions and Problems

Group A

1. What are the two general sources of fluid pressure?
2. What two factors determine the gravity pressure of a liquid?
3. Of what factors is gravity pressure independent?
4. Distinguish the terms pressure and total force.
5. State Pascal's Principle, and mention three general truths which might be considered as corollaries deduced from it.
6. Find the pressure at the bottom of a vessel filled with water to a depth of 1.5 feet. Find the total force on the bottom if it is 8 × 6 inches.
   0.65 lb. per sq. in. 31.2 lb.
7. Water in a tank is 4 feet deep. Find the pressure on the bottom of the tank. Find the force on the bottom if it is 10 × 12 feet in area.
   P—250 lb./ft.^{2} *F*—30,000 lb.
8. If you dive 8 feet below the surface of fresh water, what gravity pressure do you sustain in pounds per sq. ft.; in pounds per sq. in.? (Density of fresh water 62.4 lb. per cu. ft.) Is this pressure very noticeable?
   500 lb./ft.^{2}; 3.47 lb./in.^{2}

9. How high must the water be in a standpipe to furnish a pressure of 31.25 lb. per sq. in.? (Density of water 62.4 lb. per cu. ft.) 72.2 ft.

10. A goldfish aquarium is 20 cm. by 40 cm. by 30 cm. high. It is filled with water to within 3 cm. of the top. Find the pressure and total force on the bottom of the vessel. Find the total force on each side wall.
Pb—27 g./cm.^{2} *F*—21,600 g.— 14,580 g.—7290 g.

11. A piece of aluminum weighing 54 g. is found by measurement to have a volume of 20 cc. What will it weigh when suspended in water?
34 g.

12. If a block of ice floats with 0.1 of its volume above the surface of the water, what is the density of the ice? 0.9 g./cc.

13. (*a*) On the stem of a hydrometer are calibrations for specific gravities of 0.8, 0.9, and 1.0. Make a labeled diagram to show the hydrometer with the above calibrations.

(*b*) Describe briefly another method of determining the specific gravity of a liquid.

14. A block of wood floats with 0.5 of its volume above the surface of the water. It is placed in oil and found to float with 0.1 of its volume above the surface. Find the density of the oil. .55 g./cc.

Group B

1. A cylindrical tank of radius 5 feet stands on end and is filled with water to a height of 12 feet. Find the pressure and the total force on the bottom of the tank. 5.20 lb. per sq. in.; 749 lb./ft.^{2}; 58,826 lb.

2. A tank is 4 × 4 by 8 feet high. Find the pressure at the bottom and the total force on the bottom of the tank if it is filled with oil of specific gravity 0.92. 3.19 lb. per sq. in.; 459 lb./ft.^{2}; 7344 lb.

3. A V-shaped trough with vertical ends has sides 1 foot in width and the open top of the trough has a width of 1 foot. The trough is 5 feet long. Find the total force on each side when the trough is full of water.
135 lb.

4. A large reservoir is located on a tower and water is fed into it through a vertical pipe that enters the bottom of the tank. Find the pressure at the bottom of the pipe when the pipe has a length of 85 feet and the water in the tank is 10 feet deep. (Make a diagram of this arrangement.)
5,928 lb./ft.^{2}

5. A force pump is used to pump water into the tank of problem 4. If the maximum force that can be applied to the piston in the pump is 300 lb. how large can the area of the piston be? 7.29 sq. in.

6. The small piston in a hydraulic press has a diameter of 0.5 inch and the large piston a diameter of 12 inches. Find the expected mechanical advantage. 576.

7. If a force of 50 lb. is applied to the small piston, what pressure and what total force will act on the large piston of problem 6?
255 lb. per sq. in. 28,800 lb.

8. The movable part of a barber's chair and the customer sitting in it weigh 300 lb. The diameter of the piston attached to the chair is 5 times that of the piston in the pump. What force must be applied to the piston to move the chair and the customer? 12 lb.

9. Make a diagram of a lever arranged to operate the piston in the above problem so that, neglecting friction, a force of 5 lb. will be sufficient to operate the chair. M.A. 2.4.

10. The small piston of a hydraulic press has a diameter of one inch; the large piston, a diameter of 20 inches. What is (*a*) the area ratio of the two pistons? (*b*) their pressure ratio? (*c*) their total force ratio? (*d*) their distance of travel ratio? (*a*) $\frac{1}{400}$; (*b*) 1/1; (*c*) $\frac{1}{400}$; (*d*) $\frac{400}{1}$.

11. A service station lift utilizes compressed air at a pressure of 40 lb. per sq. in., as the force applied to the small piston. What must be the diameter of the large piston in order to lift a maximum load of four tons? 16 in.

12. A piece of iron weighing 500 grams in air is found to weigh 435.8 g. when suspended in water. What is the density of the iron? 7.79 g./cc.

13. A piece of brass known to have density of 8.5 g. per cc. is found to weigh 90 g. in air and 79.42 g. when suspended in water. What is the volume of the brass? Do you need all of the data given? 10.58 cc.

14. A piece of aluminum weighing 54 grams and having a volume of 20 cc. is found to weigh 40 g. in a liquid of unknown density. Find the density of the liquid. 0.7 g./cc.

15. A piece of metal weighs 90 g. in air, 72 g. in water and 78 g. in a liquid of unknown density. Find the density of the metal and the density of the liquid. 5 g./cc.; 0.66$\frac{2}{3}$ g./cc.

16. A piece of metal weighs 480 g. in air, 420 g. in water and 432 g. in an unknown liquid. Find the specific gravity of the metal and the specific gravity of the unknown liquid. What might the metal be? The liquid? (*a*) 8; (*b*) 0.8.

17. How much volume would be occupied by 1 lb. of cork of specific gravity 0.25? How much buoyant force would act on it when completely submerged in water? 0.064 cu. ft.; 4 lb.

18. A man weighing 200 lb. finds that his density is just equal to that of water. He wishes to make a life preserver out of cork and of sufficient dimensions so that 10 per cent of his body will float out of the water. Neglect the other material of which the life preserver is to be made and compute the amount of cork required. 0.427 cu. ft.

19. A bronze ball, which has a specific gravity of 8 and weighs 72 g., is floated in mercury, specific gravity 13.6. Then water is poured on to cover the top of the ball.

Find: (*a*) The volume of the ball in cc. 9 cc.

(*b*) The volume of mercury displaced before pouring on the water. 5.3 cc.

(*c*) The volume of the mercury displaced after pouring on the water. 5 cc.

(*d*) The volume of water displaced. 4 cc.

Experimental Problems

1. Put a few lead shot in the bottom of a long test tube so that it will float upright. Mark the water line.

Add weight to the tube, about 10 grams at a time, and mark the water line after each weight addition, continuing until the top of the tube is near the water level. Compare the distances between water lines with the corresponding increases in weight. What conclusions can you draw about the relation between pressure and depth in a liquid?

2. For each of several hydraulic devices—such as barber's chairs, freight elevators, service station lifts, etc.—find out the pressure used and how it is applied. Also, determine the approximate area of the large piston and calculate the total force exerted by it.

3. Weigh several samples of insoluble solids in air, then in water in an overflow can. Catch and weigh the water displaced in each case, and also determine the apparent loss of weight in each case. How do these two quantities compare?

Calculate the specific gravity of each solid, and compare your results with accepted values in a table of physical constants. What are some sources of possible error in your experiment?

Float several solids in water in an overflow can, catching and weighing the displaced water. Weigh each solid and compare weights with water displacements.

Chapter 10

BEHAVIOR OF GASES

This chapter carries the subject of hydrostatics from the liquid case discussed in the preceding chapter to the case of gases.

We first consider the nature of pressure in a gas and find that the pressure is due to the impacts of the moving molecules which compose the gas. The variation of gaseous pressure with altitude is discussed especially in relation to the earth's atmosphere.

Problems involving both liquid and gaseous pressures are described and the principles found to apply here may be used for the construction of pressure gauges. A gauge constructed especially to measure the atmospheric pressure is called a barometer.

A special type of pressure gauge, called an aneroid barometer, is shown to be free from some of the disadvantages of the liquid type of gauges. Its application to determining heights, especially for aerial navigation, is described.

Buoyant effects in gases similar to those found in liquids exist, and the particular application of this effect to the case of balloons receives considerable attention.

The behavior of relative pressures and volumes in a fixed quantity of a gas is described and is summarized as Boyle's Law.

The chapter closes with some problems—air pumps, lift pumps, siphons—that are of a nature involving the science of moving as well as static fluids.

1. Pressure in a Gas

As stated in the previous chapter the chief characteristics of material in the gaseous state as compared to the liquid or solid states is first the tendency of the gas to diffuse without limit and to fill all the available space, and second the readiness with which it may be compressed.

The molecules of a gas move to and fro, bumping into one another and against any solid surfaces that may be in the neighborhood. They rebound from each other or from the surfaces on which they strike and this change in momentum of the molecules implies a force of the molecule against a sur-

face, and force per unit area on a surface resulting from such a molecular bombardment constitutes a pressure.

The gas with which we are most familiar is the air that constitutes the earth's atmosphere. It is a mixture of several gases chief of which are oxygen and nitrogen. Traces of argon and helium as well as varying quantities of carbon dioxide and water vapor are also to be found.

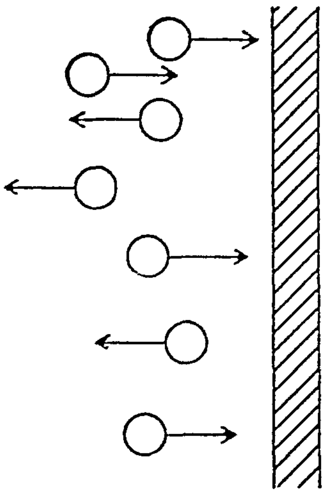


FIG. 75.—The bombardment of a surface by molecules in a gas causes pressure against the surface. For simplicity only horizontal motions of the molecules are indicated here.

2. Atmospheric Pressure

Let us imagine a tube extending straight up from the surface of the earth for many miles. All of the air molecules in this tube will be bumping into one another and also into the side walls of the tube. The latter action does not give them any support against their own weight. The former results in upward motion for some of the molecules and downward motion for the others. However, any layer of molecules may be thought of as having to hold up the layers above it against the force of gravity.

The result of this state of affairs is that the molecules near the bottom of the tube will be pretty much crowded together in comparison to those at the top of the tube. In other words, there are more molecules in a given volume near the earth than there are higher up in the earth's atmosphere. Or we may say the density of the earth's atmosphere is greatest near the earth and falls off to zero at sufficiently great heights. At a height of 3.5 miles above sea level the atmospheric pressure is down to approximately one-half of its value at sea level, at five miles it is about 35 per cent, and at 10 miles about 10 per cent of the sea level value.

The air molecules at the bottom of our tube in the above discussion must bombard the earth (or the bottom of the tube

if it is closed) with a force equal on the average to the weight of the whole column of air that is above it. The force per unit area is called the atmospheric pressure at the level at which it is measured. As in the case of liquids, the pressure at any one point has the same value in all directions.

The actual value of the atmospheric pressure at any given location varies from time to time due to air currents and other

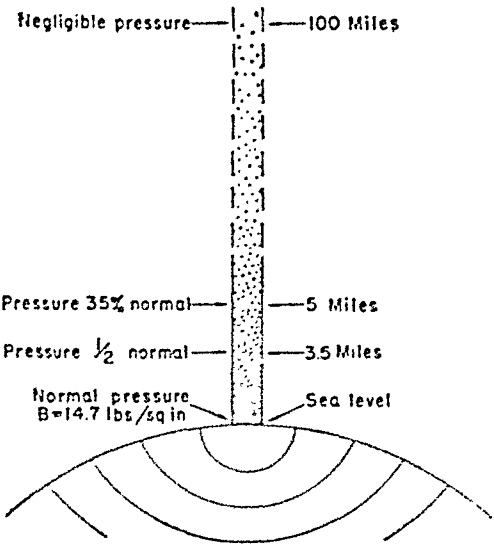


FIG. 76.—The density of the atmosphere and the atmospheric pressure fall off rapidly with altitude.

atmospheric disturbances. A somewhat arbitrary value for the normal atmospheric pressure at sea level has been adopted. It is 1034 g. per sq. cm. or 14.7 lb. per sq. in.

3. Atmospheric and Liquid Pressures

Suppose that a long tube is filled with a heavy liquid such as mercury and then inserted mouth down in a vessel containing a quantity of the same liquid. (See Figure 77.) It will be found that the mercury begins to flow from the tube into the vessel but stops at some height, *h*. The space between

the top of the mercury and the top of the tube is virtually a vacuum except for a slight amount of mercury vapor.

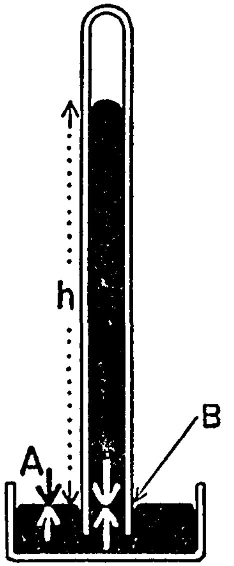


FIG. 77.—Mercury column supported by atmospheric pressure by transmission of the pressure through the liquid.

Let us now consider two points at the level of the surface of the mercury in the main part of the vessel. At A the earth's atmosphere exerts a downward pressure, say P_1. The liquid must exert an equal and opposite upward pressure. Since the point B is at the same level, the pressure in any direction at B must be the same as that at A. But the downward pressure at B is due to the weight per sq. cm. cross section of the column of mercury in the tube. In other words, the mercury is kept in the tube because of the atmospheric pressure on the surface of the liquid in the container. This pressure is communicated to the point B in the closed tube and can exert an upward force to balance the downward force that is the weight of the liquid in the tube.

4. Mercury Barometer

The above facts suggest a device for measuring the atmospheric pressure, for the weight per sq. cm. of the liquid column is equal to the atmospheric pressure. Since the density of mercury at 0° C. is 13.6 g. per cc., the student can readily show that normal atmospheric pressure as defined above will result in a column of mercury standing 76 cm. high.

An arrangement of this type is called a mercury barometer. Any other liquid may be used, but with relatively light liquids inconveniently long columns would result. For example, a water column under the above conditions would be 1034 cm. high (33.9 ft.).

Mercury columns are very extensively used for the measurement of the atmospheric pressure, since the height of the

mercury column above the level of the mercury in the lower vessel may be read directly and accurately. This type of measurement is so common that pressures are often given in terms of centimeters of mercury, although the actual dimensions of pressure are force per unit area.

5. Aneroid Barometer

The chief disadvantage of the mercury barometer is its size and the fact that it needs to be mounted with the tube in a vertical position before it can be read. These objections are overcome in an instrument called an aneroid barometer.

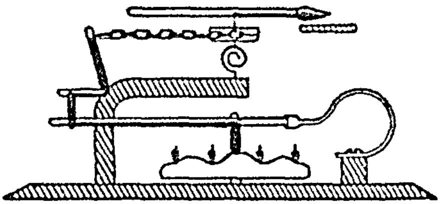


FIG. 78.—Simplified diagram of the aneroid barometer. Short red arrows indicate atmospheric pressure; the solid red line indicates transmission of force through a lever system.

The operation of this device is indicated in the drawing of Figure 78.

A small metal box is built with substantial bottom and sides but a flexible top. If the air is pumped out of this box the top will tend to collapse due to the atmospheric pressure on the outer side and the lack of gaseous pressure on the inside. It may, however, be supported by a coiled spring inside the box or by some external arrangement of springs.

When a change of pressure in the air outside the box occurs the spring will be compressed or it will expand.

An increase of pressure in the air outside the box will cause some depression of the top of the box whereas a decrease in outside air pressure will allow the spring to expand and raise the top of the box. *The extent of this motion can be amplified* by means of a lever so pivoted that the short arm is connected

to the top of the box through a drive rod. The end of the longer arm of the lever then moves through greater distances than the top of the box.

Still further amplification of this motion may be transmitted to a pointer by the arrangement shown in Figure 78. A string connected to the long end of the lever is wound around an axle to which is fastened the pointer. The string is held taut by a spring as shown. A small motion of the string results in a large angular motion of the axle to which the pointer is attached.

The scale may be calibrated to read pressure in pounds per square inch, grams per square centimeter, centimeters of a mercury column, or other units.

This type of gauge is readily portable, will work in any position and can be made very sensitive. Differences in atmospheric pressure at small changes in height, such as on two consecutive floors of a building, are readily shown. This fact suggests the possible use of this form of barometer as an altitude indicator.

In this capacity the aneroid barometer has found almost universal application in the field of aviation. The meter may be calibrated to read in terms of feet above sea level or in feet above any other arbitrarily chosen reference level.

The obvious defect of a barometer for use as an altimeter lies in the fact that changes in atmospheric pressure due to varying weather conditions will change its readings and hence make the calibration in terms of altitude inaccurate. On established air lines pilots are kept informed of atmospheric changes by radio signals from ground stations. This enables the pilot to allow for the variation or to readjust his altimeter.

6. Extent of the Earth's Atmosphere

In an earlier section in this chapter we discussed the weight per square centimeter or per square inch on the earth's surface of all the atmosphere above. The question might be raised as to how high the atmosphere extends.

There is no definite answer to this query because, since the atmosphere is a gas which is made up of molecules that do

by simply saying (see page 126), "The apparent loss in weight of any object immersed in a *fluid* is just equal to the weight of the displaced fluid."

8. Flotation in a Gas

In the case of liquids, it is a simple matter to find solid objects with densities of the same order of magnitude as those of the liquids. Hence the change in the apparent weight and even the floating of objects in water or other liquids is common experience.

Similarly striking effects can be expected with gases only if we can find objects with densities of about the same value as those of the gases. If a small flask such as is commonly used in chemical experiments is fitted with a good stopcock, it is possible to pump the air from the flask and keep it out. The flask can be weighed then with no air inside and again when the stopcock has been opened. The change in weight is readily measured.

If it were possible to make a flask of sufficiently light material, the buoyancy effect might exceed the weight of the flask just as the buoyancy effect in water exceeds the weight of a piece of cork; but it does not seem possible to make such a flask and still have it strong enough to withstand the air pressure on the outside without collapsing.

This difficulty can be met by filling the vessel with a gas whose molecules are lighter than those of air. They will provide gas pressure on the inside of the container which will balance the atmospheric pressure on the outside. Under these conditions the walls of the container may be made of very light material.

This is the technical description of the device that we call a balloon. Rubber, or rubber fabric is commonly used for the bag and either hydrogen or helium for the inflating gas. The density of air at normal pressure and a temperature of 0° C. is 0.001293 g. per cc. and that of hydrogen 0.0000898 g. per cc. The first of these is then the buoyant effect of the atmosphere on a cubic centimeter of hydrogen, and since the hydrogen weighs less than the buoyant effect, it tends

to float. The difference in the two weights (0.001203 g.) is the excess upward force on each cubic centimeter of hydrogen provided that the hydrogen and the air are at normal pressure and temperature.

This excess buoyant effect on the gas in a balloon may be sufficient not only to lift the weight of the material of which the bag is made, but may also carry a useful load.

The density of helium under standard conditions as discussed above is 0.0001769 g. per cc. so that the excess lift on helium in air is 0.001116 g. per cc., a bit less than the .001203 g. per cc. available with hydrogen. The only reason for sometimes using helium is that it does away with the fire hazard attendant on using hydrogen. On the other hand, the expense of obtaining helium is rather prohibitive.

As a balloon gains altitude it moves to regions where the air is less and less dense. The maximum height to which it can go will be at a level where the buoyant effect is reduced to a point such that it just equals the combined weight of the gas, the bag, and the useful load of the balloon.

Balloons usually carry ballast of either sand, powdered lead, or water. This ballast may be dumped when the navigator wishes to gain a greater height. The balloon also has gas valves so that gas may be permitted to flow out of the bag when a descent is desired.

Balloons may also be equipped with engines and propellers after the manner of airplanes, so that greater flexibility of motion may be obtained.

Example. Compute the amount of useful load that a spherical balloon may carry if the rubber bag is 150 cm. in diameter, weighs 750 g. and is filled with hydrogen. Make the computations for normal atmospheric pressure and temperature.

The volume of the bag is

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(75)^3 = 1{,}768{,}000 \text{ cc. approx.}$$

Since the excess lift per cubic centimeter was found above

to be 0.001203 g. in the case of hydrogen, the total excess lift is

$$1{,}768{,}000 \times 0.001203 = 2{,}126 \text{ g.}$$

Of this amount the bag weighs 750 g., leaving

$$2{,}126 - 750 = 1{,}376 \text{ g. for equipment.}$$

9. Pressure and Volume Relations in a Gas

We have seen that pressure in a gas is due to the change of momentum of the molecules as they rebound from one another or from a surface. It is therefore reasonable to expect that if the number of molecules of a gas in a given container is increased the pressure will increase also and it will increase in proportion to the relative increase in the number of molecules in the space.

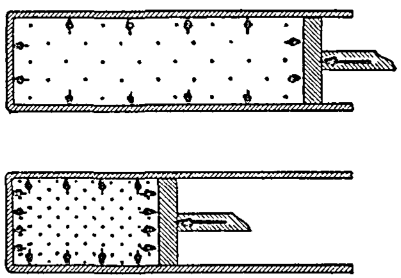


FIG. 79.—Expansion or compression of a gas. If there is gas in the cylinder as shown, and the gas is pushed into one half the volume originally occupied, it will be found on measurement that the pressure of the gas has doubled.

These predictions are borne out by experience. For example if there is gas in a cylinder as shown in Figure 79 and the gas is pushed into one-half the volume originally occupied, it will be found on measurement that the pressure of the gas has doubled. This effect can be summarized in the statement,

"At constant temperature the product of the pressure and volume of a given quantity of a gas remains constant."
In symbols this statement becomes

$$P_1V_1 = P_2V_2$$

where P_1 and V_1 are the pressure and volume existing at one instant, and P_2 and V_2 the pressure and volume existing at another time for the same quantity of the same gas.

This expression may also be written

$$PV = C$$

where P is the pressure and V the volume of a quantity of gas at any instant and C is a constant remaining always the same while the pressure and the volume of the gas vary from time to time.

This description of the behavior of gases at constant temperature is called *Boyle's Law*.

From this discussion it may be seen that the density of a gas at constant temperature varies directly as the pressure.

In a later section on heat we shall see that when the temperature changes, the motion of the molecules changes, so that pressure changes may then occur without the necessity of volume or density changes. In all of the material in this section, as well as that in this entire chapter, it is assumed that the temperature of the gas remains constant. If the temperature changes, either or both the pressure or volume of a given quantity of gas changes also. These temperature effects on gases are discussed in Sections 8, 9, 10, and 11 of Chapter 16.

10. Gas Pumps

A pump of the same general design as the force pumps used for liquids (see page 123) may be used for pumping gas through a line or into a container. If the gas is pumped into such a container (usually called a tank) at moderately high pressure, the pump is often called a compressor. Combinations of pump and tank of this type are used to supply air for filling tires at automobile service stations, for operating elevator doors and for many other purposes.

If a pump such as just described is connected at its input to a closed vessel it will tend to exhaust the gas from that vessel. When a pump is so used it is often called a vacuum pump. The name is something of a misnomer, since it is gas and not vacuum that is pumped.

Where large quantities of air are to be moved to or from a place, and where the pressures involved are small, fans may be used to effect the transfer. The fans may be of a simple

height less than the height of the liquid column that can be supported as a result of the atmospheric pressure, the liquid will pass through the pump so that the pump, although first acting as an air pump, is now functioning as a liquid pump.

Pumps of this type are commonly used in rural communities where good supplies of water for household purposes are often found at levels of less than the critical distance below the surface of the earth to which the water must be raised. For deeper wells, force pumps are used and the pumping mechanism is located at the necessary distance down the well.

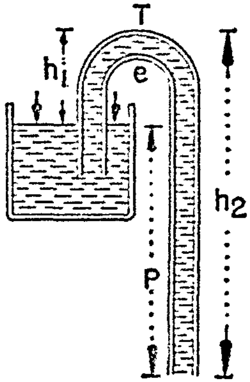


FIG. 81.—The siphon. Since the end of the tube is open to the atmosphere, the pressure at *e* toward the left will be equal to the atmospheric pressure minus that due to the weight of the liquid in a tube of height h_2. Pressure to the right at the same point *e* will be equal to the atmospheric pressure on the surface of the liquid in the vessel minus that due to the liquid in a tube of height h_1. P indicates pressure difference.

12. Siphon

Suppose that a bent tube *T* is first filled with liquid and then immersed as shown in Figure 81 with its longer end at a level below the level of the liquid in the main part of the container. Since the end of the tube is open to the atmosphere, the pressure at *e* toward the left will be equal to the atmospheric pressure minus that due to the weight of the liquid in a tube of height h_2. Pressure to the right at the same point *e* will be equal to the atmospheric pressure on the surface of the liquid in the vessel minus that due to the liquid in a tube of height h_1.

It follows that so long as h_2 is greater than h_1 the pressure at *e* toward the right will be greater than that toward the left and a flow of liquid from left to right will result.

It is seen that the action of a siphon is not a simple problem in hydrostatics, but also involves the motion of fluids.

The action of pumps could also be classified in the same manner.

A siphon has no value where the height of *e* above the level of the surface of the liquid is greater than the height of liquid that can be supported by atmospheric pressure. For water we have seen that this is about 33.9 feet.

Some Important Facts

1. Since gases diffuse without limit, their densities and pressures vary widely.

2. The density of the atmosphere decreases as altitude above sea level increases and pressure decreases as altitude increases. At standard sea level the pressure is about 14.7 lb. per sq. in. or 1034 g. per sq. cm.

3. Atmospheric pressure may be balanced against the gravity pressure of a liquid column. Standard sea level pressure is equivalent to that of a water column about 34 feet high or a mercury column about 30 inches or 76 centimeters high. The mercury barometer functions on this principle.

4. In the aneroid barometer, a flat corrugated, evacuated metal box, kept from collapsing by a coiled spring, changes volume inversely as the atmospheric pressure. These volume changes are multiplied and communicated by a compound lever system to a registering pointer, which moves over a dial calibrated in inches or centimeters of mercury, pounds per square inch, feet of altitude, etc.

5. Although the atmosphere may be regarded as extending out from the earth indefinitely, its pressure and density fall off rapidly with altitude. About half of the atmosphere's mass is within 3 to 4 miles of sea level, and at an elevation of about 50 miles the pressure is no more than that of the fairly high vacuum used in neon signs.

6. Since gases have weight and exert gravity pressure, they also exert a buoyant effect on bodies immersed in them. If the overall density of an immersed body is less than that of the gas, it is buoyed up enough to float. Balloons filled with helium or hydrogen are able to displace their own weight of air and so float in the atmosphere.

7. For any given quantity of gas, the product of its pressure and volume is constant at constant temperature.

8. Piston pumps, similar in principle to liquid force pumps, are often used to exhaust gases from, or compress them into, closed containers. Rotary or fan type pumps are also much used, especially for large quantities of gas at relatively low pressure gradients.

9. In the lift pump, atmospheric pressure pushes liquid up an intake pipe—to a possible maximum height of 33.9 feet for water—and through a valve at the bottom of a cylinder. As a piston descends, the cylinder valve is forced shut, but one in the piston opens to allow liquid to pass

through. As the piston ascends, its valve closes and it lifts some liquid which runs out of a spout due to gravity. During this upstroke atmospheric pressure pushes more liquid up into the cylinder and the cycle is repeated.

10. Liquid flows from one end of a siphon due to its own gravity pressure, the pressure head being the vertical distance of the liquid surface above this outlet end. Liquid is pushed into the other end by atmospheric pressure to a possible maximum height of 33.9 feet for water.

Generalizations

Pressure in a gas is due to bombardment of any surface by the atoms or molecules of the gas. The greater the concentration of the molecules and the greater their velocity, the greater the pressure. In general, the principles and applications of pressure in gases are similar to those found to hold for liquids.

Questions and Problems

Group A

1. What important physical difference exists between gases and liquids?

2. Why is the formula, $P = hd$, of little use in dealing with atmospheric pressures?

3. Express standard sea level atmospheric pressure in as many ways as you can.

4. Illustrate the action of the mercury barometer by a labeled diagram.

5. Explain how the aneroid barometer works.

6. How does the barometric pressure vary with the altitude?

7. How does the barometric pressure vary with changing weather conditions?

8. How does the volume of a confined gas vary with the pressure exerted?

9. Why are you not conscious of air pressure on your body?

10. Why does external pressure on the chest become noticeable when you take a deep dive in water?

11. A mercury barometer reads 75.8 cm. at 8:00 A.M., 75.2 cm. at 10:00 A.M. and 74.4 at 12:00 noon. What kind of atmospheric disturbance would you anticipate? Why?

12. Find the height of liquid that can be supported in a tube by atmospheric pressure for the cases of glycerin (density 1.25 g. per cc.), olive oil (density 0.91 g. per cc.). 27.2 ft.; 37.36 ft.

13. An aviator uses a barometer type altimeter, as he flies from one city to another. He wishes to keep 1000 feet above the ground at all points. His altimeter is correctly adjusted for the level of the airport he is leaving.

What two other things does he need to know in order to make the flight at the planned elevation above the ground?

14. Find the useful load that a spherical balloon may carry if the rubber bag is 150 cm. in diameter, weighs 750 g. and is filled with helium. Make the computations for normal pressure conditions at sea level. 1223 g.

15. Why will a commercial lift pump in actual practice not lift a liquid as high as the ideal predictions indicate?

16. (a) Explain the cause of the flow of a liquid through a siphon. Use a labeled diagram to help make your explanation clear.

(b) Can water be siphoned over an elevation of 40 ft.? Explain.

Group B

1. A small balloon filled with hydrogen at normal atmospheric pressure is found to be just capable of floating. If the balloon has a diameter of 30 cm., find the amount that the material of the balloon must weigh. (Suggestion: The material of the balloon must have a weight just equal to the excess buoyant effect.) 16.97 g.

2. It is desired to build a balloon that will carry 200 kilograms of material including the weight of the bag. If hydrogen is the gas used, find the minimum size of a bag of spherical shape. 682 cm. diameter.

3. In order to expand a toy balloon it is necessary to put hydrogen in at a greater pressure than that of the surrounding atmosphere. If the hydrogen pressure in the balloon is 90 cm. of mercury, find the lift on a spherical balloon that has a diameter of 40 cm. 39.79 g.

4. When you say that you fill the tire of an automobile to a pressure of 30 lb. per sq. in. is this pressure measured with respect to a vacuum or with respect to atmospheric pressure?

5. If atmospheric pressure on the day in question is 14.5 lb. per sq. in. and a tire is pumped till the gauge reads 29 lb., what volume would be occupied by all the air in the tire if it were allowed to expand to a pressure of 14.5 lb. per sq. in.? $3V_t$

6. If a good lift pump is placed on a pipe at a point 38 feet above the level of water in which the lower end of the pipe is immersed, describe quantitatively just what will happen when the pump operates.

7. How high can a perfect lift pump raise oil of density 0.92 if the atmospheric pressure is normal? 36.95 ft.

8. A stratosphere balloon rises to a height of 15 miles where the barometric pressure is 1 in. The gas bag is inflated with 20,000 cu. ft. of helium at a ground pressure of 30 in. What allowance must be made for expansion of the bag? 580,000 cu. ft. allowance.

9. (*a*) Dr. Beebe descended to a depth of about 3000 feet beneath the surface of the ocean in a closed metal sphere, within which normal atmospheric pressure was maintained. He looked out through a window having

an area of about 100 sq. in. Compute the total force of the water against the window. (Assume density of sea water 63 lb. per cu. ft.)

131,250 lb.

(*b*) Professor Piccard ascended in a closed metal sphere, within which normal atmospheric pressure was maintained, to a height of about 10 miles, where a mercury barometer would read about 2 in. Compute the pressure per sq. in. on the outside of the window of his sphere. 1 lb./in.^{2}

(*c*) If the windows were to give way, in which direction would they move?

Experimental Problems

1. If an air pump and Magdeburg Hemispheres are available, exhaust the air from the hemispheres. Assume that about 90 per cent of the air is removed and calculate the force necessary to pull them apart. Actually attempt to pull them apart.

2. Put about 10 cc. of water in a rectangular oil or syrup can. Bring the water to a boil. Then quickly cork, or otherwise close, the can and pour cold water over it. Observe and interpret the motion of the sides of the can.

3. Make daily records of the barometric pressure and accompanying weather conditions for at least one week, drawing any conclusions which seem justified.

4. Examine a siphon and a lift pump in operation to determine the parts played by air pressure and by liquid pressure.

Chapter 11

FORCES AND MOTION

Our common experience tells us that any object tends to stay in the position in which we find it unless some force acts upon that object. A more careful examination of our surroundings shows that if an object is moving in a straight line it continues to do so unless some force, for example friction, acts upon it so as to change that motion.

We find that our ideas as to the nature of force and the inertia of matter are quite overlapping, and it is hard to define one except in terms of the other.

The first exact statements of the relations between force, the mass of an object, and the acceleration of the object are credited to Isaac Newton, and they bear his name.

It is fairly easy to understand the general principles involved, since our everyday experience provides us with many examples. However, if we wish to make quantitative calculations relating force, mass, and acceleration, we find it necessary to use a special system of units with which to measure the force, although our old system for measuring time, distance, and mass is still satisfactory.

This chapter contains many examples of the relations of force, mass, and acceleration.

1. Forces Produce Changes in Motion

Experience teaches us that if a body is at rest, it will tend to remain at rest until some force acts upon it. Everyday experience also tells us that when some force does act on the object, the object will begin to move, and the rate at which it picks up velocity will depend on the magnitude of the force. Common sense and observation also tell us that for a given force the effect will be greater on an object of small mass than on an object of large mass.

For example, we may consider a light automobile standing on a perfectly level floor. We push on one end of the car and it picks up speed at a rate that depends on how great is the force with which we push. If the automobile is a heavy one we get similar but not nearly so large results.

In an experiment of this kind friction is always present,

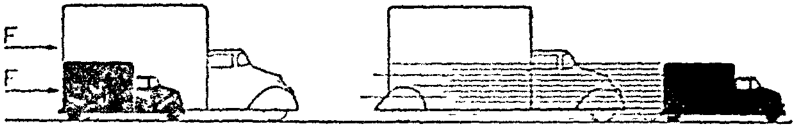


FIG. 82.—A light car picks up speed faster than a more massive car when equal forces are applied to each.

and it is only the unbalanced part of the force that causes the acceleration of the object. By using light cars on good bearings we can reduce the effect of friction, or we can apply a separate force that will just keep a car moving at constant speed. This latter force will then balance the retarding effect due to friction, and a different force can be applied to accelerate the object.

It will be hard to make accurate measurements on the forces and the accelerations in such experiments, but if a large number of them are carried out we can draw some definite conclusions along the lines of the general impressions given at the beginning of this section.

For the more simple experiments we start with objects initially at rest. However, we also find that if a body is moving at a uniform speed in a straight line, it will continue in this state of motion unless acted on by an external force, just as the body which was at rest remains at rest without the action of a force. Also we find that if a force does act on a moving body it tends to change the motion of the body, just as such a force affects a body previously at rest.

General conclusions both for the body initially at rest and a body initially in motion are as follows: (1) An object will not be accelerated without the application of a force. (2) When a force is applied to an object, the object will be accelerated in the direction of the force by an amount proportional to the magnitude of the force and inversely proportional to the mass of the object.

These relations between forces and masses were first expressed formally by Newton, and the statements in the following forms are known as Newton's first and second laws of motion.

NEWTON'S FIRST LAW OF MOTION: *Every body continues in its state of rest or of uniform motion in a straight line unless it is compelled to change that state by the application of a force.*

Except for the more formal wording this statement is essentially the same as our statement (1) and the more lengthy description above.

NEWTON'S SECOND LAW OF MOTION: *When a force is applied to a body, the motion of the body is changed in the direction of the force by an amount directly proportional both to the force and to the time the force acts.*

At first glance this statement appears to differ from (2) above, for it introduces the term "motion of the body" and mentions the time that the force acts. Newton was interested in the total change of motion that a body experienced and our observations tell us that this is greater the longer the time that the force acts. Today we are frequently more interested in the rate at which the motion changes. The following discussion may further clear up the formal statement of Newton in comparison to the informal statement (2) above.

The term "motion of a body" as used by Newton includes both the mass of the body and its velocity and is given as the product of the mass and its velocity (mv). This quantity has now received the name "momentum" and will be described further in Chapter 13.

Newton's second law as given above can be expressed in equation form as follows:

$$\text{Force} \times \text{time} = (\text{mass} \times \text{velocity})_2 - (\text{mass} \times \text{velocity})_1 \quad (1)$$

$$Ft = (mv)_2 - (mv)_1 \quad (2)$$

Where t is the time that the force F is applied and $(mv)_1$ is the quantity of motion (momentum) before the application of the force and $(mv)_2$ is the momentum after the force has been impressed for the time, t.

With the velocities that we ordinarily meet in every day life, the masses of objects do not change; so we can assume that the mass, m, in the above expression for momentum is a constant and we can rewrite equation (2)

$$Ft = m(v_2 - v_1) \tag{3}$$

If we solve for F, the force, we have

$$F = m\frac{v_2 - v_1}{t} \tag{4}$$

The latter part of this expression is a change in velocity divided by the time in which the change took place. This quantity we have already called an acceleration. So we may write

$$\text{Force} = \text{mass} \times \text{acceleration} \tag{5}$$
$$F = ma \tag{6}$$

We may consider equations (1) and (2) above to be expressions of Newton's second law in the manner in which he gave it. Similarly the derived equations (5) and (6) may be considered a modified form of Newton's second law which, in equation form, agrees with the informal statement (2) on page 153.

If we rewrite equation (5), solving for mass, we have

$$\text{Mass} = \frac{\text{Force}}{\text{acceleration}} \tag{7}$$
$$m = \frac{F}{a} \tag{8}$$

This is a defining expression for mass in terms of inertia, for it tells the relationship of a force to an acceleration for that particular piece of matter.

On the other hand equation (5) may be considered the defining expression for a force in terms of mass and acceleration. This point of view will be discussed in Section 2.

We see at once that the nature of force and the inertia

characteristic of mass are so interrelated that we cannot define either one independently of the other.

Suppose that we consider using equation (7) for defining mass. Experimentally we must apply a force that we can measure accurately and then we must make accurate observations on the acceleration produced. In actual practise we find, in the first place, we can never entirely get rid of opposing frictional forces; and in the second place, accurate measurements of forces and accelerations are difficult under the conditions of this experiment. Consequently we usually determine the mass of an object by determining its weight.

The weight of an object is due to the fact that it has a mutual attraction with the earth. This mutual attraction of objects is the second important characteristic of all matter, and this characteristic is relatively easy to determine in terms of some standard object.

Weight is, in reality, a force, whereas mass is inertia. However, we can use the same name for units for each of these two quantities. So, for example, if an object weighs one pound, we say also that it has one pound of mass. However, we must remember that one pound of weight is a force and one pound of mass is inertia.

2. New Units of Force

The units of force which we have used in earlier chapters were the pound and the gram. These force units were based on weight, which means that they have a special meaning with respect to measurements made on the earth. On the other hand, the mass of an object, that is, its inertia, does not depend on whether the object is on the earth or some place else in the universe. Similarly, seconds which we use for time, and centimeters or feet which we use to measure distance will be the same no matter whether we are working on the earth or at some other place.

If we examine the equation expressing the relation between force, mass, and acceleration, we see that it is only the quantity called force which is limited to earth operation by the unit which we have chosen. So it becomes necessary, in

order that this equation may have general truth, to develop a new unit that will be independent of the earth with which to measure force.

One way to find the proper new unit would be to drop an object and to observe the acceleration with which it falls due to its weight with respect to the earth. For example, if we take the expression

$$f = ma \tag{1}$$

and if we drop, let us say, a 1 pound mass, we will observe that its acceleration is 32.2 ft. per sec. per sec. If we put these values for mass and acceleration in this equation and solve for force, we find that

$$f = 32.2$$

Now, the only force acting on the object was the weight of the object, and since a 1-lb. mass also weighs 1 lb., we would originally have said that the force acting was 1-lb. weight. This experiment, however, tells us that a force of 32.2 units of some kind is acting. So we can conclude that there is a new unit of force smaller than the pound by the factor of 32.2 or, in other words, 32.2 of these new units of force are equal to 1 lb. of weight. This new unit of force is called the *poundal*.

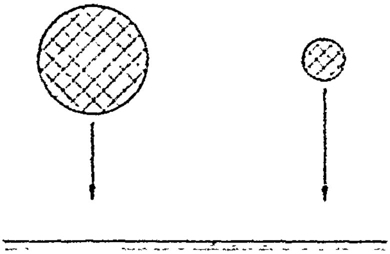


FIG. 53.—When objects are dropped their accelerations may be measured. If we may neglect air friction the acceleration of all bodies, regardless of mass will be the same. This acceleration measured in English units is 32.2 ft. per sec. per sec. and in metric units, 980 cm. per sec. per sec., approximately.

We can repeat this experiment and make our measurements in metric units. Suppose, for example, that we drop a mass weighing 1 gram. We would observe the acceleration to be 980 cm. per sec. per sec. Consequently, the equation tells us that the force is equal to 980 of the new units. However, we know that the force is also equal to 1 gram of weight. Therefore, we say that 1 gram of weight is equivalent to 980 of the new units. The new units are given the name *dynes*.

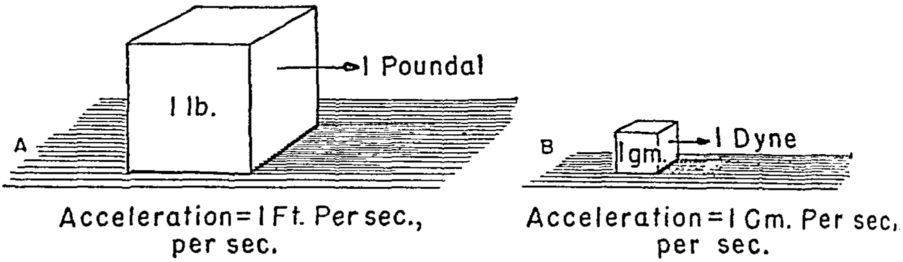


FIG. 84.—*(Left)* A force of 1 poundal will accelerate a mass of 1 pound 1 ft. per sec. per sec. *(Right)* A force of 1 dyne will accelerate a mass of 1 gram 1 cm. per sec. per sec.

One objection to finding the new units by the simple experiments described above is that slightly different answers will be obtained on different parts of the earth, for we know that the value of gravity differs slightly in different places on the earth. So we may look for the new units by another method.

Suppose that we apply a force whose size can be varied to any object and that we accurately measure the acceleration produced. For simplicity, let us again take a mass of 1 lb. Suppose that we vary the force until we observe an acceleration of 1 ft. per sec. per sec. to take place. This force, then will be of unit size in the new system; that is, it will be 1 poundal.

Similarly, we may apply a force to a mass of 1 gram. We may then vary the size of this force until we observe an acceleration of 1 cm. per sec. per sec. We can then say that we are applying 1 unit of the new force. This is the force which we call the dyne.

This method for determining absolute force units amounts to defining them as follows:

A poundal is a force capable of imparting an acceleration of one foot per second per second to a mass of one pound.

A dyne is a force capable of imparting an acceleration of one centimeter per second per second to a mass of one gram.

This suggested experiment for finding the size of the poundal and the dyne is the better method from the ideal point of view since it follows the definitions closely. It is, however, not as easy to perform as the method of dropping objects which we suggested above. The size of the poundal and the dyne found by either of these methods is the same as that found by the other method.

If we now look at equation (6) on page 155, we will observe that f is a force which is to be measured in a new unit, either the poundal or the dyne, because these units are valid any place in the universe.

We must remember, however, that m standing for mass on the right hand side of this equation, is still to be expressed in either pounds or grams. The reason for this is that pounds and grams, although not absolute units of force, are nevertheless, absolute units of inertia. We could rewrite this equation in words, using the word inertia, thus:

$$\text{Force} = \text{inertia} \times \text{acceleration}$$

In all problems where forces result in the acceleration of masses, we must use the absolute unit for force as well as the proper absolute units for mass and acceleration. If we use poundals for force, we then use pounds for mass and feet per second per second for acceleration. Similarly, in the metric system we use dynes for force, grams for mass, and centimeters per second per second for acceleration.

3. Some Examples of Acceleration

Example 1. A force of 50 lb. is applied in a forward direction to the back of an automobile. Let us suppose that there is no friction and then compute the acceleration for a car weighing 2600 lb.

First we will express the force of 50 lb. in the new absolute unit. Fifty pounds is equivalent to

$$50 \times 32.2 = 1{,}610 \text{ poundals}$$

We now use the relation

$$\text{acceleration} = \frac{\text{force}}{\text{mass}}$$

or

$$a = \frac{f}{m}$$

Therefore

$$a = \frac{1610}{2600} = .619 \text{ ft. per sec. per sec.}$$

Example 2. The engine of a train is capable of supplying a force of 2000 lb. in addition to the force necessary to overcome friction. Find the acceleration that can be produced in a train that weighs 200 tons. Find also the velocity that the train can attain in 5 min. starting from rest.

First we will reduce the force to absolute units. 2000 lb. is equivalent to

$$2000 \times 32.2 = 64{,}400 \text{ poundals}$$

The mass of the train must be expressed in pounds instead of tons, so we write, 200 tons is equivalent to

$$200 \times 2000 = 400{,}000 \text{ lb.}$$

Again using the relation

$$\text{acceleration} = \frac{\text{force}}{\text{mass}}$$

or

$$a = \frac{f}{m}$$

we may write

$$a = \frac{64{,}400}{400{,}000} = .161 \text{ ft. per sec. per sec.}$$

To find the velocity at the end of 5 min. we need only remember that acceleration is the velocity gained in each second. Consequently in 5 min. we have 300 sec. in which to gain velocity at the rate of .161 ft. per sec. per sec. So we may write

$$\text{velocity} = \text{acceleration} \times \text{time}$$

$$v_f = at$$

$$v_f = .161 \times 300 = 48.3 \text{ ft. per sec.}$$

$$48.3 \text{ ft. per sec.} \equiv 32 \text{ m.p.h. approx.}$$

Example 3. Let us consider a negative acceleration. Suppose that we are riding in an automobile at 60 m.p.h. and that the car with its passengers weighs 3500 lb. How much force will be required to slow down the automobile at the rate of 2 ft. per sec. per sec.? Also, how long will it take to stop the car and how far will the car travel while it is being stopped? For the first of these questions we need only remember

$$\text{force} = \text{mass} \times \text{acceleration}$$

or

$$f = ma$$

or

$$f = 3500 \times 2 = 7000 \text{ poundals}$$

Sometimes we need to express this force in the gravitational force unit, the pound. Consequently we write

$$\text{pounds} = \frac{\text{poundals}}{32.2}$$

or

$$\text{pounds} = \frac{7000}{32.2} = 217 \text{ lb. approx.}$$

From an earlier chapter on velocity we may remember that

$$1 \text{ m.p.h.} = 1.5 \text{ ft. per sec. approx.}$$

Consequently we see that 60 m.p.h. is approximately equal to 90 ft. per sec. We may now write

$$\text{velocity} = \text{acceleration} \times \text{time}$$

or

$$v = at$$

Solving this equation we may write

$$t = \frac{v}{a}$$

or

$$t = \frac{90}{2} = 45 \text{ sec.}$$

Since the car changes from 90 ft. per sec. to 0 velocity we may write

$$\text{average velocity} = \frac{\text{initial velocity} + \text{final velocity}}{2}$$

or

$$\bar{v} = \frac{90 + 0}{2} = 45 \text{ ft. per sec.}$$

Now, the distance covered in this interval of time is found by writing

$$\text{distance} = \text{average velocity} \times \text{time}$$

or

$$d = 45 \times 45 = 2025 \text{ ft.}$$

Example 4. Interesting cases occur in elevators where people are accelerated into upward velocity or are accelerated into downward velocity. Suppose that the elevator car accelerates either up or down at the rate of 5 ft. per sec. per sec. Let us see if we can discover how much a 150-lb. man weighs as the car starts upward and again as it starts downward. In order to accelerate the man upward, the floor of the elevator must push against his feet with a force in addition to that which is necessary to support his weight. So we can write

$$\text{force} = \text{mass} \times \text{acceleration}$$

or

$$f = ma$$

or

$$f = 150 \times 5 = 750 \text{ poundals}$$

We may now change this 750 poundals to the practical unit, the pound. So we write

$$\frac{750}{32.2} = 23.3 \text{ lb. approx.}$$

This is the additional force with which the elevator must push up against the 150-lb. man in order to give him the upward acceleration of 5 ft. per sec. per sec. Consequently, the man must appear to weigh a total of

$$150 + 23.3 = 173.3 \text{ lb.}$$

If the elevator had been accelerating downward at the same rate it would have needed an unbalanced force on the man to the extent of 23.3 lb., the same as in the case of accelerating him upward. To get this force, we must depend on the man's weight. In other words, the floor of the elevator appears to drop out from under the man just sufficiently so that the man presses against the floor with a force of 23 3 lb.

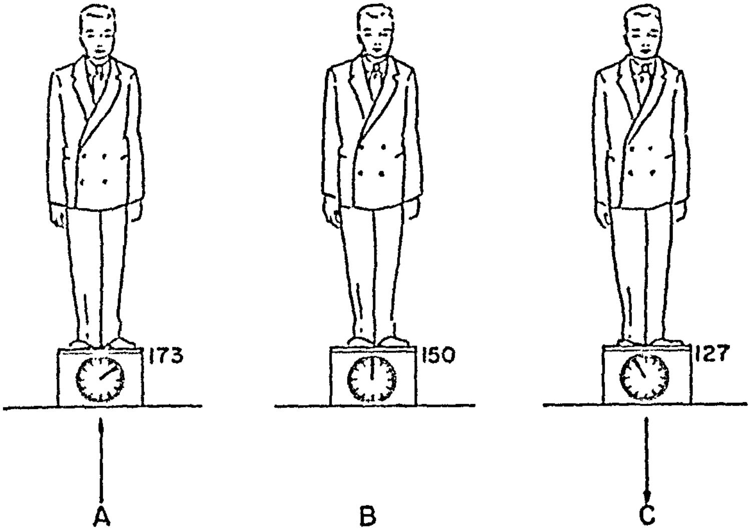


FIG. 85.—A man in an elevator appears to have increased weight as the car accelerates upward (A) and decreased weight as it accelerates downward (C) in comparison to his true weight (B).

less than his normal weight. In other words, in the downward acceleration he appears to weigh

$$150 - 23.3 = 126.7 \text{ lb.}$$

Example 5. In the case of automobile accidents where a car strikes something fairly substantial, it must come to rest in a very few feet. Consequently, the negative acceleration to stop the car must be very large. Of course, this also means that the stopping force is very large and hence we see that much damage can be done. Suppose that we consider an automobile at the rather slow speed of 20 m.p.h. bumping directly into a concrete abutment. Suppose that the entire car with its occupants stops in a distance of 3 ft. This, of course, means that the front of the car is shoved in somewhat. We can now attempt to find the stopping force.

In an earlier chapter on acceleration, velocity, and distances covered, we learned that these quantities are related

to one another through the equation (p. 56)

$$\text{velocity}^2 = 2 \times \text{acceleration} \times \text{distance}$$

In symbols we write

$$v^2 = 2ad$$

If we solve this equation for a we may write

$$a = \frac{v^2}{2d}$$

Since 20 m.p.h. is approximately 30 ft. per sec., we may write for this equation

$$a = \frac{30^2}{2 \times 3}$$

or $$a = 150 \text{ ft. per sec. per sec.}$$

The student will at once notice that this is almost 5 times the acceleration of a freely falling object due to gravity. The force on a 125-lb. boy may be computed by writing

$$\text{force} = \text{mass} \times \text{acceleration}$$

or $$f = ma$$

or $$f = 125 \times 150 = 18{,}750 \text{ poundals}$$

To find the force in pounds we write

$$\frac{18{,}750}{32.2} = 582 \text{ lb. approx.}$$

The extent to which the boy may or may not be injured depends on his ability to resist a force of 582 lb. which behaves as though it were trying to pull him into whatever part of the car is immediately in front of him.

Example 6. A somewhat more amusing case is to consider a passenger in a street car who tries to keep his balance without hanging on to anything. Suppose that the street car stops with a negative acceleration just equal to that due to gravity. At what angle will the man need to brace himself in order not to fall?

In Figure 86 we show such a man standing at an angle with a vertical arrow through his center of mass indicating his weight and with a horizontal arrow through the same point indicating his reaction due to the negative acceleration. In order that these two forces may be equal the student should see at once that the man must stand at an angle of 45° to the vertical. For other rates of stopping the man will need to stand at different angles.

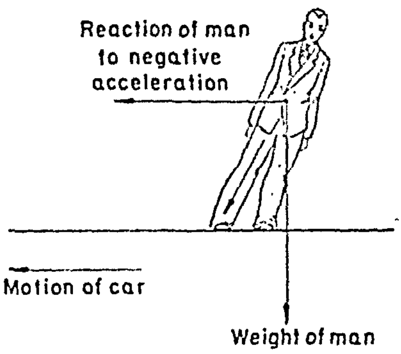


FIG. 86.—A man can brace himself as the car in which he is riding is being stopped.

Some Important Facts

1. The motions of masses and the effects of forces on their motions were described by Newton as follows:

First law: Every body continues in its state of rest or of uniform motion in a straight line unless it is compelled to change that state by the application of a force.

Second law: When a force is applied to a body, the motion of the body is changed in the direction of the force by an amount directly proportional both to the force and to the time that the force acts.

2. A force which imparts unit acceleration to unit mass is one absolute unit of force. In the English system this is the poundal and in the metric system, the dyne.

3. Absolute units of force are independent of location.

4. Absolute units of force are smaller than gravitational units. Hence a larger number of units in the absolute system are required to express an equivalent force in the gravitational system. For example: 32.2 poundals are equivalent to 1 pound and 980 dynes are equivalent to 1 gram.

5. Commonly experienced examples of acceleration of masses due to the application of forces may be observed in the starting and stopping of elevators, automobiles, buses, and other conveyances.

Generalizations

So far as the motion of matter is concerned, the fundamental nature of inertia and of force are overlapping. Either may be defined in terms of the other and the related acceleration.

Problems

Group A

1. Define inertia and give several examples.
2. State the Law of Inertia.
3. Define the term momentum.
4. Mention several instances which will illustrate each of Newton's first two laws.

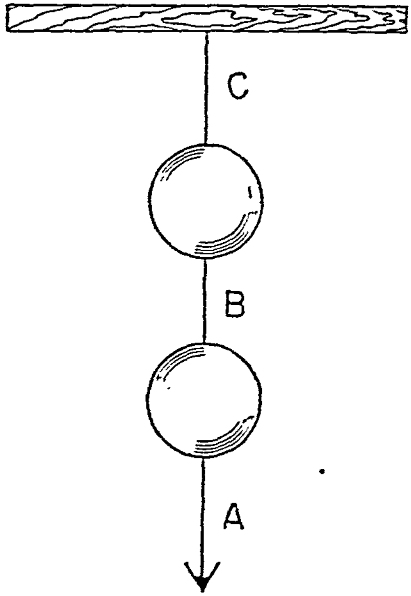


FIG. 87.—Heavy masses are suspended by strings for the experiment of problem 5.

5. Two heavy balls are suspended as indicated by sections of the same kind of string. A steady pull on the end of the string will break the string at *C*, while a sudden jerk will break it at *B* or possibly *A*. See Figure 87. Discuss the effect.

6. The existence and location of the planets Uranus, Neptune and Pluto were suspected some time before their actual discovery from slight outward bulges in the orbits of planets nearer the sun. Can you explain why?

7. At separate times a ship weighing 65,000 tons. and a rowboat weighing 350 lb. strike a dock at a velocity of 5 m.p.h. Discuss the two effects.

Chapter 12

THE WORK DONE IN PRODUCING MOTION

We have already seen that objects are put into motion by having forces applied to them. Since the forces move through distances in this type of operation, work must be done. The work done by forces in this manner is stored in the motion of the object and is called kinetic energy. In order to stop an object this kinetic energy must be removed from it.

The amount of kinetic energy in an object is found to depend on the square of the velocity.

Energy may also be stored for future use. It is then known as potential energy. Some familiar examples are objects so placed that they can fall. Water, for example, at the top of a waterfall is a potential source of energy. This energy can be converted into kinetic energy by permitting the water to drop.

The rate at which work is done is called power. It is often of importance to know the rate at which work can be done as well as the total amount of work which is done.

Many examples of these various effects of energy are given in this chapter.

1. Motions

Our experience with the everyday world about us teaches us that when a force is applied to an object, a motion of that object results, unless, of course, some other force opposes that motion. In Chapter 11 we tried to organize our knowledge about the behavior of forces and masses and we concluded that not only does a force produce motion of an object, but that the velocity of the object increases as the force is applied for a greater and greater length of time. Another way of stating these facts is to recall that for a given size force on a given object a definite acceleration is produced. Then we will recall, of course, that the longer this acceleration lasts, the greater the velocity will be.

If a force causes a body to move, then of course the force must move along with that body if it is to continue to push or pull it. In this case we have a force moving through a

distance, and therefore we must expect that the force is doing work and we may write

$$\text{Work} = \text{force} \times \text{distance}$$

as in Chapter 5. Of course this conclusion is in line with our general experience in the matter of pushing or pulling objects.

We see that the longer a force is applied, the greater the velocity it can give to an object. Also we see that the longer the force is applied, the greater the distance through which it will have moved. So now we can conclude that the longer the force is applied to a given object, the greater the amount of work that will be done.

We now ask ourselves what becomes of the energy which the force expends in moving the body. We will try to keep the problem simple by assuming that there is only negligible friction while we perform this experiment with forces and objects. In this ideal case there is no way by which the energy can escape, and so we can jump to the conclusion that the work done by the force must be stored in the motion of the object.

This conclusion seems to be the more reasonable if we consider what happens when we try to stop a moving object. Suppose, for example, that an automobile is coasting along a level road at a speed of perhaps 30 m.p.h. If it suddenly runs into a concrete abutment the car will, of course, stop, but in the process of stopping it may be quite completely wrecked. It takes energy to do all this work in wrecking the car, and the energy could have come from no other place except the motion of the car itself.

We again consider the case of an automobile and this time assume that we stop it with brakes instead of by the violent process of having it crash. If we apply the brakes, we find that the brake drums on the wheels of the car warm up as the car stops. Heat, we know, is another form of energy. In this case the energy has been taken out of the motion of the car and converted into heat in the brake drums. The brake

drums can then cool off by losing their heat to the air around them.

This experiment with stopping an automobile permits us to study a little further the nature of the energy stored in the motion of the car. Suppose, for example, that we measure the distance that it takes to stop a car from a speed of 30 m.p.h. Then suppose that we repeat this experiment by stopping the car from a speed of 60 m.p.h. We discover that it takes very much more than twice the distance to stop the car from its 60 m.p.h. speed than it did from its 30 m.p.h. speed. In fact we find that roughly it takes four times the distance to stop the car from the higher speed than from the lower one.

Our knowledge of the damage done in accidents where cars are stopped more abruptly than by brakes also bears out this idea that the amount of energy in a moving car is much greater in a fast moving car than in a slow moving one. Of course, these ideas carry over into the motion of other objects as well as that of automobiles.

The energy stored in the motion of objects is called *kinetic energy*. Careful observation on the actual amount of energy that can be taken from moving objects results in the expression

$$\text{Kinetic Energy} = \frac{1}{2}\,\text{mass} \times \text{velocity}^2$$

or

$$\text{K.E.} = \frac{1}{2}\,mv^2$$

The mass is that of the moving object and the velocity is the velocity of that object. In the English system of units the mass would be expressed in pounds and the velocity in feet per second. Kinetic energy will come out in this equation in terms of a new unit called the *foot poundal.*

In the metric system of units, the mass would be expressed in grams, the velocity in centimeters per second and the work unit would be the *dyne centimeter.* This work unit has received the special name, *erg.* The foot poundal and the erg are the absolute units of work that correspond to the absolute force units of Chapter 11, Section 2.

2. Deriving the Kinetic Energy Equation

The kinetic energy equation of the last section was obtained from experimental observation. This, of course, is a good common sense method for finding out how things happen. However, it is sometimes possible to predict an equation from what we already know.

Suppose, for example, that we recall the discussion in the first part of the last section, namely that when a force accelerates an object, it also moves through a distance. Then if we recall that work equals force times distance we may write in symbols

$$W = F \times d \tag{1}$$

F will be the force which we have applied to the object and for kinetic energy calculations it must be expressed in absolute units, poundals if we are using the English system (or dynes in the metric system). Then d will be expressed in feet (or centimeters).

We now recall the expression from Chapter 11,

$$\text{Force} = \text{mass} \times \text{acceleration}$$

or

$$F = ma \tag{2}$$

In this equation F is the force in absolute units applied to an object to accelerate it, m is the mass of that object, and a is the acceleration produced.

Suppose now that the object upon which the force is acting moves through a distance, d, in the interval of time while we are observing it. Suppose that we multiply both sides of equation (2) by d. This gives us

$$Fd = mad \tag{3}$$

and of course we can say that Fd is the work which the force has done on the object whose mass is m.

To keep the problem simple we will suppose that the object was at rest before the force started acting on it. In this case the expression relating the distance which an object covers in the time t starting from rest is (see Chapter 4, Sec. 10, formula (7) p. 56):

$$\text{distance} = \frac{1}{2}\,\text{acceleration} \times \text{time}^2$$

In symbols we may write

$$d = \frac{1}{2}\,at^2 \tag{4}$$

Suppose now that we take this value of d and substitute it in the right-hand side of equation 3. Then we can write

$$Fd = m \times \frac{1}{2} \times a^2t^2 \tag{5}$$

Now we must recall that acceleration multiplied by time is the final velocity of an object if it started from rest. Consequently a^2t^2 is the square of the final velocity, so that we write

$$Fd = m\,\frac{1}{2}\,v^2$$

or, rearranging,

$$Fd = \frac{1}{2}\,mv^2$$

But Fd is the work which the force has put into the motion of the object. Therefore we can write

$$\text{work} = \frac{1}{2}\,mv^2$$

or, since we are going to call the energy, kinetic energy, after it is stored in the motion of the object we can write

$$\text{K.E.} = \frac{1}{2}\,mv^2$$

So by linking together ideas with which we are already familiar, we are able to predict the equation relating the kinetic energy of an object to the mass and velocity of that object.

3. Energy Required to Change from One Velocity to Another

In the two sections which we have considered so far, we have talked about the simple case of starting an object from

rest and bringing it up to some velocity, or of taking an object moving at some velocity and bringing it back to rest. Of course energy changes of the same type are required if we have an object already moving and cause it to move faster. Similarly a rapidly moving object will need to have energy taken out of it in order to slow it down even though we do not bring it to a complete stop. Suppose that we perform two separate experiments on any object, for example, the familiar automobile. In one case let us bring the car from rest up to some particular velocity which we will call v_1. In the second case we will bring the car from rest up to some other velocity which we will call v_2. In the first instance the kinetic energy will be

$$\text{K.E.}_1 = \frac{1}{2} m v_1^2$$

and in the second case it will be

$$\text{K.E.}_2 = \frac{1}{2} m v_2^2$$

The difference in the kinetic energy at the velocities v_1 and v_2 must be

$$\begin{aligned} \text{Change in K.E.} &= \text{K.E.}_2 - \text{K.E.}_1 \\ &= \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2 \\ &= \frac{1}{2} m(v_2^2 - v_1^2) \end{aligned}$$

So we see that changing the velocity of an object either up or down involves an energy change proportional to the difference between the squares of the two velocities.

A study of the simple equation for kinetic energy from Sections 1 and 2 and of this expression tells us that as we change velocity by equal amounts, the energy required for the change goes up rapidly in comparison to the changes in velocity. For example, a certain amount of energy is required to bring a car from rest to 10 m.p.h. A much greater amount of energy must be added to take it from 10 m.p.h. to

20 m.p.h. although the change in velocity is the same. These facts also make it seem more reasonable that it takes about four times the distance to stop a car from any given speed as to stop it from one-half that speed; or for another similar case, a car travels much farther as you slow from 60 to 50 m.p.h. than it does while one slows from 30 to 20 m.p.h., for the same application of the brakes.

4. Some Effects of Kinetic Energy

In the preceding section we saw that if we wish to accelerate an object we have to put energy into it. On the other hand if we wish to slow down or stop the object we have to take energy out of it. The automobile was cited as a common example of these effects. Of course, in an automobile we are never entirely free from friction. Also we do some hill climbing and some coasting down hills. However, these effects are all in addition to the kinetic energy problem as we have looked at it so far in this chapter.

Much energy is wasted in driving a car in city traffic due to the excessive number of stops and starts. Energy must be supplied by means of burning gasoline in the engine to make the car pick up speed. However, when we slow down or stop the car we take the energy out by means of the brakes. This system converts the energy into heat and the heat in turn is lost to the surrounding air. Consequently, every time we stop we throw away the amount of energy involved in getting up to the desired speed. This fact accounts in part for the lower mileage per gallon of gasoline which we obtain in city driving as compared to country driving.

There are many other examples of the use of kinetic energy. For example, water flowing over the top of a high falls attains considerable speed as it reaches the bottom. This kinetic energy of the water can be used to operate water wheels, which in turn supply energy.

The ability of a bullet to penetrate an object is also due to kinetic energy. In this case we ordinarily have a rather small mass, but an extremely high velocity.

Whether an object comes to destruction or not when its

velocity is changed, depends largely on how abruptly the change is made. Our own example of the automobile shows this clearly. When we stop the car with brakes, damage does not result. When we stop the car with a concrete abutment damage does result. Similarly, we might consider the case of a person jumping from a high building. If he lands on the pavement, he will probably be killed. If he lands in a net put up by firemen his fall will be stopped much less abruptly and he may survive without serious injuries.

5. Some Simple Kinetic Energy Problems

In the first section of this chapter, we saw that kinetic energy problems are among the type where absolute units for force must be used. Consequently, in the English system, we must use poundals for force instead of pounds and our unit of work becomes the foot poundal.

Similarly, in the metric system of units we found that the unit of force is the dyne and the unit of work is the erg.

You will recall that a dyne is a very tiny amount of force, and so we expect to find that a dyne times a centimeter, or an erg, is a very tiny amount of energy. It is quite common to consider a larger energy unit called the *joule*. It consists of 10,000,000 ergs. This unit is often used in English speaking countries in calculating electrical problems, and consequently the student should notice it here.

Below you will find a few examples worked out in the English system of units.

Example 1. A boy and a bicycle together weigh 150 lb. Find the kinetic energy of the two when moving at the rate of 20 ft. per sec.

$$\begin{aligned}\text{K.E.} &= \frac{1}{2} mv^2 \\ &= \frac{1}{2} \times 150 \times (20)^2 \\ &= 30{,}000 \text{ ft. lbal.} \\ &= \frac{30{,}000}{32.2} \text{ ft. lb.} \\ &= 932 \text{ ft. lb. approx.}\end{aligned}$$

Example 2. How much kinetic energy is there in a 2600-lb. automobile traveling at the rate of 30 m.p.h.?

We must first convert the m.p.h. to ft. per sec. You will recall that there are approximately 1.5 ft. per sec. for every m.p.h. This will give us a value of 45 ft. per sec. in the place of 30 m.p.h. A more accurate calculation shows that there are nearer 44 ft. per sec., but we will use the approximation as has been done throughout this text.

$$\begin{aligned}\text{K.E.} &= \frac{1}{2}mv^2\\ &= \frac{1}{2} \times 2600 \times (45)^2\\ &= 2{,}630{,}000 \text{ ft. lbal. approx.}\\ &\equiv 81{,}700 \text{ ft. lb. approx.}\end{aligned}$$

Example 3. A bullet weighing $\frac{1}{8}$ lb. is traveling at the rate of 2000 ft. per sec. Find its kinetic energy.

$$\begin{aligned}\text{K. E.} &= \frac{1}{2}mv^2\\ &= 250{,}000 \text{ ft. lbal.}\\ &\equiv 7764 \text{ ft. lb. approx.}\end{aligned}$$

Example 4. A force of 50 lb. is applied to a car weighing 2200 lb. The force pushes the car a distance of 75 ft. Neglect friction and find the velocity of the car.

We can find the kinetic energy of the car by finding the amount of work done.

$$\begin{aligned}\text{Work} &= \text{force} \times \text{distance}\\ \text{or in symbols}\quad W &= fd\\ &= 50 \times 75\\ &= 3750 \text{ ft. lb.}\\ &\equiv 3750 \times 32.2 \text{ ft. lbal.}\\ &= 120{,}000 \text{ ft. lbal. approx.}\end{aligned}$$

This work is, of course, equal to the kinetic energy of the moving car. We can now write

$$\text{K.E.} = \frac{1}{2}mv^2$$

$$120{,}000 = \frac{1}{2} \times 2200 \times v^2$$

$$v^2 = 109 \text{ approx.}$$

$$v = 10.4 \text{ ft. per sec. approx.}$$

$$v = 6.9 \text{ m.p.h. approx.}$$

6. The Rate at Which We Work—Power

In the early part of this chapter a good bit of emphasis was placed on whether or not a moving object was stopped abruptly or more slowly. Of course we might also be concerned with whether we put an object into motion rapidly or slowly. In fact, the rate at which work is accomplished in any line of activity becomes a matter of considerable interest to us. We might get an object up to a given speed by applying a small force for a long period of time or a large force for a shorter period of time. The time involved in getting the object up to speed might be of great importance. In Chapter 5 we saw that the rate of doing work was important in many other types of problems: for example, we calculated there the rate at which objects are moved from one floor to another against gravity, since this is a common form in which energy is expended. In that chapter we had numerous other examples where the work done was concerned with work against gravity or against friction, while in the current chapter we are interested primarily in work which is involved in the changing of the motion of objects from one rate to another.

In Chapter 5 we saw that units of power could be formed from any combination of units of work and time. In that chapter the commonly used units were foot pounds for work and seconds for time.

In Chapter 5 we also saw that metric units such as the gram or kilogram could be used for units of force. In such cases we might obtain, for example, kilogram meters per second or gram centimeters per second as power units.

In the present chapter we are more concerned with absolute units of force and work than with gravitational units.

For absolute units of work, in the English system we have the foot poundal and in the metric system, the dyne centimeter (or erg). These work units yield the names foot poundals per second as a unit of power in the English system, and ergs per second as a unit of power in the metric system. Since the erg is a very tiny amount of energy, 10,000,000 ergs are ordinarily used as a unit of work and called the joule. The power unit then becomes the joule per second, and this particular unit has been given the special name *watt.* Even a watt is a rather low rate at which work is accomplished in many of the ordinary activities of life, and consequently the term *kilowatt*, which is 1,000 joules per second, is very frequently used.

If the application of a force to an object results in the acceleration of that object, the distance through which the force moves in each second keeps changing as the velocity of the object changes. In many cases we find it convenient to compute the average rate at which a force does work over some specified period of time.

Example 1. Suppose that a force of 50 lb. acts on a 1000-lb. object. Since we recall that

$$\text{force} = \text{mass} \times \text{acceleration}$$

we may write

$$\begin{aligned} 50 \times 32.2 &= 1000a \\ a &= \frac{50 \times 32.2}{1000} \\ &= 1.61 \text{ ft. per sec. per sec.} \end{aligned}$$

If this force acts for ten seconds, the object will have a velocity of

$$\begin{aligned} \text{velocity} &= \text{acceleration} \times \text{time} \\ &= 1.61 \times 10 \\ &= 16.1 \text{ ft. per sec.} \end{aligned}$$

The kinetic energy will be

$$
\begin{aligned}
\text{K.E.} &= \frac{1}{2}\,mv^2 \\
&= \frac{1}{2} \times 1000 \times 16.1^2 \\
&= 129{,}600 \text{ ft. lbal.} \\
&\cong 4025 \text{ ft. lb.}
\end{aligned}
$$

The average rate at which this force did work is then

$$
\begin{aligned}
\text{power} &= \frac{\text{work}}{\text{time}} \\
&= \frac{4025}{10} \\
&= 402.5 \text{ ft. lb. per sec.} \\
&\cong \frac{402.5}{550} = 0.73 \text{ horsepower}
\end{aligned}
$$

The amount of work done in the above case could also have been determined by finding the distance through which the force moves. If the final velocity was 16.1 ft. per sec. then the average velocity must have been 8.05 ft. per sec., and since the object moves for ten seconds the distance covered would be

$$
\begin{aligned}
\text{distance} &= \text{average velocity} \times \text{time} \\
&= 8.05 \times 10 = 80.5 \text{ ft.} \\
\text{work} &= \text{force} \times \text{distance} \\
&= 50 \times 80.5 = 4025 \text{ ft. lb.}
\end{aligned}
$$

which is the same result as that obtained above.

7. Stored Energy—Potential Energy

Up to this section in this chapter we have considered energy in a rather active state. For the most part we have considered energy of motion, which we call kinetic energy, or we have considered the rate at which energy is being taken out of or put into an object. It often happens, however, that

energy is stored in more or less stationary fashion awaiting its turn to be used.

An example from electricity is a charged battery, which can supply energy as soon as it is connected to the proper circuit. Another example would be the energy stored in the steam of a steam boiler before one starts to use it. Still another example would be any heavy object on top of a building from which it might be dropped. A similar example is an automobile standing still at the top of a long grade. Another example of a similar type is water at the top of a waterfall. In this case the water is usually in motion also, but most of its energy, in case the fall is a high one, will be due to the distance through which the water can fall.

Examples of the latter three kinds are very common in the physical world, and are often used as energy sources. When an object falls from the top of a building to the ground, or when an automobile rolls down a grade, or when water falls over a waterfall, the energy by virtue of the position is converted into energy of motion. We refer to the stored energy as *potential energy*.

In the gravitational cases the potential energy is equal to the force on the object, which means the weight of the object, multiplied by the distance through which it can fall. So we may write

$$\text{Potential Energy} = \text{weight} \times \text{height}$$

or

$$\text{P.E.} = wh \tag{1}$$

In the English system this answer will come out in foot pounds. If the answer is needed in foot poundals we must write

$$\text{P.E.} = whg \qquad \text{where} \qquad g = 32.2 \tag{2}$$

We will now get an answer in foot poundals.

Of course, if in these examples the metric system is used, we will have the gram centimeter for the unit of energy corresponding to equation (1). In equation (2) g has the value 980 and the potential energy will be given in ergs.

Since in these simple cases the potential energy at the top of a fall is all converted into kinetic energy at the bottom of the fall, it is rather easy to make calculations.

Example 1. Consider an automobile weighing 2200 lb. at the top of a hill which is 100 ft. higher than the bottom of the hill. We can then write

$$\begin{aligned} \text{P.E.} &= 2200 \times 100 \\ &= 220{,}000 \text{ ft. lb.} \\ &= 220{,}000 \times 32.2 \text{ ft. lbal.} \end{aligned}$$

If we neglect friction due to all causes we can easily find how fast the car will be traveling at the bottom of the hill, for we can write

$$\begin{aligned} \text{P.E.} &= \text{K.E.} \\ 220{,}000 \times 32.2 &= \frac{1}{2} \times mv^2 \\ &= \frac{1}{2} \times 2200 \times v^2 \end{aligned}$$

Solving this expression for v we find that $v = 80$ ft. per sec. approx.

Example 2. Consider a pound of water at the top of a falls that are 165 ft. high. Then, neglecting the velocity with which the water goes over the falls, we can see at once that it will acquire kinetic energy in falling equal to the potential energy at the top. The potential energy at the top is

$$\text{P.E.} = 1 \times 165 = 165 \text{ ft. lb.}$$

Energy from falling water can be used to operate water wheels. Modern water wheels may have efficiencies of 80 per cent or higher. Consequently we see that it is possible to compute the amount of water needed per unit of time to develop any specified amount of power.

8. Absolute and Gravitational Units of Force and Work

In Section 6 on the rate of doing work and Section 7 on potential energy we used both absolute units and gravitational units of force and work.

In Chapter 11, Section 2, we considered the reasons for going to absolute units of force and saw that their use was required in problems involving forces and the accelerations of masses that those forces could produce.

In Chapter 12, we find that absolute units of energy always result from calculations where absolute force units are used. It is also necessary to use these absolute energy units for either kinetic energy or potential energy whenever these items have to be used in determining accelerations or the velocities that objects attain or lose.

In Chapter 13, on momentum, it will again be found that absolute units for force and work are required whenever these items are involved in momentum calculations, for here again we are concerned with changes in velocities and accelerations.

On the other hand, if we simply want to know the rate at which energy can be supplied or used, or the amount available (as is often true in potential energy problems), we may use any kind of units that we choose. It has become common practice to use gravitational units in the English system in such cases, although absolute units or decimal multiples of absolute units are commonly used in the metric system. So, in the English system, we use the gravitational units, pound and foot pound, for force and work unless the nature of the problem demands absolute units. In the metric system we often use the absolute unit, dyne, but also the gravitational unit, gram (or kilogram) for force, while for energy we usually use the absolute unit, the erg (or 10,000,000 times this unit, the joule).

It is easy to change from one set of units to the other since one need only remember that it requires approximately 32.2 of the absolute units to equal the gravitational unit in the English system and 980 in the metric system.

Some Important Facts

1. When a force does work on a body to increase its motion, energy is added to the body. To decrease the motion of a body, energy must be removed from it. The energy of a body due to its motion is called kinetic energy. The exact relation between the kinetic energy of body and its

mass and velocity is

$$K.E. = \frac{1}{2}mv^2$$

Kinetic energy is expressed in foot poundals when mass is in lb. and velocity in ft. per sec. Kinetic energy is expressed in ergs when mass is in grams and velocity in cm. per sec.

2. The above expression for kinetic energy may be found by observation on experiments or it may be derived from previous knowledge involving the ideas that (*a*) a force moving through a distance does work, (*b*) a force acting on a free object accelerates it, and (*c*) distances moved by objects being accelerated can easily be calculated.

3. Changes in the kinetic energy of an object vary directly as the differences in the squares of the different velocities of the object, that is

$$\text{Change in K.E.} = \frac{1}{2}m(v_2^2 - v_1^2)$$

4. When kinetic energy changes rapidly, large forces are involved.

5. Power is the rate at which work is done and is commonly measured in watts or horsepower. A watt is one joule (10,000,000 ergs) per sec. A horsepower is 550 ft. lb. per sec.

6. Potential energy is energy stored up and available for possible future use. Charged batteries, compressed springs and masses elevated against gravity are examples.

7. Whenever force or energy is involved in calculations on accelerations or changing kinetic energies, absolute units of force or energy must be used. In most other cases either absolute or gravitational units may be used.

8. Note that the poundal and the dyne are absolute units of force, but that the pound and the gram are absolute units of mass (although they are also gravitational units of force).

Generalizations

Energy

Energy is the capacity for doing work and is measured in work units—foot pounds, foot poundals, gram centimeters, ergs, joules.

Energy is called kinetic when actually doing work.

Energy is called potential if available but not actually in use.

Power

Power is the time rate of doing work and is usually measured in horsepower, watts or kilowatts.

Questions and Problems

Group A

1. Define the term work as properly used in mechanics. Mention several examples.

2. In what English and Metric units is work measured? Explain the meaning of these units.

3. Distinguish potential from kinetic energy and give examples of each.

4. What is power, and in what units is it measured?

5. Do you think the average modern horse can develop more or less than one horsepower? Give reasons for your answer.

6. How much work is required to raise a 500-lb. piano from the ground to the floor of a truck 4 ft. above the ground?

7. How much work do you do climbing a flight of stairs if the vertical rise is 15 ft.? If you run up these stairs in 3 sec., what horsepower do you develop?

8. If the head of a pile driver weighs ½ ton and is elevated 10 ft., how much potential energy does it have? How much kinetic energy will be realized when it hits?

9. Is time a factor in calculating work? Power?

10. Find the kinetic energy of a baseball that weighs 0.55 lb. when it is moving at the rate of 60 ft. per sec. 990 ft. lbal.

11. Find the kinetic energy of a 100-lb. projectile when traveling at the rate of 1800 ft. per sec. 162×10^6 ft. lbal.

12. A cannon weighing 5000 lb. shoots a 125-lb. projectile with a muzzle velocity of 2000 ft. per sec. Find the kinetic energy of the projectile. 250×10^6 ft. lbal.

13. Find the potential energy of a 100-lb. bag of cement on a window ledge 150 ft. above the ground. 15,000 ft. lb.

Problems

Group B

1. Find the energy that must be supplied to an automobile weighing 3000 lb. to raise its velocity from 40 ft. per sec. to 60 ft. per sec. 93,170 ft. lb.

2. Find the kinetic energy of a 900-lb. speed boat when traveling at the rate of 20 m.p.h. (Suggestion: First reduce the velocity to terms of ft. per sec.) 12,580 ft. lb.

3. At the bottom of a 75-ft. waterfall, 1000 lb. of water are available every second for running a turbine. Find the energy delivered to the turbine in 1 sec. and in 1 min. 75,000 ft. lb. 4,500,000 ft. lb.

4. Find the potential energy of a 2900-lb. automobile at the top of a hill 3000 ft. long, where the road drops 1 ft. in every 12 ft. of road. 725,000 ft. lb.

5. An automobile traveling 70 ft. per sec. has its power cut just as it starts to climb a hill. Neglect friction and find the vertical height that it can rise. (Suggestion: Equate the kinetic energy at the bottom of the hill to potential energy that the car will have when it has attained its maximum

vertical height, *h*. Notice that the mass of the car does not have to be known.) 76 ft.

6. If the hill in problem 5 rises 1 ft. in every 9 ft. of roadway, how far up the hill does the car coast? 684 ft.

7. If the car in the above problem could coast back down the hill without friction, how fast would it be going at the bottom?

8. Show that the speed of the car in problem 7 will be just the same at the bottom of the hill as if it had fallen over a cliff and dropped the same vertical distance as the vertical drop in the road.

9. A 2-ton car is traveling at the rate of 80 ft. per sec. What is its kinetic energy? From what vertical height need it drop, as a freely falling body, to acquire the same kinetic energy? 397,500 ft. lb. 99.4 ft.

10. Calculate the amount of energy that must be put into a 3700-lb. automobile to change its speed from 20 to 30 m.p.h. 64,030 ft. lb.

11. The car of problem 10 travels 90 ft. while its brakes slow it down from 40 to 30 m.p.h. If the brakes apply the same stopping force, how far will the car travel while they slow it down from 30 to 20 m.p.h.? 64 ft.

12. Find the minimum power rating of a motor that has to lift an elevator weighing 500 kilograms and 6 people weighing an average of 70 kilograms each through a height of 30 meters in 20 sec. (Suggestion: Reduce the units to grams and centimeters, respectively. In what units will this give power? Convert to ergs per second and then to watts, kilowatts and horsepower.) 13,500 watts. 13.5 K.W. 18.1 H.P.

13. A cannon with a barrel 21 meters long shoots a projectile with a muzzle velocity of 600 meters per second. (*a*) How long was the projectile in the barrel? (*b*) How much kinetic energy does the projectile have if it weighs 400 kilograms? (*c*) What useful power did the cannon develop?
0.07 sec. 720×10^{12} ergs. 10.3×10^5 K.W. (13.8×10^5 H.P.)

14. A speed boat weighing 1200 lb. has 20 horsepower available to accelerate it. (*a*) Find the velocity at the end of the first second starting from rest. (*b*) Find the velocity at the end of the second second. (Suggestion: Express the energy available in (*a*) one second or (*b*) in two seconds in terms of foot poundals and equate to the kinetic energy of the boat at the end of one second or two seconds.) 24.3 f.p.s. 34.3 f.p.s.

Experimental Problems

1. Firmly anchor a grooved pulley, which has a handle for turning. Fit a belt on the pulley; and on one end of the belt hang a 5-lb. weight, on the other a 15-lb. weight. Practice turning the pulley so as to raise the 15-lb. weight and lower the 5-lb. one, until you have found the speed at which the weights remain even with each other. If necessary to decrease the friction of the belt and pulley, apply a little oil.

When you have mastered the knack of even operation, turn the pulley steadily for three or four minutes counting the total number of turns.

The circumference of the wheel in feet times the number of revolutions gives the total distance through which your effort acts. The effort may be taken as 10 lb., (15 lb. − 5 lb.). The total work is then the product of the two quantities.

By dividing this work by 550 times the number of seconds, calculate your developed horsepower as of the conditions of the experiment.

Do you think you could develop more horsepower by some other method? If so, give details.

Chapter 13

ACTION AND REACTION

A third law of motion given by Newton—that action and reaction are always equal and oppositely directed—is based on the observed fact that whenever forces act, two bodies are involved, the one delivering the force and the one receiving it.

We have seen in earlier chapters that the application of a force to a mass results in its acceleration if it is free to move; and that the velocity attained by the mass depends on the length of time that the force acts. It is shown that Newton's third law applies to such actions as well as to cases where two objects exerting forces against one another remain stationary with respect to their surroundings.

One application of Newton's laws of motion is to impacts, where, by the nature of the problem, the duration of the action of the force is usually short. Familiar examples are numerous, jumping to or from a sled or boat, a ball bouncing on a sidewalk, the recoil of a cannon when a projectile is fired.

The subject is illustrated with a number of problems.

1. Action and Reaction—Newton's Third Law of Motion

In Chapter 11 we studied the relations between forces and masses and we examined the formal statements on this subject known as Newton's first and second laws of motion. Newton added one additional statement to the first two and it may be given as follows.

> NEWTON'S THIRD LAW OF MOTION. *To every action there is an equal and opposite reaction.*

> Another form for this statement is: The mutual actions of two bodies are always equal and oppositely directed.

This third law is a recognition of the fact that forces cannot be observed except as one body reacts on another. For example, you cannot push against nothing. Always two objects are involved, the one that delivers the force and the one that is acted on by the force. Always these two effects are equal but oppositely directed.

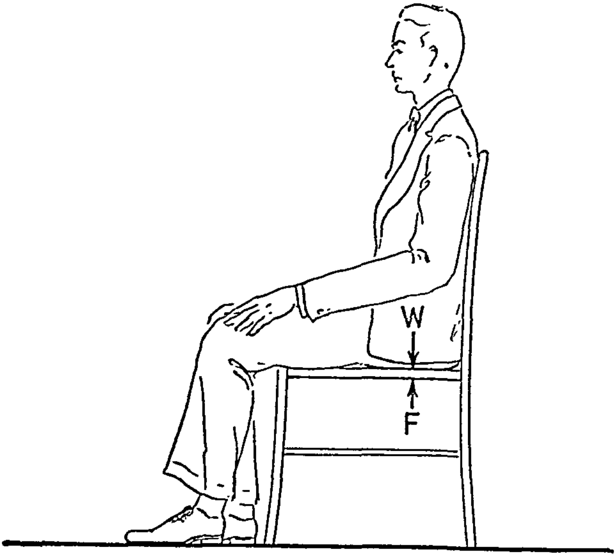


FIG. 88.—The man presses against the chair with a force equal to his own weight. The chair resists this action by pushing back equally hard. The two objects are in direct contact.

Familiar cases are those where no relative motion of the two objects results. For example, when you sit on a chair you press against the chair with a force equal to your own weight (if your feet are off the floor). However, the chair resists this force by pushing back equally hard. In this case the two objects are in direct contact.

Sometimes a medium is used. A simple example is the rope by which you might pull a sled. The sled pulls back on the rope with a force equal to that with which you pull forward. This statement may seem more reasonable if you imagine a spring scale in the center of the rope. It will make no difference in the reading whichever way you turn the scale. (You move forward with the sled because of another pair of forces. You are pushing the earth backwards with your feet and the earth is pushing you forward through your feet.)

There are three important examples where one body reacts on the other without contact or connecting medium in the simple sense described in the above two paragraphs. The

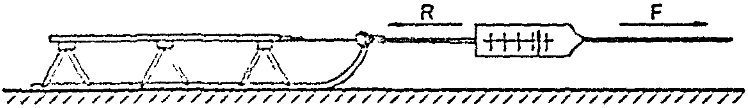


FIG. 89.—Some object pulls forward on the rope with a force F. The sled pulls backward equally hard. The pulling object and the sled are connected by a medium, in this case the rope.

most common of these is gravity (as between the earth and any object near the surface of the earth). The other two are electrical forces between electrically charged particles not touching one another and magnetic forces between, for

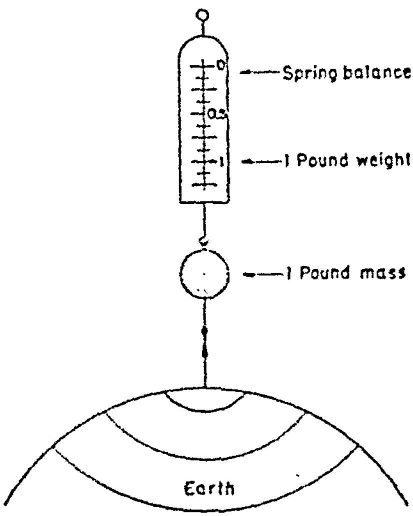


FIG. 90.—An object suspended above the earth attracts the earth and the earth attracts the object. The two are not in contact and there is no known connecting medium. The nature of the force is called gravitation.

example, a magnet and a piece of iron that are not necessarily in contact.

Our state of knowledge about the world in which we live is not yet complete enough to explain fully the manner by which the forces in these three cases are transmitted from one

object to another, but the fact that two objects are involved is still always true.

In all of the examples considered above, it is conventional to call the applied force and its behavior the *action* and the opposing force and its behavior the *reaction*. Of course,

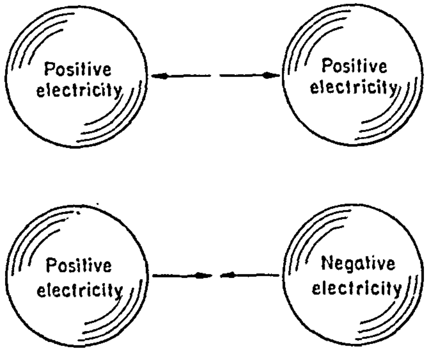


FIG. 91.—Charges of electricity exhibit mutual forces and again there is no known connecting medium.

either force may be called the applied force, so to which object we apply the term action and to which the term reaction depends on the point of view.

In the above cases we had examples where there was no relative motion of the two objects involved and where there was no motion of either object with respect to its surroundings (for example, a person sitting on a chair). We also had examples where there was no relative motion of the two objects but where both were moving with respect to the surroundings (for example, a person dragging a sled over snow).

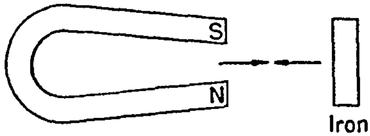


FIG. 92.—Magnetic forces without a known connecting medium exhibit mutual forces of action and reaction.

In the latter kind of case we did not consider whether the system of person and sled were moving at a steady rate or whether they were accelerating with respect to the ground. Newton's law, that action equals reaction, would hold equally

well for either case. If there is no acceleration we can say

$$\text{Applied force} = \text{Frictional force}$$

In case the applied force exceeds the frictional force there will be part of the applied force available to accelerate the system. We can now write

$$\text{Applied force} = \text{Frictional force} + (\text{mass} \times \text{acceleration}).$$

If the frictional force is negligible we may write

$$\text{Applied force} = \text{mass} \times \text{acceleration}$$
$$F = ma$$

This expression is in the familiar form in which we applied Newton's first two laws of motion in Chapter 11.

The important point in this discussion is that Newton's third law of motion applies both to static cases (the ones in which there is no acceleration as a result of the action) and to dynamic cases (those in which acceleration is produced as a result of the action).

2. Impact

There is an important group of cases to which Newton's third law applies where motion of the two objects with respect to one another takes place as a result of the action. Familiar examples include a ball bouncing on a floor, a boy jumping from a boat with the boat free to move away as he jumps, a projectile in a cannon, which through the medium of the gases in the gun barrel, pushes backward on the cannon while the cannon, through the same medium, pushes forward on the projectile. All actions of this type may be called impacts.

Some further appreciation of what goes on in cases of this kind may be gleaned from considering the case of a boy who runs over a dock and jumps into a row boat which is initially standing still. The boat now moves off in the direction in which the boy was traveling. This motion shows that the boy's impact on the boat resulted in applying a force to the boat. This was the action. But the motion of the boy and boat is now at a slower rate than that of the boy as he jumped.

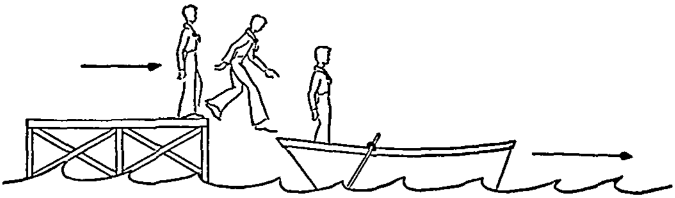


FIG. 93.—When a boy jumps with forward motion into a boat he imparts motion to the boat and the boat slows down his motion.

So we conclude that the boat must have pushed back against the boy to slow him down. This was the reaction.

In the above example, and in all other cases of impact, the quantity of motion (momentum, mv) of each object is changed as a result of the impact.

The product of the force acting with the time during which it acts is called the impulse of the force.

$$\text{Force} \times \text{time} = \text{Impulse of the force}$$

The quantity of motion of an object free to move will be changed in an impact (where frictional and other forces are negligible) by an amount proportional to the force and to the time that the force acts. If our units for force, time, and momentum are properly chosen we may say that the change in momentum will be equal to the product of the force and time it is acting.

$$\text{Force} \times \text{time} = \text{Change in momentum}$$

3. Total Momentum Before and After Impact

Let us consider further the boy and boat problem. This time we will suppose that the boy is on the boat and is about to jump off. We know from experience that the boat will move backward as he jumps forward from it. The force between his feet and the boat as he jumps will be the same against his feet as against the boat, but will be oppositely directed.

This force, F_1 will accelerate the boy according to

$$F_1 = m_1 a_1$$

FIG. 91.—When a boy jumps from a boat, the boat recoils in the opposite direction.

where m_1 is the mass of the boy and a_1 is his acceleration, as we know from the preceding chapter.

Similarly, for the boat we may write

$$F_2 = m_2 a_2$$

where m_2 is the mass of the boat and a_2 is its acceleration.

Since the forces F_1 and F_2 are equal in magnitude we may write

$$m_1 a_1 = m_2 a_2$$

Now let t be the length of time the boy's shoes are in contact with the boat as he jumps. We can multiply the above equation by t and obtain

$$m_1 a_1 t = m_2 a_2 t$$
$$m_1 v_1 = m_2 v_2 \quad (1)$$

where v_1 is the velocity of the boy and v_2 that of the boat immediately after the jump.

The quantity, mv (mass $\times$ velocity), is, as we have seen, called momentum. Since the two momentums in the above example are equal but oppositely directed, it follows that the resultant momentum of the whole system of boy and boat after the jump is zero, the same as it was before the jump.

Example 1. A boy weighing 150 lb. jumps from a boat weighing 300 lb. Find the velocity of recoil of the boat if the boy's forward velocity is 10 ft. per sec. In equation (1) above we write

$$150 \times 10 = 300 \times v_2$$

From which $v_2 = 5$ ft. per sec.

We may now consider the boy and boat problem when the boy jumps on to the boat. See Figure 93.

In this case, when the boy exerts a force in landing on the boat and the boat moves under this force, the boat is obliged to carry the boy with it; so that the mass of the boat plus that of the boy must be considered. Equation (1) above becomes

$$m_1v_1 = (m_1 + m_2)v_2 \qquad (2)$$

The right-hand side of the equation is the momentum after the impact of the boy and the boat and the left-hand side is the total momentum before the impact, for the boat was assumed to be standing still initially, so that only the boy had momentum. Again the momentum before impact equals the momentum after impact.

Example 2. A boy weighing 150 lb. jumps from a dock with a velocity of 10 ft. per sec. to a 300-lb row boat initially standing still.

From equation (2) above

$$150 \times 10 = (150 + 300)v_2$$
$$1500 = 450v_2$$
$$v_2 = \frac{1500}{450} = 3.33 \text{ ft. per sec.}$$

It is important to emphasize, as has been already mentioned, that the total momentum before impact is equal to the total momentum after impact. In the problem where the boy first stands on the boat and then jumps off, the momentum is zero before the jump and after the jump there is equal momentum in opposite directions. The net final momentum is therefore zero. In the second boat case the total momentum before impact is retained after impact, although the velocities are altered in magnitude.

Example 3. A cannon whose barrel weighs 65 tons shoots a projectile weighing 1200 lb. with a muzzle velocity of 2000 ft. per sec. Find the velocity of recoil of the barrel.

$$65 \text{ tons} = 65 \times 2000 = 130{,}000 \text{ lb.}$$

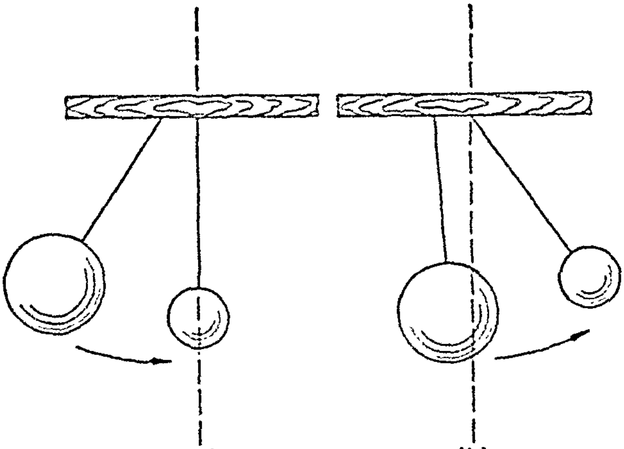


FIG. 95.—An impacting sphere imparts forward motion to a lighter sphere and then continues to move forward at reduced velocity.

From equation (1), p. 193, $m_1v_1 = m_2v_2$

$$1200 \times 2000 = 130{,}000v_2$$

$$v_2 = 1200 \times \frac{2000}{130{,}000}$$

$$= 18.46 \text{ ft. per sec.}$$

4. Elastic and Inelastic Impact

The above example of the boy jumping on to a boat and then moving with it is called inelastic impact as distinguished from a case like that of a bouncing ball where the impacting object rebounds, or at least does not stick to the object struck. The latter type is called elastic impact.

We may study this case by examining an experiment where two spheres are hung on strings. One sphere is pulled back and then released so that it will strike the other sphere.

If the impacting sphere is more massive than the second sphere, both will move forward after the impact, and the momentum will be divided between the two. See Figure 95.

If the spheres have equal masses, the first is completely stopped in the impact and the second moves forward with velocity (and momentum) equal to that of the impacting sphere. See Figure 96.

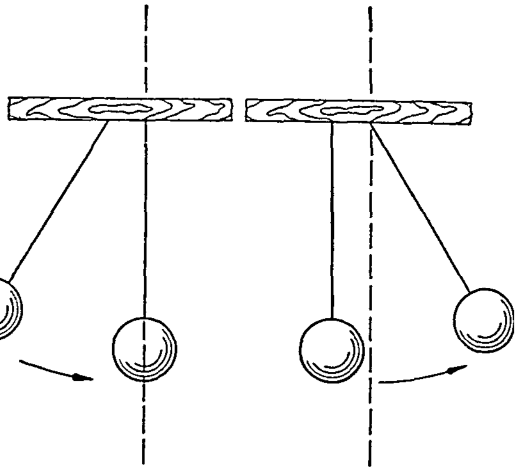


FIG. 96.—An impacting sphere imparts equal velocity to a sphere of its own mass and comes to rest itself.

If the impacting sphere is less massive than the second sphere, its motion is reversed while the more massive second sphere moves forward. The original momentum of the first sphere is equal to the foward momentum of the second sphere minus the reverse momentum of the rebounding first sphere. See Figure 97.

If the impacting sphere is very much less massive than the second sphere (consider a small rubber ball falling on the earth) we observe that the velocity of recoil of the light impacting sphere is approximately equal to its initial forward velocity. In actual practice this may be observed by noting that the ball will bounce back almost to the height from which it was dropped. (Failure to return to the same height may be used as a measure of departure from perfect elasticity.)

The law of impact, namely that total momentum before impact is equal to total momentum after impact, holds for all cases of elastic impact as well as for inelastic impact.

Although no momentum is lost in impact, the energy stored in the motion of the objects is partly dissipated in most cases. If no energy is lost, the impact is said to be a perfectly elastic one. A familiar example of partially elastic impacts

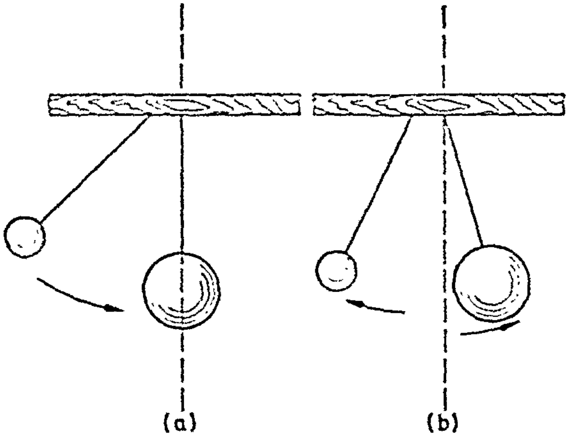


FIG. 97.—An impacting sphere imparts forward velocity to a more massive sphere and at the same time it rebounds.

may be observed by comparing the heights to which an old and a new rubber ball will bounce when dropped from the same height on to a hard surface.

The energy lost from the motion in impact is converted into heat, sound, or in some cases, light. No impact of ordinary objects is entirely free from energy losses of one or more of these types, even when the materials of the objects themselves are perfectly elastic.

5. Force Resulting During Impacts

We know both from experience and from the developments in the preceding chapter that if an object is in motion it can be stopped by the proper application of a force which may be either large or small provided it is applied for sufficient time. Of course, a small force will have to be in action longer than a large one. The gist of these statements is that the product of force and time can result in a change of momentum of the object to which the force is applied. In symbols this can be stated thus:

$$Ft = m(v_2 - v_1) \tag{8}$$

where F is the force applied, t is the time of application and

v_1 and v_2 are the velocities of the mass at the beginning and end of the time t; that is, $(v_2 - v_1)$ is the change in velocity.

From this equation we can solve for F

$$F = \frac{m(v_2 - v_1)}{t} \tag{4}$$

The right-hand side of this equation is the rate of change of momentum; so that in the case of an impact we find it convenient to say that the force acting on either object is equal to the rate of change of momentum.

If the value of m, the mass, does not change with velocity (as we assume here) we can write

$$F = m\frac{v_2 - v_1}{t} \tag{5}$$

or

$$F = ma \tag{6}$$

This is the same expression for Newton's laws that we learned in Chapter 11.

Example 1. A 2800-lb. automobile traveling 30 m.p.h. is equipped with a special spring-type bumper which stops the car in 0.1 second on impact with a solid wall. Find the force against the wall.

We first reduce the velocity to ft. per sec.

$$30 \text{ m.p.h.} \equiv 45 \text{ ft. per sec. approx.}$$

The final velocity is zero. Hence the change in momentum is simply equal to the momentum before impact. In equation (4) we may write

$$F = \frac{2800 \times 45}{0.1} = 1{,}260{,}000 \text{ lbal.}$$
$$\equiv 39{,}100 \text{ lb. approx.}$$

Example 2. A baseball weighing 0.55 lb. travels at the rate of 70 ft. per sec. It meets a swinging bat in such a manner that the direction of motion is reversed and the ball moves back at the rate of 90 ft. per sec. If the ball was in contact with the bat for 0.01 sec. find the average force against either

the bat or the ball. (Neglect any change in momentum of the bat.)

The total change in momentum is

$$(0.55 \times 70) + (0.55 \times 90) = 88 \text{ lb.} \times \text{ft. per sec.}$$

Since this change in momentum takes place in 0.01 sec. the rate of change is

$$\frac{88}{0.01} = 8800 \text{ lb.} \times \text{ft. per sec. per sec.}$$

But the rate of change of momentum, as we have seen, is equal to force so that the force in question on ball or bat is 8800 poundals.

If the force is desired in pounds we divide this result by 32.2

$$\frac{8800}{32.2} = 273 \text{ lb. approx.}$$

Notice that the rate of change in momentum determined as in the above example always gives force in absolute units; namely, poundals or dynes.

Some Important Facts

1. Newton's Third Law: Whenever a force acts on a body, it must be opposed by another equal and oppositely directed force; that is, to every action there must be an equal and opposite reaction.

2. Both action and reaction may be stationary or they may move together or they may result in the relative motion of the two objects with which the action and reaction are associated. In other words, action and reaction may be either in static or dynamic equilibrium.

3. In the case of collision or impact between two bodies, the total momentum is the same after impact as before.

4. If the colliding bodies stay together after hitting, the impact is called inelastic. If they bounce away from each other, the impact is called elastic. Many impacts are only partially elastic. In all cases the total momentum is the same after as before impact; that is, Newton's third law applies to elastic as well as to inelastic impact. However, energy may be and generally is dissipated in any impact.

5. Whenever a force moves a body, the force, in absolute units, equals the rate of change of momentum; that is,

$$F = \frac{m(v_2 - v_1)}{t}$$

Since rate of change of velocity is acceleration, this equation reduces to the familiar, $F = ma$

Generalizations

For a force to act on a body it must have opposition due to the inertia of the body, friction or another force. If the opposition is due to the inertia of the body, the force equals rate of change of momentum; that is, mass times acceleration. This action may involve elastic or inelastic impact of bodies, in which case total momentum after impact is the same as before impact.

Questions and Problems

Group A

1. Distinguish between the terms action and reaction.

2. State the Law of Action and Reaction.

3. Mention several instances which will illustrate each of Newton's three laws of motion.

4. Why do ocean liners seldom dock under their own power?

5. With the same ammunition, which would kick more, a 6-lb. or a 9-lb. rifle? Explain.

6. A boy whose mass is 125 lb. jumps from a sled weighing 50 lb. with a velocity of 8 ft. per sec. Neglect friction between sled and ice and compute the velocity of recoil of the sled. 20 ft. per sec.

7. Imagine yourself to be located on a perfectly smooth surface so that the coefficient of friction between yourself and the surface is zero How could you travel over the surface?

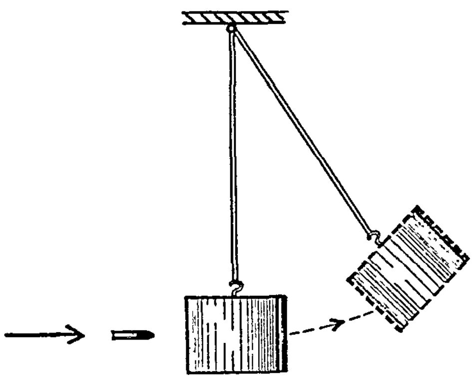


FIG. 98.—Firing a bullet into a suspended block.

Group B

1. A block weighing 2000 grams is hung as shown in Figure 98. A bullet weighing 30 grams is fired into the block and the latter is observed to start moving with a velocity of 8 meters per sec. Compute the velocity of the bullet. 541 m. per sec.

2. A bullet weighing 0.1 lb. traveling 2000 ft. per sec. strikes a bird flying in the same direction as the bullet with a velocity of 50 ft. per sec. The bird weighs 2.0 lb. Assume that the bullet lodges in the bird and compute the velocity of the bird immediately after being shot. (Suggestion: Write on the left hand side of the equation the total momentum of bullet and bird just before impact and equate it to the total momentum immediately after impact.) 143 f.p.s.

3. Two spheres are hung as shown in Figure 97. Sphere A weighs 100 grams and sphere B weighs 250 grams. Sphere A is pulled to one side and falls against B with velocity of 30 cm. per sec. It rebounds with velocity of 18 cm. per sec. Compute the velocity of sphere B immediately after impact. 19.2 cm. per sec.

4. A 128-lb. boy standing on the seat of a 64-lb. canoe jumps to a dock 2 ft. distant. If he exerts a force of 100 lb. (3200 poundals) for $\frac{1}{25}$ of a second, what velocity does he give (*a*) himself? (*b*) the canoe? How long must he be in the air to reach the dock? Will he probably make it?

(*a*) 1 ft. per sec.; (*b*) 2 ft. per sec.

5. A bullet weighing half an ounce is fired from a rifle weighing 10 lb., with a muzzle velocity of 3000 ft. per second. Calculate (*a*) the velocity of the rifle; (*b*) the momentum of the rifle; (*c*) the momentum of the bullet.

(*a*) 9.375 ft. per sec.; (*b*) and (*c*) 93.75 lb. ft. per sec.

6. An automobile weighing 3700 lb. traveling 40 m.p.h. collides with a concrete abutment of a bridge and comes to a complete stop in 0.08 sec. Find the average force which the abutment had to withstand.

86,180 lb.

7. A machine gun shoots 500 bullets per minute, each bullet weighing 25 grams. The muzzle velocity is 650 meters per sec. Find the average force necessary to hold the gun in place against the tendency to recoil. (Suggestion: Find the number of bullets fired per sec. The mass of this number of bullets multiplied by the velocity will give the *rate* of change of momentum.) 13.82 Kg.

8. A stream of water from a fire hose deposits 100 gallons of water per minute horizontally against a wall at a velocity of 20 ft. per sec. If it rebounds with equal velocity, find the average force against the wall. (Suggestion: Find the mass of water striking the wall per second. In computing the total change of momentum per second remember that the water rebounds with velocity equal to the initial velocity.) 17.3 lb.

9. If the foot of a player is in contact with a football for 0.04 seconds, how much average force must he exert to give a 2.5-lb. ball a velocity of 60 ft. per sec.? 116 lb.

10. A football player tackles and hangs on to a ball carrier. The former weighs 190 lb., the latter 165 lb. The speed of the former is 20 ft. per sec. and that of the latter is 15 ft. per sec. Find the average force on either player if the impact lasts 0.3 sec. and (1) is head-on, or (2) from the rear.

329 lb. 45.7 lb.

11. In problem B13 of Chapter 12 find the momentum of the projectile and the cannon.

12. A 3600-lb. car traveling 50 m.p.h. has a head-on collision with a 4300-lb. car traveling 30 m.p.h. The cars get stuck together in the impact. Find the velocity of the wreck. Compare the kinetic energy of the cars before impact with that of the wreck after impact. What became of the rest of the original kinetic energy?

9.68 f.p.s. 450,000 ft. lb. 11,500 ft. lb.

Experimental Problems

1. Suspend by fine threads 10 or 12 spheres of approximately identical mass so that they just touch in a straight line. Withdraw some 45° or so from the line, first one, then two, then three of these spheres, in each case letting those that are withdrawn fall against the line. Note results and draw any conclusions which seem justified.

2. Grasp any spring balance which will register about 30 lb. with one thumb through the hook. With arms extended straight in front, pull the balance to full scale registry.

Hook two such balances together and repeat.

When you make one balance register full scale, say 30 lb., what force do you think you are exerting with each hand?

When you make both balances register full scale, what force are you exerting with each hand?

As a check experiment, hang one balance from some firm support, and hang the second balance on the hook of the first. Then hang weights on the hook of the second balance, recording both balance readings for each weight.

What general conclusion is justified?

Chapter 14

SOME SPECIAL PROBLEMS IN MECHANICS

This chapter is concerned with motions that do not take place in straight lines. Common examples are riding in a curve on a bicycle or in a car, or riding in complete circles as on a merry-go-round.

Variations on the simple experiment of twirling a stone or other small object on the end of a string are suggested for studying the relations between forces and such motions.

Practical applications in the handling and use of central acting forces are to be found in the banking of roads, the action of spinning type clothes dryers, the cream separator, and the speed governor for engines.

A further study of the behavior of rotating systems permits us to consider angular acceleration, angular inertia, angular momentum and angular kinetic energy, and the torques associated with them in a manner similar to that in which corresponding items in linear motion are covered by Newton's laws of motion.

The special case of trying to rotate the axis of rotation of an object in a direction at right angles to itself gives rise to a new phenomenon which is called gyroscopic action. This effect is discussed but briefly in this text.

The chapter closes with the special problem of shooting projectiles where the force of gravity causes a downward acceleration of the projectile while the projectile attempts to move forward as a result of its initial muzzle velocity.

Emphasis in this chapter is placed on a description of these various motions and a study of them from observation rather than on the formal development of the theory involved.

1. Motion in Curved Paths

In all of our study of moving bodies thus far, we have been concerned with motion in a straight line; yet much of our everyday experience involves traveling in curves, sometimes even in complete circles. One early experience in life is to stand in a bus or street car when it suddenly starts to go round a curve. We grab hastily for a support on which to hold, or we lean in, for otherwise our bodies tend to travel in the straight line in which we had been moving, while the bus or car follows the curve. The practical result, unless we support

ourselves properly, is that we fall towards the outside of the curve—sidewise with respect to the bus or car.

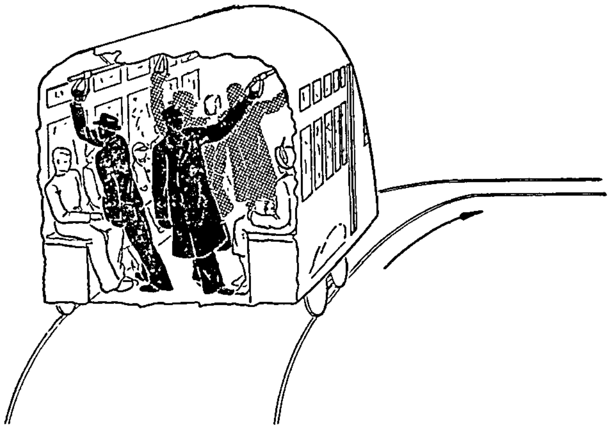


Fig. 99.—As a street car goes round a curve, people grab for straps.

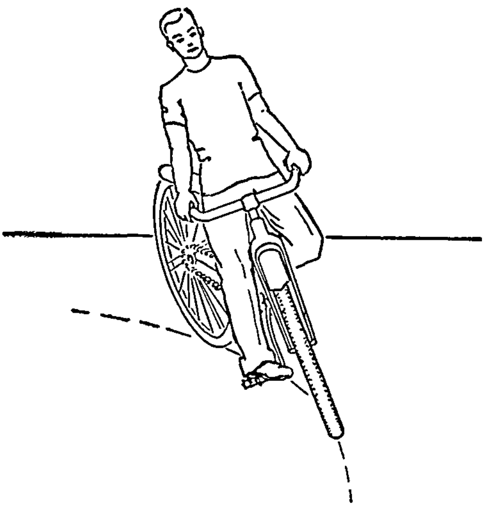


Fig. 100.—When a bicycle and rider travel in a curve, the two lean inward.

Similarly when one learns to ride a bicycle he finds that leaning in is necessary to go around a curve. Also it is easier when riding in an automobile to travel in a curve if the

road is banked; that is, a road where the outer edge is higher than the inner. If the banking is just right for the speed at which one takes the curve, he gets none of the sidewise effect with respect to the car.

An example of motion in a complete circle is twirling a small object tied to the end of a string. So also is riding on a merry-go-round. A modification of the object on the string experiment is to try swinging a bucket full of water in vertical circles with the hope that the water will not fall out on one's head.

2. Central Forces

Examples like twirling the object on the string tell us that it is necessary to exert a force on the object by means of a pull transmitted through the string to keep the object going around in the circle. This force that must be exerted on the object to keep it going in a circle is called a *centripetal force*.

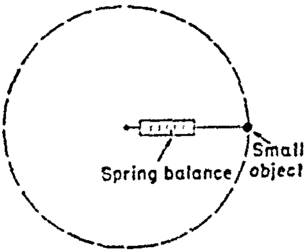


Fig. 101.—Twirling an object on a string suggests a group of experiments for studying angular motions.

Twirling the object on the end of the string suggests a whole series of experiments for making a study of centripetal forces. For example, we can keep the length of string constant and always twirl the object at the same speed. Meantime we can try objects of various masses. We can measure the force with which we have to pull on each object of different mass by means of a spring scale in the string. In practice it will be hard to do this experiment accurately, but assuming that it can be done, we will find that the force is directly proportional to the mass of the object.

Next we repeat the experiment keeping the length of the string constant and using the same object for all the experiments. This time we will vary the rate of twirling. We find that double the speed of the object in its path requires four times the pull to keep it following the curved path. Three times the speed takes nine times the pull. So we conclude

that the centripetal force changes as the square of the speed in the path.

Now we repeat the experiment once more, this time using always the same object and the same path speed, but different lengths of string. (We have to use different rates of rotation in order to keep the path speed the same for differing string lengths.) We find now that the shorter the string the greater the force that is required.

If we put together the results of these three experiments, we can write

$$\text{Centripetal Force is proportional to } \frac{\text{Mass} \times (\text{velocity})^2}{\text{radius}} \quad (1)$$

If we examine the results still more carefully we find that we can take out the "is proportional to" and write "equals" if we change the force as read from the ordinary spring balance into absolute units of force (dynes or poundals) about which we learned in Chapter 11. So we may write

$$\text{Centripetal Force} = \frac{\text{Mass} \times (\text{velocity})^2}{\text{radius}} \quad (2)$$

$$F_c = \frac{mv^2}{r} \quad (3)$$

From the fact that absolute units of force are required for central force calculations, we conclude that it is the mass of the object (that is, its inertia) that counts, not its weight. This we have anticipated by writing mass on the right-hand side of the above equation.

In order to keep gravity out of the problem of the twirling object as far as possible, it would be well to swing the object in circles in a horizontal plane.

The pull of the finger in this example on the object through the medium of the string, which we have called centripetal force, may be considered as the "action" in Newton's laws type of analysis of forces and masses. The "reaction" to this force, of course, is the force with which the mass pulls back (also through the string) on the finger. It is called *centrifugal force.*

If one cuts the string, both forces cease to exist simultaneously, and the object continues to move in a straight line in whatever was the direction of its motion at the instant the string was cut. In other words, it moves tangentially to the curve in which it had been moving. A simple demonstration of this effect is to put water on a wheel and then give the wheel a spin. As the water leaves the edge of the wheel and no longer experiences any centripetal force, it will be seen to fly off tangentially. Similarly, the burning powder from a Fourth of July pin wheel is a beautiful example of the same type of tangential motion.

In the above discussion we have seen that the expression for centripetal forces may be found from a study of things that we can observe, in this case objects twirled on a string. It is also possible to derive this expression from material that we have studied in earlier chapters on motion, force, and force and motion. As a feat of logic, this would be the more elegant way to develop a formula, and the interested student will find this development in many other text books in case he cannot think it through himself. But the finding of a law of nature by direct experimentation as suggested above is equally valid and perhaps more exciting.

3. The Bucket of Water Demonstration

In the experiment of swinging a bucket of water over one's head, the bucket should be swung in circles in a vertical plane, since the whole point of the trick is to show that water can be kept in a bucket when the bucket is upside down.

When the bucket is at the top of its path, the whole weight of the water contributes to the centripetal force required to keep it traveling in the circle. At the bottom of the path centripetal force is conveyed from the person's hand to the bottom of the pail which must then exert a force on the water equal both to the weight of the water and to the centripetal force necessary to keep the water traveling in the circle.

At the critical speed, the water at the top of the path does not push against the bottom of the pail and it does not tend

to run out; while at the bottom of the path it pushes against the bottom of the pail with twice its weight; and the bottom of the pail must supply an equal and opposite force in order to support the weight of the water and also supply the necessary centripetal force. At the top of the path the person's arm does not need to supply any force, and at the bottom of the path it supplies a force equal to twice the weight of the water. (For simplicity we have neglected the weight of the bucket in this discussion.)

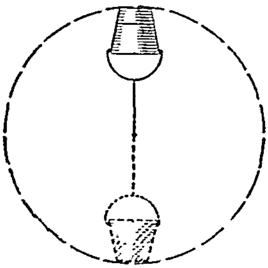


FIG. 102.—A bucket of water can be swung in a vertical circle without the water running out.

To perform this experiment safely from the point of view of keeping your head dry, it is well to be sure that you twirl the bucket of water at higher than the critical speed.

4. Banked Roads

An application of this knowledge of central forces to the construction of roadbeds either for cars or trains is suggested from our experience with bicycle riding. We all know that to take a curve on a bicycle it is necessary to lean in. Otherwise our bodies and the mass of the bicycle itself tend to continue in a straight line while we try to steer the bicycle around the bend. The effect is that we fall sidewise with respect to the bicycle; that is, outwards with respect to the curve.

When we lean in, the center of mass of the system of bicycle and rider is no longer over the points of contact of wheels and earth but is displaced towards the inside of the curve. In Figure 103 the bicycle and rider are indicated by a slanting line and the roadbed by a horizontal line. The arrow suggests the direction of the curve. Superimposed on this sketch is a force diagram showing the weight, W, of the bicycle and rider as a vertical line and showing also the thrust, T, of the road against the wheels of the bicycle. We imagine this thrust to be split into an upward component equal and

opposite to the weight of the moving system and a horizontal force directed inward to provide the centripetal force necessary to maintain the motion in the curve.

Of course the bicycle and rider react to these two components, the downward force due to the weight of the system being equal to the upward component of the road's thrust,

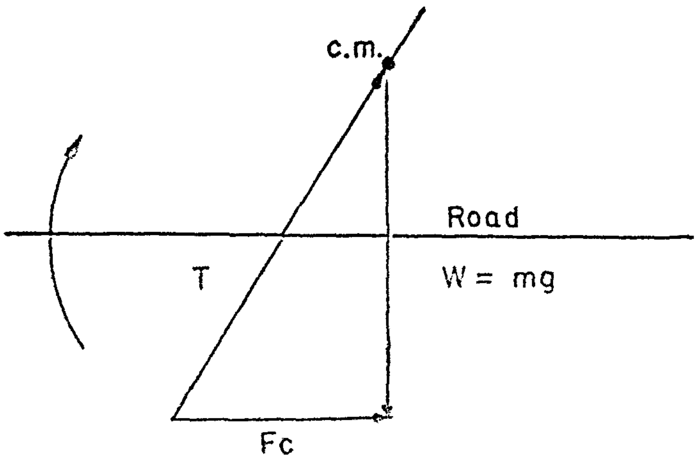


FIG. 103.—When a bicycle rider leans in on a curve, the horizontal component of his thrust against the road is balanced by frictional force between road and tires. This force is the centripetal force that keeps him traveling in the curve.

and the outward thrust of the moving system (centrifugal force) being equal and opposite to the horizontal component of the road's thrust (centripetal force).

From experience we know that the matched horizontal forces cannot exist if the road is completely slippery. So in this simple case we depend entirely on friction between the bicycle wheels and the surface of the road.

Next we try the experiment of tipping the road edgewise so that the outer edge is higher than the inner. Such a situation is shown in Figure 104. In this particular case we have shown the road surface to be at right angles to the bicycle. Again by a superimposed force diagram the weight of the bicycle and rider is indicated as is also the thrust of the road.

Again the thrust of the road can be divided into components, one upward to support the weight of the moving system and one horizontal to provide the needed centripetal force. But since the thrust of the road is perpendicular to the roadbed itself, it follows that there is no component of the thrust parallel to the road, and so friction is not a factor. The bicycle will not slide sidewise on the road and the rider

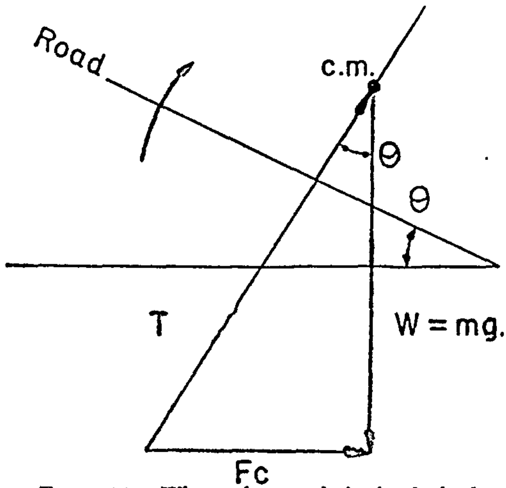


Fig. 104.—When the road is banked the centripetal force required to keep the bicycle and rider traveling in the curve is provided by the horizontal component of the road's reaction to the thrust and frictional forces are not required.

will have no sensation of tendencies to pull him off sidewise with respect to the bicycle. His only sensation, which he will hardly notice at ordinary speeds, is that he sits a little heavier in the saddle.

Now since central forces change both with speed and curvature, it follows that the above state of perfection will not exist if either of these items change, for we cannot conveniently change the tilt of the road to suit every rider.

With a bicycle the problem is simple—the rider simply leans in or out and adds this method of supplying centripetal force to what the banked road supplies. He does depend now

partly on friction, but not to the same extent that he would if the road were not banked at all.

With an automobile, no such change in inward leaning is possible, for the general construction of the car forces it to stay parallel to the road. If the speed is wrong for the banking on that particular curve, friction between the roadbed and the wheels must supply the extra centripetal force for speeds that are too high, and must prevent the car from skidding inward for speeds that are too low.

If one studies Figure 104 and compares the space diagram with the force diagram, he will see that the angles marked θ are equal (since their sides are mutually perpendicular). The horizontal component of the road's thrust is the centripetal force, mv^2/r, and the vertical component is equal to the weight of the moving system. So, for the tangent of the angle θ we can write

$$\text{Tan } \theta = \frac{\frac{mv^2}{r}}{mg} \tag{4}$$

Note again that for the weight of the system we write mg rather than m, for the force of the moving system due to its weight must be expressed in absolute force units, since the centripetal force, mv^2/r, is in absolute force units. This equation reduces to

$$\text{Tan } \theta = \frac{v^2}{gr} \tag{5}$$

and so tells us how to compute the angle of banking of a road for a specified speed and radius of curvature.

This banking of roadways is used extensively on both highways and railroads. In exaggerated form we find it in motor dromes, where motorcycle riders at high speeds and low radii of curvature climb almost vertical walls.

An interesting item always used on railroads, and which should be used more in highway construction, is that a sharp curve is never begun abruptly. The curve starts very gently, that is, with a long radius of curvature. The banking then

begins gently also. This construction at each end of a curve is called an *easement*. It reduces sudden jolts to equipment and makes the ride smoother.

5. Some Applications of Central Forces

A practical application of central forces may be found in the rotating clothes dryer. Here wet clothes are spun in a cage. Heavy droplets of water do not receive sufficient centripetal force from the clothing to keep them traveling in

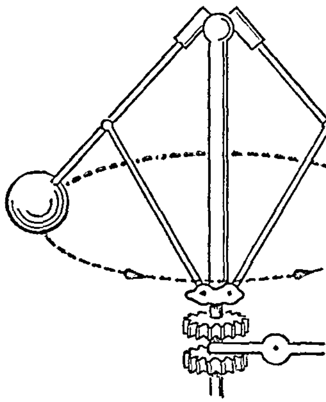


FIG. 105.—A centrifugal speed governor such as is used on a steam engine.

circles, and they tend to move tangentially to the motion of rotation. This means that droplets of water throughout the wet clothing move towards the cage through which they escape.

Similarly if a mixture of two fluids of different densities, such as milk and cream, is spun in a container, the heavier particles tend to occupy positions toward the outer part of the system. This means that a concentration of skim milk occurs near the outer section of the container and a concentration of cream near the center. Openings from these two regions will respectively deliver skim milk and cream.

Another application of central forces is the speed governor. Two large masses are suspended on arms that are free to swing and rotate. When the angular speed of the device

increases, the required centripetal force on the masses increases and they swing up on the arms so that a larger component of the tension in the supporting arms is available to supply the centripetal force. It is easy to have this motion of the masses move a lever that in turn can be used to control the speed of the engine supplying the rotatory motion to this governor.

6. Rotational Inertia

In studying the problems of central forces up to this point, we have assumed that the massive object performing the motion was small in comparison to the radius of curvature. If we now substitute for the small object on the end of a string in Section 2 a stick weighing the same as the object but extending all the way from the hand to the position formerly occupied by the small object, we can again perform a series of experiments and arrive at similar but not quite as simple results. It is easy to see that the inner end of the stick, although going around at the same number of revolutions per second as the outer end, is traveling at a very much lower speed.

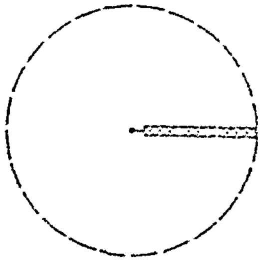


FIG. 106.—If a stick is used in place of the small object in Fig. 101, we find that the distribution of the mass of the moving object complicates our results.

This situation leads to the conclusion that in circular motions, the distribution of the mass of the object with respect to the center about which the rotation takes place is important.

This problem can be studied further by applying torque to a massive wheel. Let us suppose that we have two large wheels such as are used on some stationary engines for fly wheels. Each one is mounted so that it can turn with very little friction. Let the wheels be similar in construction but let one be more massive than the other. In turn we push on each wheel at a point equally distant from the center in both

cases. We notice that the lighter wheel experiences greater angular acceleration than the heavier. This does not surprise us because we have learned to associate inertia with mass.

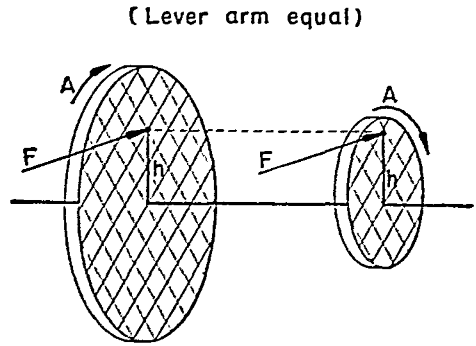


FIG. 107.—Equal torques (in this case equal forces and equal lever arms) give greater angular acceleration to a light wheel than to a heavier one of similar construction.

Now we arrange for some very special wheels. This time each will have the same mass, but one will have most of its mass in the rim which will be supported by light spokes. The

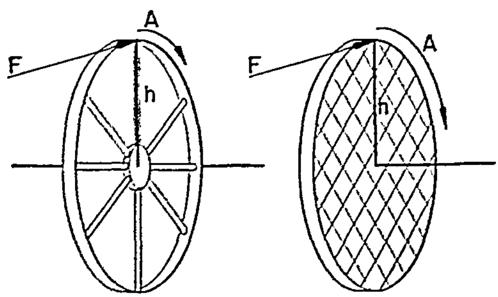


FIG. 108.—Equal torques give greater angular acceleration to a disk type wheel than to a rim type where the two are of equal diameters and equal masses.

other wheel will be built more like a solid disk. This time we find that the disk type wheel gets the greater acceleration from our effort, and since the actual mass is the same in both cases we are driven to the conclusion that the inertial effect

of an object to rotatory motion depends not only on its ordinary inertia (mass) but also on the distribution of this mass.

The inertial property of an object to rotatory motion is sometimes called angular inertia and more commonly *moment of inertia.* This name has no special significance to the beginner in the study of mechanics, but it has to be accepted because of its general use. A discussion of the units in which moments of inertia are expressed will be found in more advanced texts.

7. Torques and Motions of Rotation

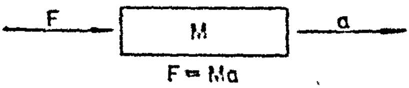
a. Angular Acceleration

In the experiments described in Section 6 we applied a force at a lever arm after the fashion described in Chapter 7. The product of a force and its lever arm we have learned to call a torque. The application of this torque changed the state of angular motion of the object. In other words, an angular acceleration was produced.

The relation between torque, angular acceleration and the moment of inertia of a system is similar to the simple expression of Newton's laws for linear motions. (See page 154.) For linear motions we have

$$\text{Force} = \text{Mass} \times \text{acceleration} \tag{6}$$

$$F = ma \tag{7}$$



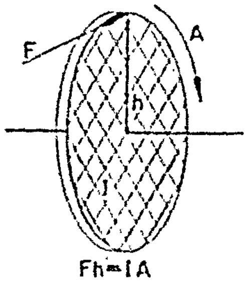


FIG. 109.—(*Left*) A force accelerates a mass in a linear motion.

FIG. 110.—(*Right*) A torque gives angular acceleration to a mass free to rotate. The inertia of the mass to angular acceleration depends on its distribution as well as its amount

For angular motion we may write

$$\text{Torque} = \text{Moment of Inertia} \times \text{Angular Acceleration} \quad (8)$$
$$T = IA \quad (9)$$

If we know any two of the items in equation (9) we can easily solve for the third.

Example. What angular acceleration can be expected if a force of 50 lb. is applied at a radius of 2 ft. to an object with 2,500 units of moment of inertia in the English system? What velocity will it acquire in 10 sec.?

Remembering that all forces must be expressed in absolute units we write for the torque

$$\begin{aligned} \text{Torque} &= \text{force} \times \text{radius arm} \\ T &= 50 \times 32.2 \times 2 \\ &= 3220 \text{ poundals} \times \text{feet} \end{aligned}$$

In equation (9) we substitute this value for torque and insert also the numerical value of the moment of inertia.

$$3220 = 2{,}500A$$

From which $A = \dfrac{3220}{2500}$

$$= 1.29 \text{ radians per sec. per sec.}$$

In 10 sec. the angular velocity will be

$$\begin{aligned} \text{Ang. Vel.} &= \text{Ang. Acc.} \times \text{time} \\ &= 1.29 \times 10 \\ &= 12.9 \text{ radians per sec.} \\ &\equiv 2.05 \text{ turns per sec.} \end{aligned}$$

b. Angular Momentum

Further studies in motions of rotation can be made by continuing to use analogies with linear motions. Always we let torque correspond to force, angular inertia (moment of inertia) to simple inertia (mass), angular velocity to linear velocity, and angular acceleration to linear acceleration. In most calculations it is necessary to express angular velocity in radians per second rather than in turns per second, and forces must be expressed in absolute units.

We have already seen in (*a*) above how the familiar expression of Newton's laws for linear motion can be modified for angular motion to show the relations between torque, moment of inertia and angular acceleration.

We can make a similar development for angular momentum. In Chapters 11 and 13 we learned that momentum in the linear case is given by

Momentum = mass × velocity

So in rotatory motion we write

Angular momentum = Moment of inertia × Angular velocity

We also saw that within a closed system linear momentum remains constant. Similarly we may expect angular momentum to remain constant within a system so long as it is not acted on by torques from without the system.

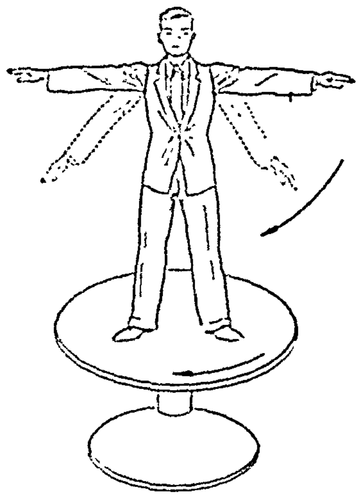


FIG. 111.—A man on a turntable can change his rate of rotation by raising or lowering his arms.

An interesting example in angular momentum may be observed by watching a person standing on a turntable. Have him stretch his hands out from his body and give him two heavy books to hold. Then spin him (but not too fast). He will have momentum depending on his speed of rotation and his moment of inertia as we have seen above. So we write

$$(\text{Angular momentum})_1 = (\text{Moment of inertia})_1 \times (\text{Angular velocity})_1$$

Now we tell him to lower his arms, and we observe that his rate of rotation increases most amazingly. His angular momentum is now

$$(\text{Angular momentum})_2 = (\text{Moment of inertia})_2 \times (\text{Angular velocity})_2$$

Since nothing has been added from outside the rotating system, the angular momentums in the two cases must be equal. In view of the observed fact that angular velocity is greater in the second case we conclude that the moment of inertia has decreased. The lowering of the man's arms and thus the placing of part of the rotating mass closer to the center of rotation accounts for this decrease.

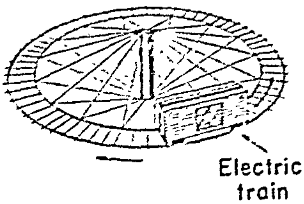


FIG. 112.—An electric engine stands on a track mounted so that it can rotate. If the train is started in one direction, the track will start to rotate in the other.

Another example of the conservation of angular momentum may be seen in the behavior of an electric train on a circular track which is mounted so that the track can rotate. (See Figure 112.) Before the train is started the angular momentum of the system is zero. When the train is started we observe that the train starts in one direction while the track starts to rotate in the opposite direction. Both train and track have angular momentum, but they are oppositely directed; and since in the ideal case the two are also equal, the net angular momentum is still zero.

Interesting variations of this problem consist in (1) loading the train with additional mass, (2) loading the track with additional mass, (3) holding the track so that it cannot move, (4) holding the train so that it cannot move. The first two of these variations, together with the original experiment, demonstrate still further that in isolated systems angular momentum stays constant.

Examples (3) and (4) above are not isolated systems. Part of the system becomes fixed, and energy is expended by the electric motor of the train in building up kinetic energy in only one part of the two parts of the system. Hence only that one part develops momentum.

c. Angular Kinetic Energy

For kinetic energy in the linear case we had

$$\text{K.E.} = \frac{1}{2}\,\text{mass} \times (\text{velocity})^2$$

and for the rotatory case we may write

$$\text{Angular K.E.} = \frac{1}{2}\,\text{moment of inertia} \times (\text{Ang. Vel.})^2$$

A demonstration of the effects of moment of inertia on kinetic energy consists in rolling two wheels down an inclined plane. The two wheels have the same mass and the same

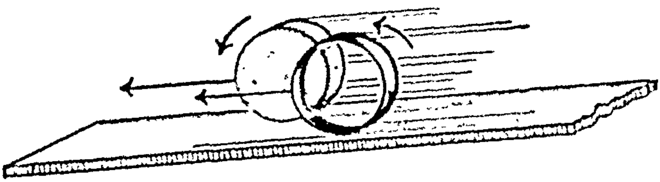


FIG. 118.—When a disk and a rim type wheel of the same mass and diameter roll down an incline, the disk wheel acquires the greater velocity.

diameter, but one is built in the form of a disk and the other as a section of a cylinder. At the top of the plane the potential energy of each wheel is the same as that of the other, for it is the product of its weight and the height of the plane. At the bottom of the plane this potential energy will have been converted into kinetic energy, and should therefore be the same for both wheels. However, the kinetic energy for each wheel will be divided between angular kinetic energy and ordinary linear kinetic energy. For the disk wheel we write the sum of these kinetic energies on the left-hand side of an equation and for the rim type wheel we write them on the right-hand side.

$$(\text{Ang. K.E.})_1 + (\text{Lin. K.E.})_1 = (\text{Ang. K.E.})_2 + (\text{Lin. K.E.})_2$$

We notice in the experiment that the disk wheel (1) reaches the bottom of the incline first and is traveling much faster than the rim type wheel (2). This means that more of its kinetic energy is in the linear motion and less in the angular

motion than in the case of the rim type wheel. The rim type wheel, having a larger moment of inertia receives less angular acceleration in coming down the plane, and even though it is on the plane longer it acquires less angular velocity, so that its linear motion down the plane is slower than that of the disk type wheel. But with its greater moment of inertia, the kinetic energy of its rotating motion is greater than that of the disk type wheel.

8. Gyroscopes

Another example of circular motion is common to our everyday experience but is less easy to study. Any rotating object free in space has a tendency to keep its axis of rotation fixed in direction. If one grabs the axle of a rotating wheel and tries to turn it, he at once observes a force tending to make it jump out of his hands in a direction at right angles to the axis and at right angles to the motion he was trying to impart. This new motion is called *precession*. The total effect is called gyroscopic action and a device capable of showing it is called a gyroscope.

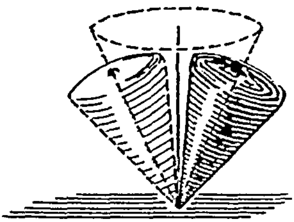


FIG. 114.—Spinning top displaying gyroscopic action.

A very common example is the toy top that we spin to amuse children. If the axis of the spinning top is displaced even slightly from the vertical, a component of gravity will try to turn the axis in the sense of making the top fall over. Instead of falling, the top performs a new slow circular motion which is its motion of precession.

More elaborately constructed gyroscopes furnish interesting demonstrations for the class room. It is also possible to use the effect as a control mechanism, and it is so used to assist in stabilizing ships. It is similarly used to stabilize instrument platforms on ships or other objects which roll or pitch.

Perhaps the most extensive use of the gyroscope is as a compass, for a properly designed gyroscope can be made to tend to keep its axis parallel to the axis of spin of the earth.

Such a compass is independent of the variations that beset magnetic compasses.

9. Projectile Problems

In this chapter we will include one case of nonlinear motion that is not a simple example of circular motion, but is nevertheless fairly common to our experience.

If we throw a ball horizontally, we observe that it drops as it travels and follows a path as indicated in Figure 115. The

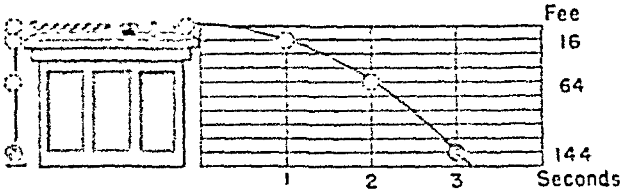


FIG. 115.—A ball with a horizontal velocity falls at the same rate as one dropped from rest.

throwing of a ball is suggested since the motion of the ball is easy to watch. An apparatus has been developed for throwing a ball in this manner and at the same time shoving a second ball off a support at the same height as the ball that is being thrown horizontally. The experiment shows that each reaches the floor at the same instant. These observations show that the ball moving horizontally acquires downward velocity at the same rate as if it were simply falling vertically.

Mathematicians can show that the curve followed under these circumstances is a parabola. In actual problems of this type we are frequently interested in the paths of projectiles rather than in slow moving objects like the one suggested above.

Gravity acts downward and at such time that it has contributed downward velocity equal to the original upward component of the velocity, the projectile will have reached the top of its path. It will then start downward and, neglecting air friction, the drawing for the last half of the flight will look like a mirror image of that for the first half.

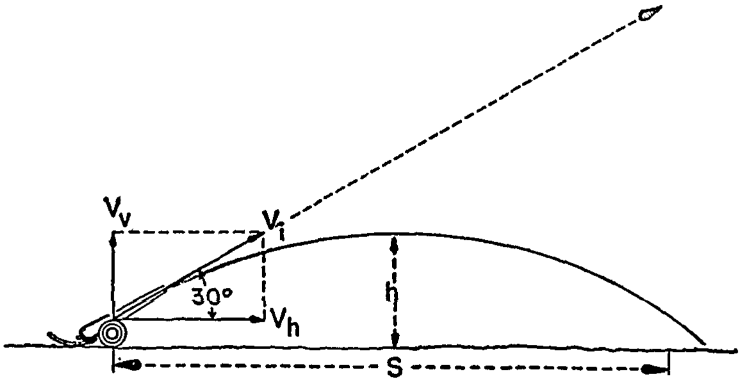


FIG. 116.—For a cannon pointed above the horizontal, the red dotted arrow shows the direction of flight as the projectile leaves the muzzle. The solid red line gives the actual flight path. V_v is the vertical and V_h the horizontal component of the muzzle velocity V_i.

Most numerical problems in the flight of projectiles require the use of some trigonometry. A rather simple example is given below.

Example. A cannon fires a projectile at an angle of 30° to the horizontal and at a muzzle velocity of 2400 ft. per sec. Find the height to which the projectile rises. Find also the range of the projectile.

Referring to Figure 117 we can see that the vertical component of velocity is

$$\begin{aligned} V_v &= 2400 \sin 30° = 2400 \times 0.5 \\ &= 1200 \text{ ft. per sec.} \end{aligned}$$

and the horizontal component is

$$\begin{aligned} V_h &= 2400 \cos 30° = 2400 \times 0.866 \\ &= 2078 \text{ ft. per sec.} \end{aligned}$$

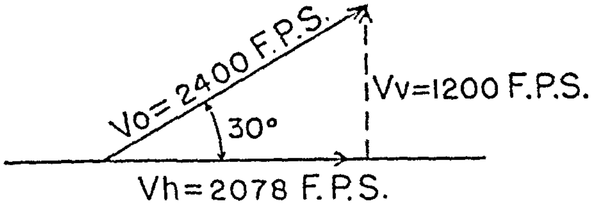


FIG. 117.—The initial velocity, V_o, may be divided into vertical and horizontal components.

At any instant the upward velocity will be the algebraic sum of the upward component of the initial velocity, V_v, and the downward component of velocity due to the action of gravity. At the top of the path the net up and down velocity is zero, so we may write

$$0 = V_v - gt$$

where t is the time required to reach the top of the path and g is 32.2 ft. per sec. per sec.

$$0 = 1200 - 32.2t$$

From which $$t = \frac{1200}{32.2} = 37.3 \text{ sec.}$$

The total time of travel is twice this amount, that is

$$T = 2t = 2 \times 37.3 = 74.6 \text{ sec.}$$

Since the starting component of upward velocity is 1200 ft. per sec. and the value at the top of the path is zero, the average velocity is 600 ft. per sec. The height to which the projectile rises is

$$\text{Height} = \text{av. vel.} \times \text{time}$$
$$H = 600 \times 37.3 = 22{,}380 \text{ ft.} = 4.24 \text{ miles, approx.}$$

The horizontal velocity remains steady at 2078 ft. per sec., so that the range is

$$\text{Range} = (\text{horizontal velocity}) \times \text{time}$$
$$R = 2078 \times 74.6 = 155{,}000 \text{ ft. approx.}$$
$$= 29.4 \text{ miles approx.}$$

Some Important Facts

1. Due to the straight-line tendency of inertia in motion, a centrally directed force is necessary to cause a body to move in a curved path.

If we keep this fact in mind we find that we can apply Newton's laws of motion to curvilinear motion as conveniently as to rectilinear motion.

2. Centrally directed forces are called centripetal ones. The oppositely directed reaction to a centripetal force is called a centrifugal force. Either a centripetal force or its centrifugal reaction may be expressed in absolute units as:

$$F_c = \frac{MV^2}{r}$$

3. Swinging a bucket of water in a vertical circle, throwing the hammer, rotating clothes dryers, cream separators, speed governors, centrifuges, banking curves, etc. are examples of the common sense recognition of the centrifugal reaction of inertia in cases of curved motion, and the necessity of providing sufficient centripetal force to keep the moving body in a curved path.

4. In banking curves, and in airplane flight, the correct angle of bank is indicated by the expression:

$$\text{Tan } \theta = \frac{V^2}{gr}$$

5. Inertia in circular motion depends not only on the mass rotating but also on the distribution of this mass with respect to the center of rotation. The further the mass is from the axis of rotation, the greater is the rotational inertia. Rotational inertia is called moment of inertia.

6. The momentum of a body in circular motion, that is, angular momentum, equals the moment of inertia times the angular velocity:

$$M_r = Iw$$

7. Angular kinetic energy equals one-half the moment of inertia times the square of the angular velocity.

$$\text{K.E.} = \frac{1}{2} Iw^2$$

8. A continuously rotating body is often called a gyroscope. Its angular momentum may be utilized in gyroscopic stabilizers and compasses; and its tendency to precess when the direction of its axis is disturbed is used in automatic airplane piloting devices.

9. Since a projectile is accelerated by gravity independently of its initial velocity, it follows a curved path or trajectory.

Generalizations

Circular Motion

1. Since the motion of a free object tends always to be in a straight line, the application of a force is always necessary to produce motion along a curved path. For uniform motion this force is always directed along a radius of the curved path. It is called a centripetal force. This centripetal force is the action and to it there is a reaction called a centrifugal force.

2. The inertia of objects to rotational motion depends both on the mass and the distribution of that mass in the object.

3. Angular acceleration, velocity, momentum, and kinetic energy may be computed by methods analogous to those for linear motion.

Projectiles

1. The velocity of a projectile constantly changes due to the contribution to its velocity of the action of gravity.

Problems

Group A

1. What is meant by uniform circular motion? Mention several examples.

2. Distinguish centripetal from centrifugal force. Mention several examples of each.

3. What is a radian? How many radians are there in one complete circle?

4. How is it possible for a cyclist to loop-the-loop without dropping off the track when he is upside down?

5. Why are curves on roads banked? What considerations determine how much a curve should be banked?

6. Explain the action of a centrifugal clothes dryer.

7. Compare simple inertia and moment of inertia.

8. Describe the motion of a simple top.

9. A projectile follows a curved path. In what ways does the behavior of the projectile differ from that of a body traveling in a circular path?

Group B

1. An automobile weighing 4800 lb. is rounding a curve of 720 ft. radius at 40 m.p.h. Find the centripetal force required in poundals and in pounds. At what angle should the curve be banked?

24,000 lbal. 745 lb. 8.8°.

2. A highway curve of 300-ft. radius is banked at an angle of 30°. What is the maximum speed for this curve? Is this a safe speed—ever? always? Explain. Would you consider this a sharp curve?

50 m.p.h.

3. A railroad track is banked at an angle of 5° for a speed of 100 m.p.h. What is the radius of the curve? 8,000 ft. approx.

4. An airplane, traveling at 200 m.p.h., makes a turn of 3600-ft. radius. What is the correct angle of bank? 37.7°.

5. A boy swings a bucket of water in vertical circles. It is 3 ft. from his shoulder to the top of the water. What is the minimum number of revolutions per second that will keep the water in the bucket?

0.52 approx.

6. What angular acceleration will be produced on a wheel of 5,000 units of moment of inertia by a force of 40 lb. applied at a lever arm of 1.5 ft?

0.386 rad. per sec. per sec.

7. What is the moment of inertia of a wheel that is accelerated 2.5 rad. per sec. per sec. by a force of 50 lb. applied at a lever arm of 2 ft.?

1,288 English units.

8. A wheel weighing 100 lb. rolls down an inclined plane whose height is 9 ft. The center of the wheel has linear velocity of 12 ft. per sec. Find the distribution of kinetic energy between linear and angular.

7,200 ft. lbal. 21,800 ft. lbal.

9. The diameter of the wheel in problem 8 is 2 ft. Find (a) the angular velocity and (b) the moment of inertia.

1.91 turns per sec. 12.0 rad. per sec. 302.5 English units.

10. A projectile is fired at an angle of 45° to the horizontal with muzzle velocity of 1,800 ft. per sec. Find the time to reach the top of the path. Find the height to which it will rise. Find the range.

39.6 sec. 25,200 ft. 100,800 ft. (19 miles approx.).

Experimental Problems

1. Place a small truck, such as a flatcar from a toy train, on a turntable in such a direction that it can move along a radius. Use a heavy string and a spring balance to fasten it to the center of the turntable. Revolve the turntable at as steady a rate as possible. Compute the centripetal force required to keep the truck rotating in a circle for different lengths of the string and for different weights added to the truck. Compare these results with the force as determined from the readings on the spring balance.

2. Stand on a rotating platform with 5- or 10-lb. weights in each hand. Try the effect on the rate of rotation of extending the weights to arm's length and of drawing them in close. Explain the effect in terms of distribution of angular inertia.

SOME PROBLEMS IN THE MOTION OF FLUIDS

In a moving fluid, pressure is found to be least in regions of greatest velocity. This effect is common in everyday experience and may be used to explain the tendency of a person to *fall towards a moving train*, the *curving of a baseball*, the *lift of an airplane*, and many other daily occurring phenomena.

Drag on an object moving through a fluid is due largely to a reduction in pressure in the fluid at the rear end of the moving object. Turbulence in the flow of the fluid is responsible for this effect. Shaping the object to reduce turbulence is called streamlining and is very effective in reducing drag.

Internal resistance in a fluid requires energy to force a flow of a fluid over a fixed surface and is called viscosity.

Problems of lubrication of bearings indicate that a good lubricant should have low viscosity but high adherence for the materials of which bearings are made.

1. Reduction in Pressure Due to Velocity of a Fluid

Nearly everyone has felt drawn towards a fast moving train when he has been standing too close to the edge of a train platform. This effect is real and not imaginary, for there is a *reduction in air pressure* between the person and the train that accompanies the motion of the air carried along by the train. This effect can be demonstrated by suspending two light spheres from long strings as shown in Figure 118. A jet of air is now pointed between the two spheres and it is easy to see that they are drawn towards one another. Actually they are pushed towards one another as a result of the excess air pressure on the outside surfaces of the spheres as compared to the pressure on the adjacent surfaces.

Some understanding of this phenomenon can be gained by studying the motion of either a liquid or a gas in a pipe that has a varying cross section. Figure 119 shows such a pipe with a contracted section between two larger sections. Dial type pressure gauges are shown here, but manometer type

gauges may also be used. These gauges show that if the end of the pipe is closed so that no fluid is in motion, all the gauges read the same. When the end is opened all gauges drop slightly but the one in the contracted region most of all. Our common sense tells us that in this region the fluid is moving with higher velocity than at either of the other positions, for the same amount of fluid must pass this point each second as must pass points where the cross section is larger. So we associate reduced pressure with higher velocity. This result may seem the more reasonable if we argue that to make the fluid pick up speed as it goes from *A* to *B* there must be greater pressure at *A* than at *B*. Similarly if the fluid slows down in going from *B* to *C* the pressure at *C* must be greater than at *B*.

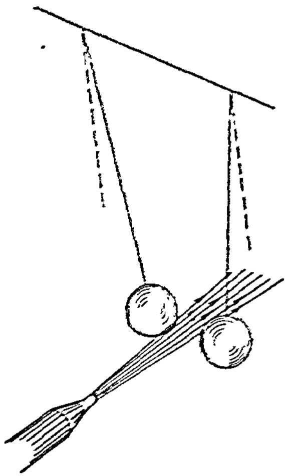


FIG. 118.—Suspended light spheres move toward one another when an air stream flows between them.

2. Bernoulli's Analysis

Still another way to examine this problem is to consider the total energy per unit volume in a moving fluid. Each unit

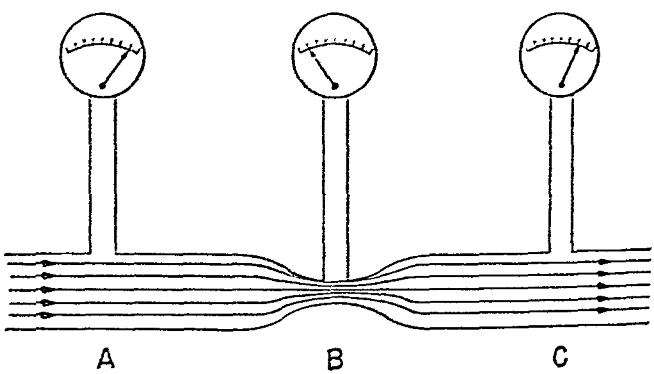


FIG. 119.—At the region of greatest velocity (B) the pressure in the fluid is least.

volume of fluid will possess potential energy due to two causes, one the height through which the fluid may fall and the other the pressure under which the fluid exists. For example in a long vertical pipe filled with water standing still it is obvious that the water at the top of the pipe has potential energy due to its position. The water at the bottom of the pipe has potential energy of equal amount per unit volume due to the pressure transmitted to it by the weight of the water above it. At any intermediate point each unit volume of water possesses the same total amount of energy but it is divided between potential energy due to position above the bottom of the pipe and potential energy due to pressure.

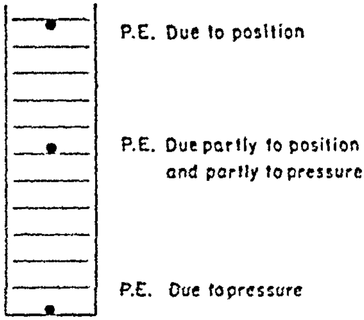


Fig. 120.—At any position in a standpipe the total potential energy in the fluid per unit volume is constant although it is divided between energy due to position and energy due to pressure.

When the fluid is in motion each unit volume has kinetic energy due to this motion in addition to potential energy.

We now return to Figure 119 and consider a unit volume of fluid at each of the points *A*, *B*, and *C*. If the pipe is horizontal, the energy due to position is the same at all points and we need consider only potential energy changes due to pressure differences and kinetic energy changes due to velocity differences. We know, of course, that the kinetic energy per unit volume at *B* is greater than at *A* or *C* because at *B* the velocity is greatest. Also, since we are not adding or subtracting energy along the pipe, we conclude that an increase in kinetic energy can be brought about only by a decrease in the potential energy due to pressure. This means a lowering of pressure.

This method of analyzing the problem was first carried through by Bernoulli and bears his name.

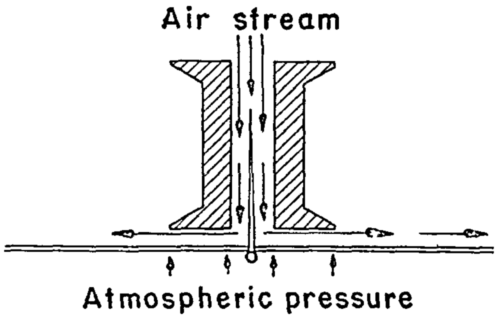


FIG. 121.—(*Left*) A piece of paper near the end of a spool is attracted to the spool when one blows through the hole in the spool.
FIG. 122.—(*Right*) A ping pong ball can be supported by a stream of air.

3. Laboratory Demonstrations of Reduction in Pressure Due to Motion of a Fluid

Several simple laboratory demonstrations using air as the fluid are easily performed. Figure 121 shows a spool with a disk of paper held loosely in place at one end by means of a pin. Blow through the spool and you find that the paper, instead of flying away, sticks fairly tightly to the end of the spool. The fast flowing air between the paper and the end of the spool is at lower pressure than the relatively still air on the other side of the paper.

Figure 122 shows a stream of air supporting a ping pong ball. Usually the ball stays somewhat to one side of the center of the stream. The reduced pressure in the stream permits the normal atmospheric pressure on the opposite side of the ball to press it against the moving air stream. The upward motion of the air stream contributes enough support to the ball to hold it up against gravity.

4. Some Applications of Reduction in Pressure Due to Motion of a Fluid

a. Flow meter

An immediate application of the effect as shown in Figure 119 is to use it as a flow meter. The construction usually consists of a thick plate with a small hole in the center. The

plate can be inserted in a slot in the pipe. Provision is made for measuring the difference in pressure between points such as A and B. The range of the meter can be changed by using plates with different size openings. The device is known as a venturi meter.

b. Gas burner

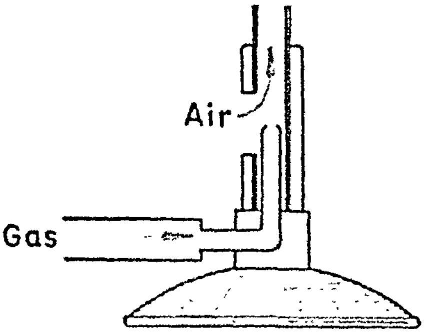
Ordinary gas burners are designed so that a high velocity stream of gas from a jet flows by openings in a pipe. Pressure is reduced below atmospheric near these holes and air comes in from outside. The air and gas then mix as they go through the pipe and a combustible mixture is delivered at the end of the pipe. (See Figure 133.)

c. Vaporizer

A simplified form of a vaporizer is shown in Figure 124. Here a fast moving stream of air reduces the atmospheric pressure at the upper end of a tube the lower end of which is immersed in a liquid. Atmospheric pressure on the liquid in the container causes liquid to rise in the tube at the top of which it is blown off in small droplets by the air stream.

d. Curved path of a baseball

Figure 125 shows a baseball spinning and at the same time moving to the right as indicated by the straight black arrow.



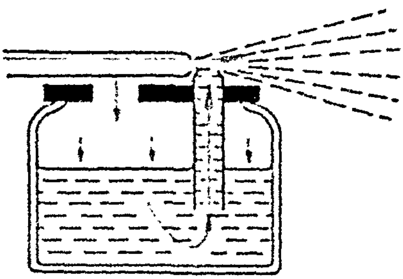


FIG. 123.—(*Left*) Air flows into a gas burner when the pressure inside the upright tube is reduced by the flow of gas.

FIG. 124.—(*Right*) Pressure at the top of the vertical pipe is reduced by the air stream, so that atmospheric pressure pushes the liquid up the pipe to be blown off in small droplets.

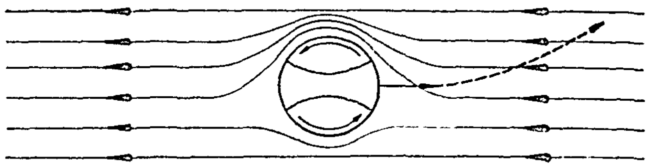


Fig. 125.—A baseball can be made to curve by giving it a spinning motion.

This motion of the ball to the right through the air is much the same so far as air effects are concerned as having the ball stand still and letting the air blow to the left. This alternative condition is indicated by the red lines. Curved arrows on the ball show its direction of spin.

The surface of the ball carries some air with it. On the top this air motion due to the ball adds to the motion of the air going by, while at the bottom of the ball this motion tends to reduce the air motion with respect to the ball as a whole. The result is that with air motion above the ball greater than below, there is reduced pressure above as compared to below and hence there is an upward force on the ball. The curved black arrow indicates that the ball tends to curve upward as it moves towards the right.

Similarly, a reversed spin of the ball will tend to make it curve downward. This latter type of spin (called top spin)

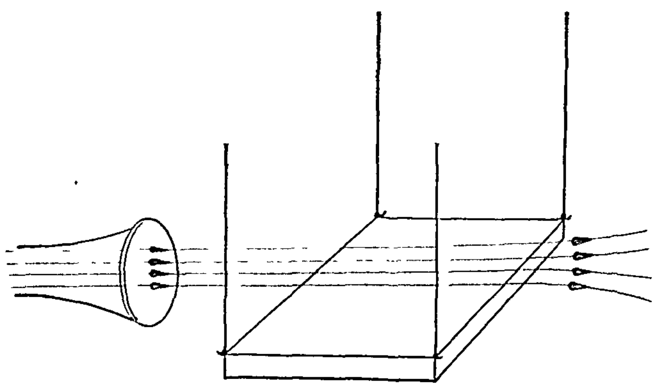


Fig. 126.—The light piece of wood is suspended by rubber bands. A stream of air over the top surface permits the block to rise due to reduced pressure on this surface as compared to normal pressure on the bottom of the wood.

is used extensively by tennis players who wish to hit a ball hard and yet have it come down fast enough to stay inside the court lines. Spin around a vertical axis will make a ball curve to the right or left. Spin around some other axis may combine an up or down curve with a side curve.

c. Lift on airplane wings

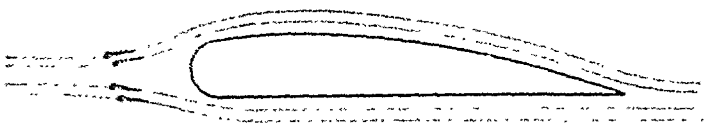
Figure 126 shows a very light block of wood suspended from long light rubber bands. A flat stream of air may be sent over either the upper surface of the block or the lower surface. In the latter case we observe that the block moves downward showing that we have reduced the pressure on the bottom side. Of course the air stream over the top of the block shows a lift on the block. This experiment gives a clue for designing airplane wings to give lift.

Figure 127 shows the cross section of a wing of such shape that when a fairly uniform air stream strikes the leading edge, the velocity of flow over the top surface is greater than over the lower surface. Reduced pressure over the top surface as compared to the lower surface is the result and the desired lift is obtained.

5. Streamlining

When any object is moved rapidly through any fluid, some resistance to the motion is observed. Actual measurements of pressure in front and at the rear of such an object show that there is a reduction of pressure at the rear. Hence it is a pressure difference on the leading and the trailing ends of the object that cause the principal resistance to the motion.

Further study made by holding an object stationary and forcing a stream of air with smoke in it to blow past the object



shows that the air does not flow smoothly around most objects but that whirlpools and eddies are formed at the rear of the object. This type of flow is said to be *turbulent.* Experimentally then we attack the problem of reducing turbulence and so obtaining smooth flow by changing the shape of an object and making measurements either on the pressure or, more simply, on the drag on the object. First we will measure

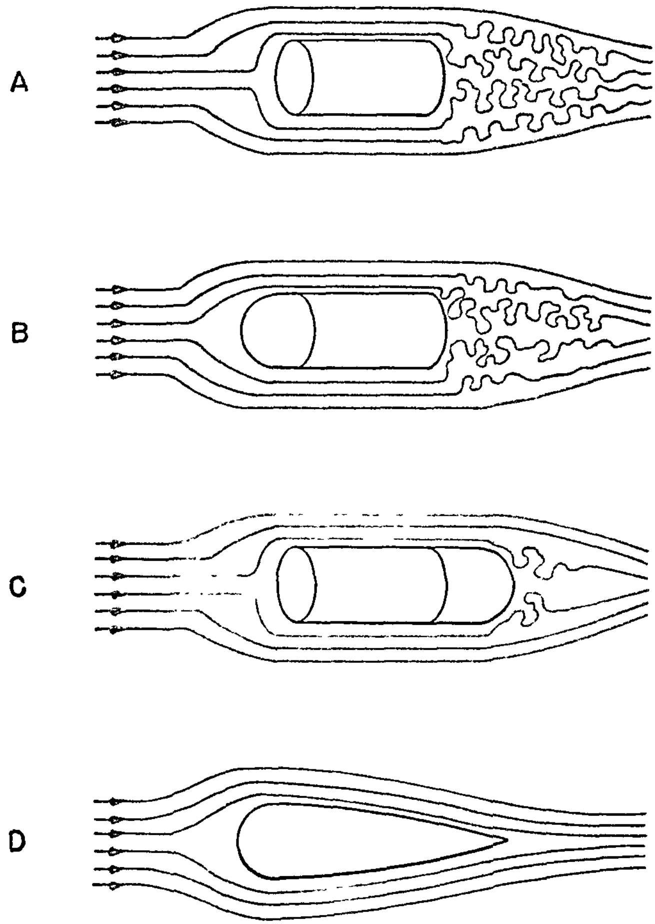


FIG. 128.—The drag on an object as it is moved through a fluid can be greatly reduced by adjusting the shape of the object until turbulence is eliminated and smooth flow of the fluid is obtained.

the drag on a section of cylinder as shown in Figure 128A. We expect (and find) considerable drag here.

Now we add first a spherically shaped front end to the cylinder and we observe some reduction in drag. Then we try this section on the rear end and get a greater reduction in drag. This result we more or less anticipated, because we have already learned from the smoke experiments that most of the trouble comes from the turbulence at the rear end of the cylinder. These experiments are shown in Figure 128B and C. We now try a variety of shapes on both the front and rear ends and finally arrive at the so-called tear drop design which is the one that gives the least drag. If we examine this shape with smoke in moving air, we will find that turbulence has been very largely eliminated.

An object with a shape especially designed to reduce turbulence is said to be *streamlined.* The drag through air with a carefully streamlined shape as shown in Figure 128D can be reduced to about 5 per cent of that obtained with a simple cylindrical construction as shown in Figure 128A.

Streamlining is not important for low speeds in air since the amount of drag is small even with a poor shape. However, streamlining is important even for medium high speeds in automobiles since smaller engines are required when the drag is low, and streamlining becomes essential for really high-speed devices such as airplanes.

6. Viscosity

Figure 129 shows a pipe of uniform cross section with pressure gauges attached at various places. When the end of the pipe is closed, the pressure gauges all read the same. When the end is opened so that fluid motion takes place there is a gradual falling off of pressure along the stream. Since the kinetic energy per unit volume remains the same along the pipe, and since the pressure is decreasing, we conclude that energy is being used up in producing the flow.

The property of a fluid that presents resistance to flow is called *viscosity.*

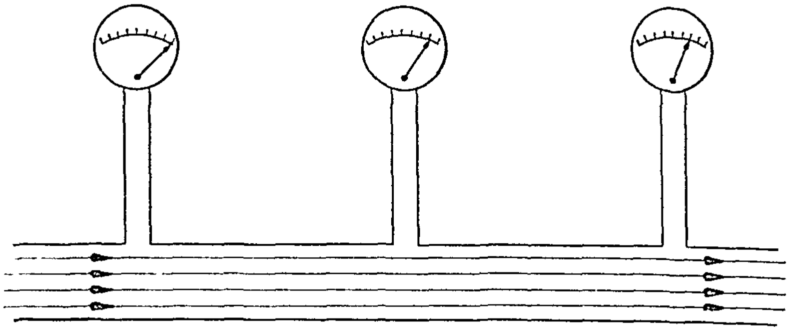


FIG. 129.—Falling pressure in a moving fluid in a horizontal pipe of uniform cross section shows that energy is being used to maintain the flow.

Take two beakers of the same size and pour oil in one and water in the other. Shake them gently to and fro and you discover at once that the oil flows less freely than the water. The water has less viscosity. Repeat the experiment with two beakers containing the same kind of oil, but precool the oil in one of them by placing it in a refrigerator for some time before the experiment. You find that the colder oil flows less freely than the warm oil. Reduction in temperature has caused an increase in viscosity.

Flow of a fluid over a surface is explained by assuming that the first layer of molecules adheres to the surface. The second layer of molecules slides over the first, but with some friction. The third layer slides over the second, and so on. Thus the fluid in the middle of a pipe flows faster than the fluid near the inner surface of the pipe and work is being done against the friction between the molecular layers.

7. Lubrication

It is well known that oil is commonly used as a lubricant in metal bearings. One may think of the oil molecules as behaving somewhat like sub-microscopic ball bearings between the two surfaces of the bearing. To reduce friction in a bearing to a minimum it would at first seem that a fluid with low viscosity should be used. Yet in general we use oil rather than other liquids such as water and alcohol which are known to be less viscous. This use is not because we would not like

low viscosity, but because it is necessary to use a fluid with great adherence for the surfaces of the bearing. Otherwise it will soon be squeezed out and there will be no lubricant left between the two bearing surfaces.

So the ideal lubricant would be one with low viscosity but with great adherence to the bearing surfaces. It would also be desirable to have a lubricant whose properties were not greatly affected by temperature changes. Present oil refining processes are supposed to eliminate some of the heavier hydrocarbons that may be largely responsible for temperature changes in viscosity.

Some Important Facts

1. Many common experiences illustrate the fact that pressure and velocity in fluids vary inversely.

Among the more common examples of this effect are gas burners, flow meters, vaporizers, the tendency to fall toward a speeding train, the curve of a baseball, the lift of an airplane.

2. Bernoulli first explained the inverse variation of pressure and velocity in terms of energy conservation. He stated that a mass of fluid has a constant total energy content which consists of three parts: (1) Kinetic energy of motion; (2) pressure energy and (3) potential energy due to position. The distribution of the energy in (1) and (2) changes with variations in velocity.

3. The relative motion of a solid in a fluid is retarded mainly by reduced pressure and turbulence behind the solid. This is greatly reduced by streamlining solids which are to move at high velocities in air, water, or other fluids.

4. Internal friction in a fluid causes loss of energy during the motion of the fluid. The property of the fluid causing this effect is called viscosity.

Generalizations

In a moving fluid, an increase in velocity is accompanied by a decrease in pressure and vice versa.

All fluids offer some resistance to flow and this resistance is called viscosity.

Problems

Group A

1. In what two ways is pressure used up in causing fluid motion?
2. State Bernoulli's Principle.

3. Mention several common examples that illustrate Bernoulli's Principle.

4. Show by means of a labeled diagram that the lift of an airfoil, such as an airplane wing. is an example of Bernoulli's Principle.

5. What is a streamlined object? Give several common examples.

Group B

1. Is pressure transmitted without loss by a fluid in motion? If any pressure is lost, how and why?

2. What quantities are involved in a complete statement of Bernoulli's Principle? How are they related to each other?

3. Why is the Bernoulli relationship a reasonable one to expect?

4. Can you deduce Bernoulli's Principle as a corollary, or special case,, under the Law of Energy Conservation?

5. Does the shape of an airfoil have any effect on the amount of lift? Can you illustrate by labeled diagrams?

6. Does the angle made by an airfoil with the direction of the relative air affect the amount of lift? Can you illustrate by labeled diagrams?

7. Draw a diagram of a perfectly streamlined object. Could a perfectly streamlined airfoil have lift? If so, explain how.

8. What animals are most streamlined? Can you explain why?

9. Not all projectiles, aircraft, boats, vehicles, etc. are as streamlined as possible. Why not?

Experimental Problems

1. Suspend a table tennis ball by a thread and, by blowing through a soda straw, direct a jet of air along one side of the ball. What happens to the ball? Why?

2. Stick a large pin through the center of a small cardboard disk. Then insert the pin into one end of the hole in a spool. Blow into the other end of the spool. Can you blow the card away from the spool? Explain.

3. Construct several small model airfoils. Suspend each by two cords passing over small pulleys so that each airfoil may be counterbalanced. Direct a flat stream of air—as by blowing through a fish-tail burner—at the leading edge of each airfoil. What can you conclude as to their relative lift characteristics?

Chapter 16

TEMPERATURE AND HEAT

Our first knowledge of heat and temperature comes to us through feeling. But such sensations are not easily measured and so other effects of heat (such as the change of size of objects) furnish us with instruments for actual measurement. The term heat is used for quantity of heat and the term temperature for the state of hotness or coldness of an object. A device used to measure temperature is called a thermometer.

Temperature scales for use with thermometers have been decided upon in a rather arbitrary manner. The two with which we are most familiar are the Fahrenheit and the Centigrade.

The linear coefficient of expansion of a substance is defined as the change per degree per unit length of the material at 0° C. The volume coefficient of expansion is defined as the volume change per degree per unit volume at 0° C.

The nature of heat energy was at first unknown and hence arbitrary units for quantity of heat were chosen. In the metric system the *calorie* was defined as the amount of heat necessary to raise one gram of water one degree Centigrade. In the English system, the *British Thermal Unit* (B.T.U.) was defined as the amount of heat required to raise one pound of water one degree Fahrenheit. We now believe that heat is a form of mechanical energy of the molecules of which the substance is made. Hence mechanical units of energy, the erg or joule and the foot pound might have been more logical choices for units of heat.

The amount of heat required to raise a unit mass of any substance one degree in temperature varies from one temperature to another even for the same substance, and it varies at the same temperature for different substances. A laboratory method is suggested for determining this amount of heat.

The chapter closes with a number of examples dealing with the heating of various objects.

1. First Knowledge of Temperature and Heat

Among our more common experiences are the sensations of heat and cold. We touch several objects and decide as to their relative hotness or coldness. Or perhaps we experience a general sensation of heat or cold as in the cases of being in an overheated room or, on the other hand, being out of doors

on a day in winter. Sometimes we say, "I am cold," sometimes we say, "The day is cold."

This matter of hotness and coldness which we interpret through our sensations we call temperature.

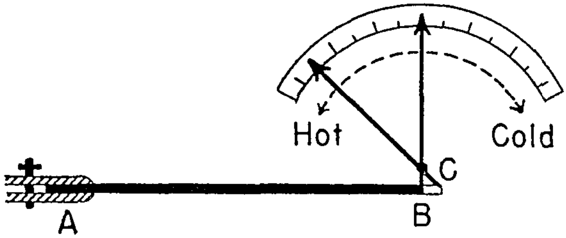
So long as the meaning of the term is restricted to our sensations a great deal of indefiniteness is attached to the idea. For example, one often feels cold in a heated room in the winter time while on many days in summer with the air at the same identical temperature, one may be uncomfortably warm. The difference in these cases may depend on the possibility that in the first case the air is dry and in the second case the air is full of moisture. In the first case a great deal of evaporation can proceed and so produce a cooling effect on one's body.

It becomes obvious that one can make greater progress with a study of temperature and heat if he can find more objective effects than simple sensations of heat and cold.

2. Objective Temperature Indicators

One simple observation is that most objects increase in size when changed from a lower to a higher temperature.

This fact at once offers a means for making an instrument to measure temperatures. A simple arrangement for accom-



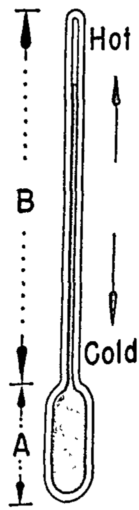


FIG. 130.—(*Top*) Changes in the length of rod AB can be used to indicate temperature variations. The motion of the end of the rod is amplified here by a pointer system.

FIG. 131.—(*Right*) Temperature variations change the volume of liquid in bulb A. These changes are easily observed by the height of the liquid in tube B.

plishing this feat is shown in Figure 130. The rod *AB* is clamped firmly at *A* so that the end *B* will move to the right when the rod is heated or to the left when it cools. The motion of the end *B* will be small, but it can be amplified by a simple lever system as shown with the pointer pivoted at *C*, or by a more complicated amplifying system as illustrated for the aneroid barometer in Figure 78, page 139.

Another temperature indicator consists of a small glass bulb *A* to which is attached a long capillary tube *B*. The bulb is filled with any liquid which will expand more with temperature rises than the material used to make the bulb and tube. Usually glass is used for the latter and either mercury or alcohol for the liquid.

This arrangement provides its own amplifying action since, if the volume of the liquid in the bulb increases by even a small percentage amount, the actual increase in volume will necessitate a great rise of the liquid into the small capillary tube.

Instruments designed to measure temperature are called thermometers, and those depending on changes in dimensions, especially the liquid glass type, are more extensively used than any others.

3. Temperature Scales

After deciding on a method for measuring temperature it is necessary to adopt units for calibrating the scale. The Fahrenheit scale as commonly used in this country is said to have had its zero point determined as the temperature of a mixture of ice and salt, such as may be used to freeze ice cream. The temperature of a human body was picked out as a high point and the temperature region between these two points divided into 100 parts called degrees.

There could have been no great accuracy used in determining these two temperatures, for the average normal temperature of human beings as measured in the mouth is now well known to be about 98.6° and not 100° F.

A somewhat more sensible temperature scale was later determined by taking the freezing point and the boiling point of water under normal pressure conditions as the two end points for the temperature scale. This temperature region was divided into 100 degrees; but in the case of the new scale, called the Centigrade, the temperature region covered is

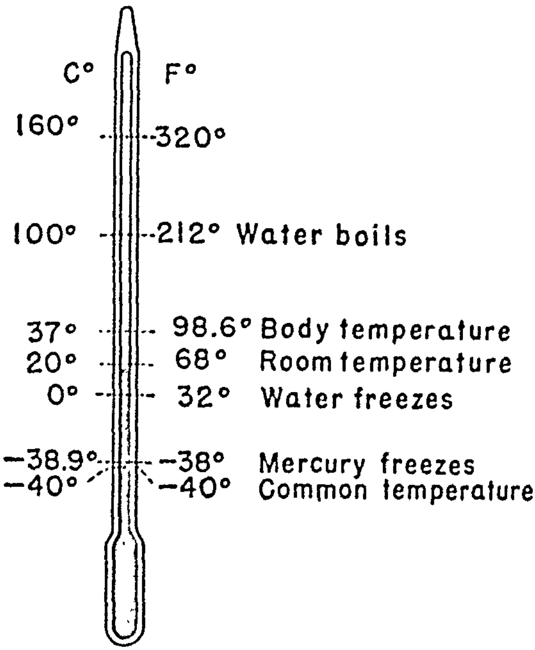


FIG. 132.—A comparison of Fahrenheit and Centigrade scales on a mercury in glass thermometer.

considerably greater and so the Centigrade degree is larger than the Fahrenheit.

Experiment shows that the freezing and boiling points of water on the Fahrenheit scale are 32° and 212° respectively, giving a difference of 180 degrees for only 100 on the Centigrade scale.

From these figures it is easy to show that a Fahrenheit degree is only $5/9$ of a Centigrade degree, or reciprocally, that a Centigrade degree is equivalent to $9/5$ of a Fahrenheit

degree. Conversion of temperature from one scale to the other may easily be made.

Some comparison of the two scales may be obtained from the drawing of Figure 132 which shows two thermometer scales, one Centigrade and one Fahrenheit. Notice that at $-40°$ the two read the same. A student clever in the use of algebra can show that equal readings at this temperature could have been predicted. It is interesting to note that $-40°$ is close to the freezing point of mercury. The latter is $-38.87°$.

The fact that the readings of temperature on the two scales agree at $-40°$ offers a simplified method for converting any temperature on one scale to the corresponding temperature on the other scale.

To change from a Fahrenheit reading to a Centigrade reading:

(1) Add 40°.
(2) Take $\frac{5}{9}$ of the total.
(3) Subtract 40°.

To change from a Centigrade reading to a Fahrenheit reading:

(1) Add 40°.
(2) Take $\frac{9}{5}$ of the total.
(3) Subtract 40°.

Notice that the only difference is in procedure (2).

Example. Change 68° F. to the corresponding Centigrade value.

(1) $$\begin{array}{r} 68 \\ +40 \\ \hline 108 \end{array}$$

(2) $\frac{5}{9} \times 108 = 60$

(3) $$\begin{array}{r} 60 \\ -40 \\ \hline 20° \text{ C.} \end{array}$$

Example. Change 20° C. to the corresponding Fahrenheit value.

$$
\begin{array}{ll}
(1) & 20 \\
 & \underline{+40} \\
 & 60 \\
(2) & 9/5 \times 60 = 108 \\
(3) & 108 \\
 & \underline{-40} \\
 & 68^\circ \text{ F.}
\end{array}
$$

A more conventional method for solving these same two problems is given below.

Example. Convert 68° F. to the corresponding Centigrade value.

68 − 32 = 36 Fahrenheit degrees above the freezing point of water which is at 0° on the Centigrade scale.

36 Fahrenheit degrees are equivalent to $5/9 \times 36 = 20$ Centigrade degrees.

Hence 20° C. is equivalent to 68° F.

Example. Convert 20° C. to the corresponding Fahrenheit value.

20 Centigrade degrees is equivalent to $9/5 \times 20 = 36$ Fahrenheit degrees.

Since the Centigrade temperature scale starts with its zero at the freezing point of water and since this temperature is 32° on the Fahrenheit scale, we add 32° to the 36° just found and obtain 68° as the Fahrenheit temperature corresponding to 20° C.

4. Coefficients of Expansion

In the use of expanding liquids and solids for temperature indicators we have tacitly assumed that the expansion is uniform; or, in other words, that equal temperature changes in different parts of the temperature region will produce equal expansions of the indicating substance.

For mercury and alcohol at moderate temperatures this

condition is near enough to being correct for practical purposes, but for many substances the variation is considerable. For example, in the case of water there is a contraction instead of an expansion as the temperature increases from 0° to 4° C. Above this point the water expands at a varying rate as the temperature rises.

One practical aspect of this peculiarity of water is the fact that when water cools in a lake to temperatures lower than 4° C., it tends to float instead of sinking. For this reason ice begins to form on the surface of the lake rather than at the bottom.

If variation per degree in the expansion of the same material in different temperature regions exists, it is even more to be expected that there will be variation of expansion as we go from one material to another.

Expansions in the case of metal rods are easily measured in the laboratory by a set-up essentially similar to that shown in the drawing of Figure 130. The rod AB may be surrounded by a steam jacket, and by means of the pointer or other mechanism, the change in length between two temperatures may be determined. From this data the average change per degree is readily found, and if the length of the rod is known, the change of each centimeter of rod per degree can be stated. This latter quantity is called the temperature coefficient of linear expansion. (To be precise the change per centimeter per degree should be referred to the length of the rod at 0° C.)

5. Volume Expansion

The coefficient of volume expansion is defined as the change in volume per degree per unit volume. (As in the case of linear expansion the comparison should be to unit volume at 0° C.; but for many practical calculations this refinement may be ignored.)

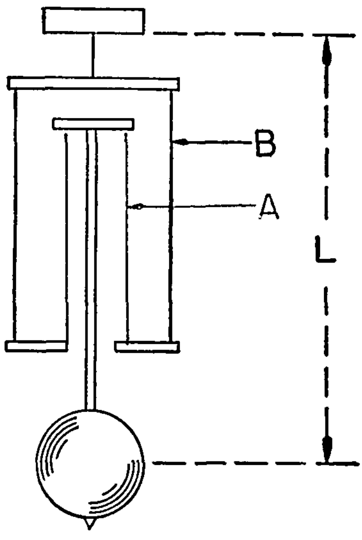
More advanced texts show that there is a simple relation between linear expansion and volume expansion. The coefficient of volume expansion for any substance is approximately 3 times the coefficient of linear expansion for that substance.

In the case of solids it is customary to determine linear coefficients by experiment and to calculate the volume coefficients in this simple manner. On the other hand the volume coefficient of expansion for a liquid is usually determined by experiment directly.

6. Differential Expansion—Linear and Volume

Figure 133 shows a pendulum system made from two types of rods. Those labeled A have a linear coefficient of expansion more than twice that of the B rods. When temperature rises or falls with such a system the distance L changes very little, if at all. Such systems have been used on some very accurate pendulum type clocks.

Figure 134 (a) shows two pieces of metal, welded or otherwise fastened together. Metal A has a greater coefficient of expansion than B. When this compound strip is heated the greater expansion of metal A forces the strip to bend as indicated at (b). If the strip is cooled, the bending is reversed as shown



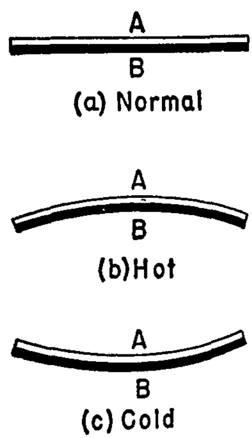


FIG. 133.—(*Left*) The length of a pendulum can be held constant during temperature changes by a system of supports having different temperature coefficients.

FIG. 134.—(*Right*) A bimetal strip of two metals each having a different temperature coefficient of expansion will bend with temperature changes.

at (c). One end of such a strip may be clamped and the other end may be used to make or break electric contacts depending on the temperature of the strip. Any temperature controlled instrument used to turn off or on other devices is called a thermostat.

In Figure 135 is shown a bulb and tube similar in appearance to those used for thermometers. In this case the bulb is relatively large and its capacity must be known accurately. The tube is calibrated in terms of volume also. The coefficient of expansion of the glass must be known.

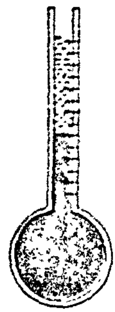


Fig. 135.—Differential expansion of liquid and container.

If a liquid having the same volume temperature coefficient as that of the glass is put into this container, no apparent change in volume of the liquid will be noted when the temperature is changed. However, if, when the temperature rises, the liquid expands at a greater rate than that of the flask, the level of liquid will rise in the tube and the apparent expansion can be determined. Of course the observed coefficient of expansion will be the difference between that of the liquid and that of the glass. However, if the latter is known, the former can be computed, for the observed effect is due to the difference between the expansion of the liquid and that of the glass. (The glass container expands as though it were a solid piece of glass, and hence the space in the glass vessel that holds the liquid becomes larger with rising temperature by an amount depending directly on the volume coefficient for the glass.)

7. Other Applications of Expansion

Expansion of solids and liquids sometimes represents a serious difficulty in mechanical structures, and under other circumstances it may be an asset. In the laying of a railroad track, provision for expansion and contraction of the rails is commonly made by allowing a short distance between successive rails and using couplings of a type that will permit the ends of the rails to slide back and forth.

On the other hand, an iron band may be securely mounted on a wheel by making the band just too small to slip on the wheel. The band is then heated until it expands to the point where it slips over the wheel easily. It shrinks on to the wheel as it cools and makes a very substantial arrangement.

Example 1. The steel span of a bridge is 1000 ft. long and the coefficient of expansion of the steel used is 0.000012 per degree C. Find the allowance for expansion between summer temperatures of 30° C. and winter temperatures of −20° C.

The expansion, e, is given by the product of the total length, the coefficient of expansion, and the change in temperature; that is

$$e = 1000 \times 0.000012 \times 50 = 0.6 \text{ ft.}$$

Example 2. Find the volume coefficient of expansion of mercury from the following experimental data. Mercury is placed in a flask as shown in Figure 135, and its temperature raised from 15° C. to 45° C. The volume of the mercury initially was 200 cc. and its apparent increase in volume is 0.948 cc. The linear coefficient of expansion of the glass is known to be 0.000008 per degree C.

The apparent increase in volume per cc. is

$$\frac{0.948}{200} = 0.00474$$

and the apparent increase per cc. per degree is

$$\frac{0.00474}{30^\circ} = 0.000158$$

The true coefficient of expansion for the mercury will be greater than this number by an amount equal to the volume coefficient for the glass. This is

$$3 \times 0.000008 = 0.000024$$

Hence the final value of the mercury is

$$0.000158 + 0.000024 = 0.000182 \text{ per degree C.}$$

8. Temperature Effects in a Gas

Variation of the volume of gases with temperature as well as that of liquids and solids also occurs. For example, a small gas-filled balloon tends to grow larger when the gas is heated. In many other cases, however, gases are kept in containers with rigid walls so that the volume cannot increase. This is approximately true for the case of air in automobile tires.

Here we find that the pressure of the gas goes up or down with the temperature while the volume is obliged to remain constant.

Returning to the case of allowing the volume to expand while the pressure is held constant, we find the volume-temperature coefficient of expansion of a gas is $1/273$ per degree as compared to the volume at 0° C. For example, 1 cc. of a gas at 0° C. becomes, at the same pressure, $1 + 10/273$ cc. at 10° C. or $1 - 30/273$ at −30° C.

9. Absolute Temperature

From the above facts one might conclude that the volume of a gas would shrink to zero at −273° C. However all known gases change to the liquid state before this temperature is reached and the volume-temperature coefficient for the liquid is quite different from that of the gas.

However, this line of reasoning leads to the idea that −273° C. may be as cold as it is possible to have any temperature. Sometimes −273° C. is called ABSOLUTE ZERO and all other temperatures are reckoned from this starting point. A temperature of 20° C. becomes 293° Absolute. The student may find some amusement in computing the corresponding values in terms of Fahrenheit degrees.

It may also be seen from the above description of the behavior of gases that the volume at constant pressure is proportional to the absolute temperature. For example, if the volume of some gas at constant pressure is 100 cc. at 10° C. we can easily find its volume at some other temperature, say 40° C., by first reducing these temperatures to the absolute

scale, ($273° + 10° = 283°$ and $273° + 40° = 313°$). Then

$$\frac{V_2}{T_2} = \frac{V_1}{T_1} \tag{1}$$

$$\frac{V_2}{313} = \frac{100}{283}$$

$$V_2 = 100 \times \frac{313}{283} = 110.6 \text{ cc.}$$

The statement of this relation between volumes and absolute temperatures of a gas as set down in equation (1) above is sometimes called Charles' law.

10. Pressure-Temperature Changes in a Gas

Experiments on the change in pressure with temperature when the volume of a gas is kept constant leads to the same coefficient ($\frac{1}{273}$) for the pressure change as was found for the volume-temperature coefficient, and hence the pressures at constant volume are proportional to the absolute temperatures. That is,

$$\frac{P_2}{T_2} = \frac{P_1}{T_1} \tag{2}$$

Example. On a day when the barometric pressure is 15 lb. per sq. in. and the temperature is 20° C. a man has his automobile tires inflated to 30 lb. per sq. in. Find the pressure in his tires if the temperature drops rapidly to 0° C.

Since a pressure gauge as used on tires reads the difference between the real pressure and the atmospheric pressure we conclude that the initial pressure in the tires is $30 + 15 = 45$ lb. per sq. in. The two absolute temperatures are $273° + 20° = 293°$ and 273°. Hence we may write:

$$\frac{P_2}{T_2} = \frac{P_1}{T_1} \tag{2}$$

$$\frac{P_2}{273} = \frac{45}{293}$$

$$P_2 = 45 \times \frac{273}{293} = 41.9 \text{ lb. per sq. in.}$$

If the barometric pressure is the same as before, the pressure gauge would show $41.9 - 15 = 26.9$ lb. per sq. in.

We also arrive at the concept of an absolute zero by considering the pressure-temperature coefficient instead of the volume-temperature coefficient of a gas as was done in Section 9. If the pressure falls off by $1/273$ of its value at 0° C. for every fall of one degree in temperature, we would expect to find zero pressure in a gas at −273° C. Actually there is no substance that stays in gaseous form at −273° C., but this line of reasoning does lead to the belief that there is a lower limit to possible temperatures and that the limit is at approximately −273° C.

11. General Gas Law

It must be obvious that a gas may be confined in such a container that it will undergo a combination of both pressure and volume changes with temperature. In reality this is what happens in the case of the gas-filled balloon cited above where for simplicity we assumed that the pressure stayed constant and only the volume changed.

A general expression for the behavior of a gas undergoing pressure, volume and temperature changes may be found by an algebraic juggling of the two expressions (1) and (2) above and Boyle's Law ($PV = C$) (see Chapter 10, Section 9, page 145). The final expression, called the *general gas law* becomes

$$PV = RT$$

where P, V, and T are the pressure, volume, and absolute temperature respectively of a given quantity of gas at any instant. R is a constant depending on the amount of gas used.

Mathematically inclined students may attempt to derive the above formula from the three formulae referred to above. The general gas law may also be derived from a consideration of the motions of the molecules of which the gas is made. Such developments are to be found in more advanced texts.

12. The Nature of Heat Energy

One of the more simple ways in which to develop heat is by rubbing two objects together. We rub our hands together

on a cold day to warm them. We can also raise the temperature of two sticks by rubbing them. In fact it is possible to raise the temperature to the point where a fire can be started in this manner—as all Boy Scouts know.

This simple fact leads to speculation as to the possibility of heat energy being of the same nature as kinetic or potential energy in ordinary mechanics. If one simply assumes that the kinetic energies of the individual atoms, molecules, or groups of molecules change with temperature, he has an explanation for the frictional experiments described here.

In by-gone years scientists thought that heat was something like a fluid that could be poured into or out of otherwise solid substances, but the above concept of molecular kinetic and potential energy is the explanation now in good standing.

Changing the temperature of a substance then, consists in changing the average molecular kinetic energy and this hypothesis is in accord with our concepts of the nature of pressures in gases (see p. 135) and the changes in gaseous pressures with temperature discussed above in the present chapter.

13. Heat Units

With this simple mechanical explanation for heat energy, we might expect to measure heat in terms of ergs or joules in the metric system, and foot poundals or foot pounds in the English system. This would indeed be a logical thing to do. But the subject of heat, like many other things in science was not discovered or developed in an entirely logical fashion. As pointed out above, speculation as to the nature of heat energy was at first widely divergent from the present views and no one seemed to connect heat energy with ordinary mechanical energy.

As a result of this state of thinking a perfectly arbitrary unit of heat was defined. It is the amount of heat required to raise a unit mass of water through one degree. In the English system this means one pound of water through one Fahrenheit degree and this unit is called the *British Thermal*

Unit (abbreviated to B.T.U.). In the metric system we raise one gram of water through one Centigrade degree and the unit so obtained is called a *caloric*.

By experiment we can find how much energy in mechanical units will be equivalent to either of these heat energy units. By computation we can also find the relative size of the B.T.U. and the caloric. These relations are listed below:

$$4.19 \text{ joules} = 1 \text{ caloric}$$
$$778 \text{ ft. lb.} = 1 \text{ B.T.U.}$$
$$252 \text{ calories} = 1 \text{ B.T.U.}$$

One difficulty with the above definitions of heat units arises from the fact that the amount of heat required to produce equal temperature changes in a substance varies from one point on the temperature scale to another. Precisely then the unit of heat in the metric system, the caloric, is defined as the heat which will change one gram of water from 14.5° to 15.5° C. since the amount of heat required for this change is about equal to the average per gram per degree from 0° to 100° C.

14. Thermal Capacity—Specific Heat

The amount of heat required to raise the temperature of one gram of any substance through one degree Centigrade (or one pound through one degree Fahrenheit) is called the thermal capacity at that temperature. To avoid confusion the term thermal capacity per gram (or per pound) is often used.

The ratio of the thermal capacity per gram (or pound) of any substance to that of one gram (or pound) of water is called specific heat. Since by definition the thermal capacity of water is precisely one at some temperatures and approximately one at all temperatures between its freezing and boiling points, we see that specific heat and thermal capacity per unit mass for any substance are numerically the same. Moreover, specific heat values are the same in either the metric or the English system, since specific heat is only a ratio and hence has no dimensions. Also, thermal capacity per unit mass is

numerically the same in either the English or metric systems although the dimensions are B.T.U. per lb. per degree Fahrenheit in the one case and calories per g. per degree Centigrade in the other.

The difference between specific heat and thermal capacity as here defined is a technical one. Specific heat is just a ratio and has no units. Thermal capacity is in calories per g. per degree C. or in B.T.U. per lb. per degree F. But the actual numbers for specific heat and for thermal capacity in either system are the same. Tables of these quantities are usually labeled "Specific Heat." But the numbers are often used as thermal capacities with the units indicated from either the English or metric systems as may be needed.

15. Measurement of Specific Heat

The measurement of specific heats of some solids forms a convenient laboratory exercise. A piece of copper or iron of known mass is raised to some moderately high temperature T_1—for example, by means of a steam bath—and then it is dropped into a known mass of cool water. A thermometer in the water indicates the initial and final temperatures, say T_2 and T_3.

To a first approximation we may say that the heat lost by the metal in cooling from T_1 to T_3 must have been taken up by the water in having its temperature raised from T_2 to T_3. From this data we can compute the thermal capacity per gram of the metal. For more precise results we must remember that the metal container holding the water also had its temperature raised from T_2 to T_3.

Example 1. 300 grams of copper at a temperature of 98° C. is dropped into 200 grams of water at a temperature of 15° C. The temperature of the water rises to 24.9° C. Find the specific heat of the copper.

We equate the heat lost by the block in cooling to that taken up by the water. Calling the thermal capacity per gram of copper S_c and remembering that the thermal capacity of water (S_w) is unity we write

$$S_c \times M_c \times (T_1 - T_3) = S_w \times M_w \times (T_3 - T_2)$$
$$S_c 300(98 - 24.9) = 200(24.9 - 15)$$
$$S = \frac{200 \times 9.9}{300 \times 73.1} = 0.0903 \text{ calories per gram per degree C.}$$

Example 2. Suppose that in the above case we are to correct for the water container and suppose that this container is made of copper like the sample and that it weighs 100 grams. The equation then becomes

$$S_c 300(98 - 24.9) = 200(24.9 - 15) + S_c 100(24.9 - 15)$$
$$21{,}930S_c - 990S_c = 1{,}980$$
$$S_c = \frac{1{,}980}{20{,}940} = 0.0946 \text{ calories per gram per degree C.}$$

Short Table of Specific Heats
(near room temperatures)

Aluminum	0.22	Nickel	0.11
Copper	0.093	Silver	0.056
Iron	0.119	Air	0.24 (at constant pressure)
Mercury	0.033		

Example 3. An iron kettle weighing 15 lb. is filled with 20 lb. of water at a temperature of 50° F. Find the heat required to raise the temperature of the combination to the boiling point of water.

Since the thermal capacity of water is one B.T.U. per lb. per degree Fahrenheit, we find the amount of heat H required for the water alone by multiplying together the amount of water in pounds, and the temperature change.

$$H_w = 20 \times (212° - 50°) = 3{,}240 \text{ B.T.U.}$$

The heat required for the kettle is computed in the same way after we have found the thermal capacity per pound of the kettle by reference to the Specific Heat Table above.

$$H_k = 15 \times 0.119 \times (212° - 50°) = 289 \text{ B.T.U.}$$

The total heat required is then

$$H = 3{,}240 + 289 = 3{,}529 \text{ B.T.U.}$$

Example 4. How much heat will be required to raise the temperature of the air alone in a room $15' \times 12' \times 9'$ from $20°$ F. to $80°$ F.? (The mean density of air at these temperatures and at constant pressure may be taken as 0.077 lb. per cubic foot.)

The volume V of the room is

$$V = 15 \times 12 \times 9 = 1{,}620 \text{ cu. ft.}$$

The mass of the air in the room is

$$M = 1{,}620 \times 0.077 = 124.7 \text{ lb.}$$

The heat required is

$$\begin{aligned} H &= MS(T_2 - T_1) \\ &= 124.7 \times 0.24 \times 60 = 1{,}796 \text{ B.T.U.} \end{aligned}$$

Example 5. A certain grade of coal is known to have a heat of combustion of 14,000 B.T.U. per lb. Find the amount of coal to be burned in a furnace to heat the air described in the preceding example if the heating has an efficiency of 10 per cent.

The amount of heat available from each lb. of coal is

$$14{,}000 \times 0.10 = 1{,}400 \text{ B.T.U.}$$

The amount of coal required is

$$\frac{1{,}796}{1{,}400} = 1.283 \text{ lb.}$$

Some Important Facts

1. Our physical sensations of heat and cold supply our first concepts of these quantities, but they are not at all quantitatively accurate.

2. Thermometers are objective devices for measuring temperature. The more common ones work on the principle that some liquids and solids expand uniformly with a rise in temperature.

3. The Fahrenheit and the Centigrade temperature scales are in common use. On the former, water freezes at $32°$ and boils at $212°$; on the latter at $0°$ and $100°$ respectively. Therefore a Fahrenheit degree is $\frac{5}{9}$ as large as a Centigrade degree.

4. Linear coefficient of thermal expansion is the increase in any linear dimension of an object per unit length per degree temperature rise.

16. The thermal capacity of a substance is the amount of heat required to raise unit mass one degree in either metric or English units; that is, one gram through one degree C. or one pound through one degree F.

17. Specific heat is the ratio of the thermal capacity of a substance in comparison to the thermal capacity of water. Specific heat is therefore an abstract number.

18. Due to the manner in which the unit of heat is defined in the metric and the English systems, the number expressing thermal capacity of a substance is the same in either system and it is numerically equal to the specific heat.

19. In heat changes that do not involve a change of state of any of the substances involved, the heat gained or lost by any part of the system is equal to the product of the thermal capacity, the mass, and the temperature change for that part of the system. In any such system the heat lost by any part or parts of the system must be equal to that gained by the other part or parts of the system.

Generalizations

Temperature, first noticed through physiological sensations of heat or cold, is an indication of the kinetic energy of the atoms or molecules of a substance. Heat is a form of mechanical energy, but it is associated with the motions of the molecules of which a substance is composed rather than with the behavior of the substance as a whole. Different substances require different amounts of heat per unit mass to experience equal temperature changes. Most substances undergo changes in dimensions to accompany temperature changes.

Questions and Problems

Group A

1. Distinguish heat from temperature. In what units is each measured?

2. Define the calorie and the British Thermal Unit. How do they compare in size?

3. How may thermometer readings be translated from Centigrade to Fahrenheit? From Fahrenheit to Centigrade?

4. How may thermometer readings be translated from Centigrade to Absolute? From Absolute to Centigrade?

5. Normal room temperature is sometimes said to be 20° C. What is the equivalent Fahrenheit temperature? 68° F.

6. Normal body temperature is 98.6° F. What is it on the Centigrade scale? 37° C.

7. Find the Centigrade reading for a temperature of 0° F. −17.8° C.

Explain the sense of this method and use it in both directions for several conversions.

7. Distinguish "Specific Heat" from "Thermal Capacity."

8. A piece of aluminum weighing 200 grams is heated to a temperature of 97° C. and dropped into 150 grams of water at a temperature of 18° C. The water container is made of copper and weighs 100 grams. Take the specific heat of copper from the tables and compute the specific heat of the aluminum if the final temperature of the mixture is 35.0° C. 0.218.

9. A coffee percolator contains 6 lb. of water at a temperature of 45° F. The percolator is made of copper and weighs 4 lb. Find the amount of heat required to bring the water to the boiling point in this percolator.

1064 B.T.U.

10. Two grades of coal selling at \$12 and \$10 per ton respectively are found to have heats of combustion of 13,000 and 11,000 B.T.U. per pound respectively. Compare the actual costs of these two grades of coal as a source of heat. What other practical considerations will be involved in a choice between these two fuels? \$10.00 coal—22,000 B.T.U./cent.

\$12.00 coal—21,666 B.T.U./cent.

11. Assume equal efficiencies and compare the cost of heating with natural gas which costs \$.65 per 1000 cubic feet and has a heat of combustion of 1400 B.T.U. per cu. ft. with the cost of heating with coal rated at 12,000 B.T.U. per lb. and costing \$11.00 per ton.

Gas—21,500 B.T.U./cent.

Coal—21,800 B.T.U./cent.

12. Repeat problem 11 on the assumption that the efficiency of heating with coal is 15 per cent and the efficiency of heating with gas is 50 per cent.

Gas—10,800 B.T.U./cent.

Coal—3,270 B.T.U./cent.

13. Can you prove algebraically that the volume coefficient of thermal expansion for any solid is approximately three times the linear coefficient? Suggestion: Let L be one unit of length; K, the linear coefficient; then, after a temperature rise of one degree, L becomes $L + LK$.

14. At absolute zero, what volume becomes zero? What volume still remains?

15. Mention several good examples of heat being given off when a gas is suddenly compressed.

16. Mention several good examples of heat being absorbed when a gas is suddenly expanded.

17. A liter of gas at 20° C. and 760 mm. pressure is cooled to −10° C. What will be its new volume if again taken at 760 mm. pressure?

900 cc. approx.

18. A man inflates his tires to a pressure of 30 lb. per square inch as registered by a tire gauge, on a day when the atmospheric pressure is

14.5 lb. per square inch and the temperature, 30° C. If the atmospheric pressure rises to 15.5 lb. per square inch and the temperature drops to 10° C., what will the gauge register when applied to a tire—assuming no leakage? 26 lb.

Experimental Problems

1. The calibration of a thermometer may be checked—or if a blank thermometer is available, it may be calibrated—by the following procedure.

(*a*) To check at the freezing point of water—0° C.; 32° F.

Mix cracked ice and water in a wooden, earthenware or other well-insulated container. While keeping the mixture thoroughly stirred, insert the thermometer to be tested and take the reading.

(*b*) To check at the boiling point of water—100° C.; 212° F.

Take the reading when the thermometer bulb is just below the surface of pure water which is boiling freely in an open dish.

Since 100° C. or 212° F. is the boiling point of pure water at a barometric pressure of 76 cm. or 30 in., the barometer reading should be taken, and the corresponding accepted value of the boiling point looked up in a table or calculated. What are other sources of error?

2. Heat about 500 grams of lead shot to about 100° C. in a double boiler. Take the temperature of the shot by stirring a thermometer bulb slowly through them. At the same time take the temperature of a carefully weighed quantity of water, say 500 grams.

Then immediately pour the shot into the water and slowly stir the mixture with the thermometer already in the water until its temperature ceases to rise.

Assuming all the heat lost by the lead to be gained by the water, solve the following equation for the specific heat of lead:

$$\text{Wt. of Water} \times \text{C}^\circ\ \text{Temp. rise} \times 1\ (\text{S.H. of water}) = \text{Wt. of Lead} \times \text{C}^\circ\ \text{Temp. drop} \times \text{S.H.}$$

Compare your experimental value with the accepted value—determining the per cent of difference, and suggesting a few possible reasons for this difference.

the molecules will result in an even average division of energy of motion among all the molecules of both objects. Then the two will be at the same temperature.

In this manner the molecular kinetic theory of heat suggested in an earlier chapter is seen to offer a very satisfactory explanation for the fact that heat may be conducted from one object to another with which it is in contact or from one part of a body to another which is initially at a different temperature.

The rate at which this kinetic energy of molecular vibrations is transmitted along a given piece of material varies greatly from one substance to another. Aluminum is an example of a substance having good heat conductivity. An aluminum tablespoon is unsatisfactory for stirring anything cooking on a stove because the handle soon gets too hot to hold. However, aluminum for the cooking kettle is very desirable for it conducts the heat away from the point of the flame and introduces heat into the thing being cooked from the sides as well as directly over the flame.

Conduction of heat in water is rather poor as may be seen by filling a test tube with water, holding it by the bottom, and placing the top in a flame until the water at the top boils. There is no difficulty in holding the bottom end of the tube for some time after the water reaches boiling temperature at the top. (See Figure 136.)

Air, or any other gas, is rather a poor conductor. For this reason storm windows are often used on houses in cold climates, since by their use a layer of still air is trapped between the regular window and the storm window. This arrangement reduces the loss of heat through the rather thin glass.

However, the molecules of air gain some energy from impacts against the relatively warm inside glass, and they give up some energy in impact against the storm window; so the heat insulation is imperfect. Better results would be obtained if all the air from the space between the two sets of glass were removed. It is impractical to do this with storm

dimensions of objects whether solid, liquid, or gaseous. In general, objects grow larger as they become heated. So it follows that a cubic foot of hot water weighs less than a cubic foot of cold water. If the hot water could be held in a weightless container and placed at the bottom of a lake, it would float to the top the same as any other substance less dense than the water of the lake.

Similarly, hot air is less dense than cold air at the same pressure. It is possible to make a paper container light enough so that it will float as a balloon when filled with sufficiently hot air. In fact the first successful balloons built by man were hot air balloons.

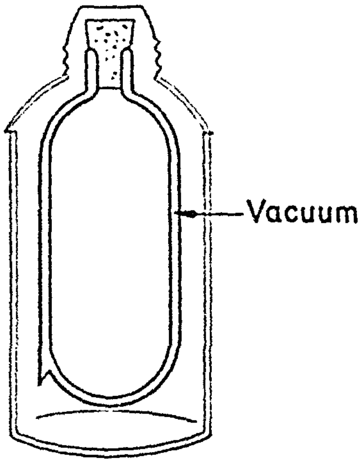


FIG. 137.—Double walled flask for keeping things hot or cold. The flask is usually protected by being placed in a metal container as shown. Heat transfer by conduction and convection is prevented by the vacuum. Heat transfer by radiation is prevented by silvering (shown red in the figure) the inner surface of the outer wall and the outer surface of the inner wall.

The motion of the heated water and air in the above cases are examples of the transfer of heat from one point to another by a simple transportation process, much as one might carry a hot brick from one place to another. However, when the action takes place in the same medium (that is, hot water in cold water, hot air in cold air) and is a result of density changes, it is called heat transfer by *convection*.

is available. Similarly hot air may rise through large pipes from a furnace to rooms above, and cold air may reenter the furnace at its bottom. The cold air is usually taken from grates in the floors of the rooms located at points some distance from those through which the hot air is introduced into the rooms.

Air may be heated on contact with the earth. This air then rises and the circulation of air so induced is largely responsible for the development of winds.

3. Transfer of Heat—Radiation

Heat may be transmitted from one place to another without the use of any material carrier. The best known example of this method of transfer of heat is found in the radiation of energy from the sun to the earth. We know that the atmosphere of the earth is negligible at a comparatively short height above the earth and that the rest of the more than ninety million miles up to the place where the sun's atmosphere begins is filled with nothing at all. So we conclude that both light and heat energy from the sun must come through space. Such a method of transfer is called *radiation* as indicated below.

Heat in the form of radiation has an electrical and magnetic nature and in this sense is related to light and also to radio waves. Some discussion as to how such waves are produced, the speed at which they travel, and their effect when received will be found in later sections on electricity, light, and related subjects.

Some heat energy is radiated from objects at ordinary temperatures as well as from hot bodies like the sun. For example, a steam radiator not only gives heat by conduction to the air as the air molecules make impacts against it, but it also actually *radiates* some heat energy. This heat energy is absorbed to a slight extent by the air through which it travels and to a greater extent by solid objects, such as chairs, tables, room walls, etc. These latter objects both radiate heat and absorb it. But if two objects are of the same size

and have similar kinds of surfaces, the hotter will radiate more and absorb less heat in comparison to the colder.

Objects with dull black surfaces are better absorbers and radiators than objects with polished surfaces. For example, a silvered mirror not only *reflects* light, but it also reflects heat efficiently and *absorbs* very little. For this reason silvering is used on "Thermos" bottles as noted above. This silvering prevents the radiation of heat into or out of the container.

Steam and hot water radiators for maximum efficiency should be painted a dull black. At least they should not be painted with anything that gives them a polished surface, for a radiator should be designed to be efficient in transferring heat to the room.

Some Important Facts

1. CONDUCTION

The kinetic energy of molecular motion, that is, heat, is transmitted from molecule to molecule by impact.

Metals in general are good conductors.

Solids of more complex molecules are in general poor heat conductors or good insulators. Mineral compounds, such as glass and organic materials such as cellulose are examples.

Any solid material is a fairly good insulator if finely porous in structure.

2. CONVECTION

Due to the fact that fluids readily expand with an increase in molecular motion, that is, decrease in density, warmer portions of a fluid are buoyed upward by surrounding colder, denser fluid.

Such flotation, or convection currents serve to transfer heat energy through fluids, provided this energy is added at the bottom of the fluid or withdrawn at the top.

3. RADIATION

Heat energy also sets up electromagnetic waves of longer wave length than visible light but shorter than the shortest radio waves. These infrared waves travel best through space containing no molecular matter, at the speed of light, 186,000 miles per second. We receive our heat as well as light from the sun solely by radiation.

Black bodies absorb radiant heat energy, but white or polished surfaces reflect much of it. The only effective insulation against heat transfer by radiation is a good reflecting surface which returns the heat waves back whence they came.

Generalizations

CONDUCTION

In conduction, the kinetic energy of molecular motion is transferred from molecule to molecule by contact.

CONVECTION

In convection, increased molecular motion results in decreased density so that convection currents are set up by flotation. Convection is entirely limited to fluids.

RADIATION

Heat, and other forms of radiant energy, can travel at 186,000 miles per second through vacuum.

Questions and Problems

Group A

1. What is heat conduction? Mention a few good examples.
2. What is convection? Mention a few good examples.
3. What is radiation? Mention a few good examples.
4. What is meant by insulation?
5. How is insulation against conduction often accomplished? Give examples.
6. How is insulation against convection accomplished? Give examples.
7. Mention several instances of useful conduction. Useful convection.
8. How can heat transfer due to radiation be retarded? Mention useful examples.
9. What do you know about the construction of thermos bottles or jugs?
10. Can you explain any systems for heating rooms in terms of convection?

Group B

1. In terms of molecular behavior—that is, the kinetic theory—explain the difference between conduction and convection.
2. What is a radiometer? Can you explain its action?
3. What useful purpose is served by polishing and nickel-plating the outside of tea-kettles, coffee pots, etc.? Why not nickel-plate the inside?
4. Should the surface of steam radiators be rough or smooth? Painted light or dark?
5. How is heat transfer by each of the three methods prevented in the vacuum type thermos bottle?

6. In an old fable, a person is under suspicion because he blows on his fingers to cool them when burnt, and also to warm them when frost-bitten. What do you think about the reasonableness of his procedure?

7. Why do even small ponds and streams seldom freeze solid in winter?

8. London with about the same North Latitude as Winnipeg—has a much smaller annual temperature range. Can you explain why?

9. Diagram a hot water heating system—so placing all parts as to best utilize convection.

Experimental Problems

1. Freeze about 50 cc. of water in a test tube, and then run about an equal amount of cold water on the ice. Apply a flame diagonally on the side of that portion of the test tube containing the water until the water boils, carefully observing the ice meanwhile. What conclusion is justified?

2. Arrange equal sized pieces of several solids as spokes in a wheel. Dip the end of each spoke in melted paraffin, until approximately equal amounts of paraffin have solidified on the end of each. Then apply a flame at the center, or hub of the wheel, and note the order in which the solids conduct sufficient heat to melt the paraffin on their ends.

CHANGE OF STATE

Any liquid, when left in an open dish, tends to decrease in volume by a process called evaporation. The effect is large for some substances, for example, ether or high-test gasoline. The effect is moderate for some well-known substances—such as water. It is small for other liquids—for example, heavy oils and mercury.

The addition of heat to a liquid not only raises its temperature but also increases its tendency to evaporate. Finally a temperature of the liquid is reached such that the addition of more heat results only in increased evaporation and not a further increase in the temperature of the liquid. This temperature is called the boiling point.

When heat is added to a solid the temperature of the latter rises until the solid begins to melt. The addition of more heat causes more melting but does not raise the temperature of what is left of the solid.

The amount of heat required to change a unit mass of a solid into a liquid at the same temperature—or vice-versa—is called the latent heat of fusion for that substance. Similarly the amount of heat required to change unit mass of a liquid into a vapor at the same temperature—or vice-versa—is called the latent heat of vaporization for that substance.

When a liquid evaporates, the molecules formerly in the liquid move about in space as the molecules of a vapor. The number of molecules in a given space may become so great that they tend to join with one another to cause condensation and so return to the liquid state. The subject of vapors is of particular interest in the case of moisture in the air and some special attention is devoted to this matter at the end of the chapter.

1. Evaporation

It is common knowledge that the amount of water in an open dish grows less and less until finally it has entirely disappeared. The same phenomenon occurs even more rapidly with high-test gasoline, much more slowly with thin oil, and so slowly as to be difficult to observe with a heavy oil or with mercury.

Tests on the surrounding atmosphere show that all of these liquids are being slowly converted from liquids into vapors

thermometer before and after the steam has been condensed in the water and so the amount of heat used for warming the water can be calculated.

Example. 200 grams of water initially at a temperature of 20.0° C. are heated by letting steam at 100° condense in the water. The final amount of water in the container is found to be 205 g. The final temperature is 34.0° C. Neglect the effect of heating the container and calculate the heat of vaporization.

The change in weight of the water shows that $205 - 200 = 5$ g. of steam condensed.

The change in temperature from 20.0° to 34.0° for 200 g. of water shows that the heat needed to warm this water is $200 \times 14 = 2800$ calories.

This 2800 calories is gotten partly from the latent heat of vaporization of the steam and partly from the cooling off of this 5 g. of hot water from 100° to 34°. The heat from this latter source is

$$5 \times (100 - 34) = 5 \times 66 = 330 \text{ calories}$$

This leaves

$2800 - 330 = 2470$ calories to be obtained from condensing the 5 g. of steam.

$$\frac{2470}{5} = 494 \text{ calories per g.}$$

An answer nearer the accepted value would have been obtained if we had allowed for heating the container which holds the water.

6. Latent Heat of Fusion

If a liquid is cooled by permitting heat to leave it and escape into the surroundings, a temperature will finally be reached where the substance begins to solidify. Extraction of further heat results in more of the substance changing state, but does not produce a further change in temperature until all of the liquid has changed to a solid.

If heat is now added to the mixture, the solid material

freezing temperature. As one steps on this ice the pressure tends to compress the ice, this layer of ice takes in heat from the surrounding ice and the pavement beneath and succeeds in melting (and shrinking in size.) The actual temperature of the water, however, is lower than before when it was in the form of ice. When one attempts to remove his foot (and hence relieves the pressure), this water immediately begins to freeze and so one's shoes appear to freeze to the icy walk.

If a substance decreases in volume on solidifying, as is true for some metals, added pressure raises the temperature at which freezing will take place.

7. Humidity

In early sections of this chapter we have seen that in a closed vessel evaporation will proceed until the density of the vapor over the surface is such that the number of molecules reentering the liquid per second is just equal to the number leaving it.

Even in an open vessel this condition of saturation of the vapor is very nearly reached near the surface of the liquid unless air currents carry away the vapor as fast as it comes from the liquid. Evaporation in the open is greatly aided by the motion of the air, for fewer molecules have an opportunity to reenter the liquid.

This effect is apparent in such a simple case as the cooling of one's body caused by the evaporation of perspiration. A breeze of dry air aids the evaporation and increases the cooling effect while still air or air that is already laden with moisture retards evaporation.

From the point of view of evaporation, the important item is not what the absolute amount of moisture per cubic centimeter may be, but what the relative amount is as compared to the saturation value for that temperature. The actual amount of moisture may be determined by passing a known volume of air through a water absorbing substance and then determining the weight of water obtained.

The relative amount of moisture, however, can be obtained

by simply measuring the cooling effect. For example two identical thermometers may be arranged so that one of them has its bulb covered by a cloth which is kept wet with water. The other thermometer is kept dry and the air is kept circulating by means of a fan. The temperature of the wet thermometer will drop by an amount depending inversely on the degree of saturation already existing in the air. From the difference in the readings of the thermometers and with the aid of a table that must have been originally prepared by absolute determinations, the "relative humidity" may be determined.

In very humid air, especially if there is no breeze, a temperature of 75° F. may seem too warm, since very little evaporation can take place from one's body. The same temperature in a heated house during cold weather might seem only moderately warm or even cool if the relative humidity were low as is often the case under such conditions.

If air containing moisture is cooled, a temperature may be reached where the amount of moisture present is more than sufficient to produce saturation. In such a case the particles may begin to cluster and condense. If this effect takes place in the air itself we have clouds and fog. If condensation of water vapor in air proceeds far enough, large drops of water form and fall as rain.

Condensation may take place on solid objects with which moisture laden air comes in contact. This formation is called dew.

The dew point is thus seen to be the temperature at which saturation of water vapor will occur and this temperature will depend on the density of moisture originally in the air. A pitcher full of ice water can collect dew on its surface even when the warm air in the room contains but a small density of water vapor.

Some Important Facts

1. As a matter of chance interchange of kinetic energy, surface molecules occasionally attain sufficient velocity to take off from the surface into space. In liquids this is called evaporation; in solids, sublimation.

liquid to gas. As heat is withdrawn from a substance the process is reversed.

During a change of state there is no temperature change, all heat energy exchanged being accounted for as latent heat of fusion or vaporization.

Questions and Problems

Group A

1. Describe the motions of molecules in gases and solutions.
2. What determines the temperature of any object?
3. How is evaporation of liquids explained on the basis of molecular theory?
4. Why does a liquid cool during the process of evaporation?
5. Why does not a liquid continue to cool indefinitely as evaporation proceeds from an open surface?
6. Compare evaporation of water from two similar dishes with the surfaces of the water open to the air in both but with one dish encased in a thick layer of material that hinders the passage of heat into or out of the dish through its sides and bottom.
7. Just what is meant by the term "boiling"?
8. What is the boiling point of a liquid?
9. What is the difference between a vapor and a gas?
10. What similarities exist between a vapor and a gas?
11. When is a vapor said to be saturated?
12. Explain the meaning of "Heat of Fusion." What is its value in calories for pure water?
13. Explain the meaning of "Heat of Vaporization." What is its value in calories for pure water?

Group B

1. If a saturated vapor has its volume reduced as would result by compression with a piston, what immediately happens?
2. What happens in a saturated vapor if the temperature is lowered?
3. Why does mist (or dew) settle on the inside of the windows of a heated automobile in the winter?
4. In most houses in the winter time the windows stay dry. Why should this be true?
5. Into 150 grams of water at 10° C. 2 grams of steam at 100° C. is condensed. Find the resulting temperature of the mixture. Neglect the heat capacity of the container. (Remember that the steam first becomes hot water at 100° C. and then cools to the final temperature of the mixture.) 18.3° C.
6. Into 150 grams of water at 30° C. 2 grams of ice at 0° C. are placed. Find the final temperature of the mixture. (Remember that the water resulting from the ice must warm up to the final temperature of the mixture.) Neglect the heat capacity of the container as before. 28.5° C.

HEAT AND WORK

The ease with which mechanical work may be converted into heat and with which heat may be changed to mechanical work has lead to the type of industrial age in which we now live. Heat energy may be used to supply mechanical work through the medium of engines of various types and these in turn operate the machinery of our factories as well as the transportation devices with which we are familiar. Man has become the director of energy instead of one of the chief sources of energy as he was in by-gone days.

The earliest type of engine to contribute greatly to our present developments was the steam engine. To some extent this engine has been displaced by steam turbines and by internal combustion engines. This chapter contains brief descriptions of the action of these devices and suggests references for further study.

1. Energy in the Form of Heat Is Equivalent to Mechanical Energy

In the first chapter on heat (see page 251) some attempt was made to show that our best explanation for the nature of heat is based on the mechanical behavior of molecules. We saw that heat could be induced by simple mechanical processes such, for example, as friction. Finally we concluded that it would be sensible to have mechanical energy units for heat units, and that the calorie and the British Thermal Unit are arbitrary units for heat. They are simple multiples of the mechanical units for energy.

Below is repeated the table showing numerical relations between the heat units and some of the mechanical units.

$$4.19 \text{ joules} = 1 \text{ calorie}$$
$$778 \text{ ft. lb.} = 1 \text{ B.T.U.}$$
$$252 \text{ calories} = 1 \text{ B.T.U.}$$

Not only can we obtain heat by mechanical processes, but we can produce mechanical work by the use of heat energy.

The best-known examples of utilizing heat for obtaining mechanical energy are found in steam engines, gas engines, and gasoline engines. A study of the conversion of work to heat and heat to work is called thermo-dynamics. On the practical applications of these energy conversions depends almost all of our industrial development.

Heat from burning coal produces steam with which steam engines and hence machinery is operated in our factories; or steam or gas engines drive electric generators which in turn supply power to motors and in some cases to heating devices. Perhaps the only important exception to these processes in our industrial life today is the use of water wheels which start with mechanical instead of heat energy and drive electric generators or other machinery.

2. Changing Mechanical Energy to Heat Energy

We saw in the first chapter on heat that friction is a common process for converting mechanical energy to heat energy. A practical application of this effect is found in the braking action on automobiles and trains.

For simplicity let us consider a car traveling on a level road. To stop the car we must remove its kinetic energy. This mechanical form of energy is converted by friction into heat in the brakes. The heated parts of the brakes gradually cool off by losing heat to the surroundings. If the brakes are applied so hard that the wheels skid, the friction takes place between the tires and the pavement and some heating of the tires and the pavement takes place.

If a car is moving down a hill it will tend to gain speed unless the friction in the machinery and in the brakes is of such a value as to convert the potential energy available in dropping down the hill into heat energy.

It is fairly simple to compute the heat developed in cases of this kind.

Example 1. An automobile weighing 3000 lb. is traveling 30 m.p.h. on a level road. Find the amount of heat developed in the brakes in stopping the car assuming all of the work produced by friction to be in the brakes.

First we find the kinetic energy of the car. Its velocity, 30 m.p.h., must be converted to ft. per sec.

$$30 \text{ m.p.h.} \equiv 45 \text{ ft. per sec.}$$

The kinetic energy is (see page 170)

$$\begin{aligned} \text{K.E.} = \frac{1}{2}mv^2 &= \frac{1}{2} \times 3000 \times (45)^2 \\ &= 3{,}030{,}000 \text{ ft. lbal.} \\ &\equiv \frac{3{,}030{,}000}{32.2} = 94{,}000 \text{ ft. lb. approx.} \end{aligned}$$

Since it requires 778 ft. lb. to produce one B.T.U. we write

$$\frac{94{,}000}{778} = 121 \text{ B.T.U. approx.}$$

Example 2. A sledge hammer weighing 2 kilograms strikes a rivet with a velocity of 20 meters per sec. It strikes 150 blows per minute. Assume that all of its energy is converted into heat and find the amount of energy in calories.

The kinetic energy of the sledge as it strikes is

$$\begin{aligned} \text{K.E.} &= \frac{1}{2}mv^2 = \frac{1}{2} \times 2000 \times (2000)^2 \\ &= 4{,}000{,}000{,}000 \text{ ergs} \\ &\equiv \frac{4{,}000{,}000{,}000}{10{,}000{,}000} = 400 \text{ joules} \end{aligned}$$

The amount of heat to which this work is equivalent may be found by noting in the table above that 4.19 joules are required to produce one calorie of heat. Hence

$$\frac{400}{4.19} = 95.5 \text{ calories per blow}$$

In one minute the heat developed will be

$$95.5 \times 150 = 14{,}325 \text{ calories}$$

Example 3. Suppose that half of the heat developed in the above example goes into the hammer itself. Neglect

losses and compute the temperature rise in one minute if the material of which the hammer is made has a specific heat of 0.09.

We set the heat available equal to the mass of the hammer multiplied by its specific heat and by the unknown rise in temperature thus:

$$H = msT$$

$$7{,}163 = 2{,}000 \times 0.09\ T$$

From which $$T = \frac{7{,}163}{180} = 39.8° \text{ C.}$$

3. Heating a Gas by Compressing It

Another example of the conversion of mechanical energy to heat energy may be found in compressing a gas. In Figure 139 is shown a cylinder filled with gas. We will suppose the walls of the cylinder are of such nature that heat cannot flow through them. Suppose that the piston is now shoved into the cylinder, thus compressing the gas to any desired amount. The rebound of the molecules against the piston as it is pushed in gives them greater velocity than if the piston were stationary. This seems to be a reasonable explanation for the fact that we find the gas to be heated, and the amount of the heat developed is equivalent to the work done mechanically in pushing in the piston.

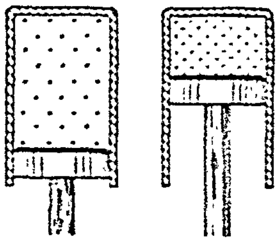


FIG. 139.—The temperature of a gas rises when the gas is compressed and decreases when the gas expands.

If we now let the gas expand to its original volume the force of the gas against the piston can do work in pushing out the piston. It will now be found that the temperature of the gas has returned to its original value.

Expansion and compression of a gas when accompanied by temperature changes as described above are called *adiabatic.*

These temperature changes with gas pressure changes are easily observed qualitatively at any auto service station where

compressed air is available for filling tires. The pump that compresses the gas becomes hot from the gas which is heated in its cylinders. On the other hand the end of the nozzle becomes cold if the air is allowed to expand and escape into the atmosphere.

4. Changing Heat Energy to Mechanical Energy

The latter part of the above section on gases suggests the possibility of obtaining mechanical energy at the expense of

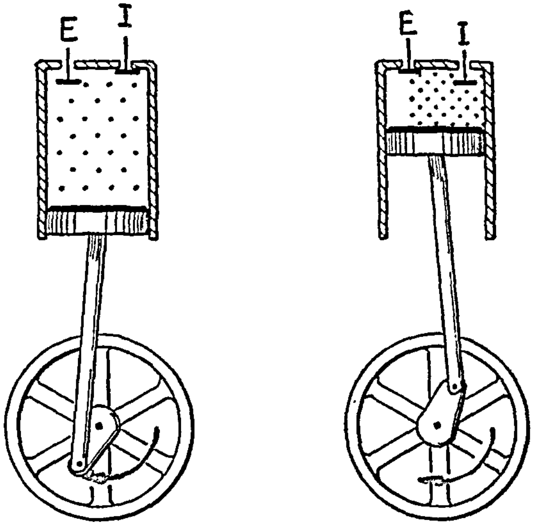


FIG. 140.—Hot compressed gas may be used to operate an engine.

heat energy by means of the expansion of a gas. This method is commonly employed in all of the engines in common use, including steam, gas, gasoline, oil-burning engines, steam turbines, and others.

For simplicity we may consider a compressed air engine built on the principle of the example in the previous section. In Figure 140 we have this cylinder with its piston connected by means of rods to a wheel in such a manner that the wheel will turn when the gas expands. Suppose that when the piston has moved out as far as it can go, a valve E in the cylinder opens so that the air will escape when the piston

returns. The piston will return due to the angular inertia of the wheel.

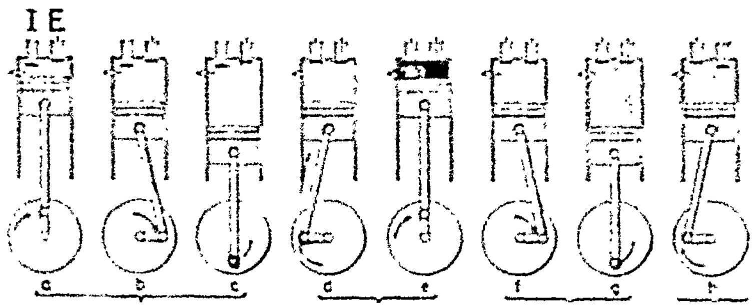
When the piston has moved into the cylinder as far as possible, valve *E* closes and new hot compressed air is introduced from a supply line through valve *I*. The process can then repeat itself.

Hot air engines as described above are seldom used, but the same principle is employed in the use of steam. The details of the valves for letting the steam in and out and for making steam act on both sides of the piston as well as other items of steam engine construction may be found in text books on heat engines and in any good encyclopedia.

5. Internal Combustion Engines

The action of internal combustion engines is related to that for steam engines and hot air engines; but instead of having steam at high pressure and temperature (or hot air) delivered to the cylinder a mixture of combustible vapor and air is allowed to burn rapidly in the cylinder itself.

The cycles through which an internal combustion engine passes may be seen from the pictures of Figure 141. Suppose that the wheel to which the drive rod is attached is turning as indicated by the arrow in (*a*). The exhaust valve *E* is closed and a mixture of vapor and air is drawn into the cylinder



through the inlet valve I as the wheel revolves and the piston moves through the position shown in (*b*) to that shown in (*c*).

As the wheel continues to turn the inlet valve I closes and the piston is pushed back through the position indicated in (*d*) until it regains the original position as shown in (*e*). During this operation the valve I as well as valve E is kept closed. The mixture of vapor and air is highly compressed during this process. A spark now occurs at the spark gap and ignites the compressed combustible mixture and raises its temperature (and hence its pressure). In the process of expanding, these hot gases force the piston out through position (*f*) and into position (*g*). During this part of the process the piston applies a torque to the wheel. The expanding gases do mechanical work until the piston reaches the end of its stroke. At this instant the outlet valve E opens and as the momentum of the wheel results in a return of the piston, the gases, somewhat cooled off as a result of the work done in expanding, escape as the piston passes through position (*h*) and finally returns to position (*a*). At this time the exhaust valve E closes and the inlet valve I opens. The entire process may now be repeated.

A little study of the above action of an internal combustion engine shows that this type of engine has energy applied to the crank shaft only during one-half of every other revolution. The continued motion of the engine depends on the energy stored in the motion of the fly wheel. The engine is called a four cycle engine because there are four distinct actions in the cylinder—(1) Filling the cylinder with a combustible mixture on the outstroke of the piston, (2) Compressing the mixture on the return stroke, (3) Burning the mixture and supplying mechanical work on the next outstroke, and (4) Exhausting the burned gases on the second return stroke.

To make an engine run smoothly it would be desirable to supply mechanical energy uniformly instead of in pulses as is done in the engine just described. If we assume that energy is being supplied one-fourth of the time by a single cylinder engine, we may connect four cylinders and their pistons to the same rotating mechanism and they could be so arranged that

one cylinder would start supplying energy as soon as another ceased. Such engines were used extensively in the earlier models of automobiles.

Greater smoothness of operation may be obtained by having still more cylinders since it will then be possible to have one start delivering energy before the preceding one has ceased. Six, eight, twelve, and sixteen cylinders have been used successfully in automobiles. At the time of this writing eight is the most popular number since it represents a good compromise between economy of operation and smoothness of running the car.

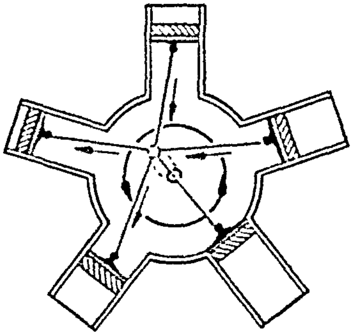


FIG. 142.—Radial type engine as used in airplanes. Five cylinders. (Valves and spark plugs are not shown.)

When a large number of cylinders are used they may be arranged side by side in a row or they may be placed in two rows which are arranged as a V.

Still another system is used for many airplane engines. This consists in arranging them radially as is shown diagrammatically in Figure 142. Here an odd instead of an even number of cylinders is used. Smaller engines use three or five cylinders and larger ones, nine or more cylinders.

6. Diesel Engines

A Diesel engine is a special type of internal combustion engine. On the stroke when a mixture of vapor and air would be drawn into an ordinary engine, only air is taken into the diesel. On the compression stroke, compression is carried to a higher degree than in most other types of engines and the temperature of the air is raised by the compression to about 500° C. Into this hot air, oil is sprayed and at once begins to burn. No spark is required. This type of engine can operate efficiently on cheap grades of oil and it is used extensively in large installations such as ocean-going boats, and Diesel-electric locomotives. It is used on some trucks.

7. Turbines

A turbine consists essentially of a tube which has fan vanes mounted in it on an axle running lengthwise through the tube in such a manner that if a stream of fluid should flow through the tube the vanes would tend to revolve.

Such a device may be made to use with water, or it may be made to operate from hot vapors such as steam. It is more efficient than other types of water wheels on the one hand, or other types of steam engines on the other.

Students especially interested in engines should consult text books on mechanical engineering and the various encyclopedias for the details of turbines as well as other types of engines covered only briefly or omitted entirely in this text.

Some Important Facts

1. *Energy as heat and as mechanical work* are readily convertible either into the other. One calorie is equivalent to 4.19 joules and one B.T.U. to 778 ft. lb.

2. Impacts, mechanical friction and compression are common methods for changing mechanical to heat energy.

3. Gases are readily heated by compression, and as readily cooled by expansion. Heat changes in gases due entirely to pressure and volume changes are called adiabatic.

4. Some devices for converting heat energy into mechanical energy, such as heat engines, utilize the expansion of hot gases.

In steam engines, advantage is taken of the expansion due to vaporization as well as the subsequent expansion of the superheated steam.

In internal combustion engines, the great increase of volume accompanying the oxidation of some volatile hydrocarbon fuel in the cylinder, and the expansion of the hot gaseous combustion products, provides the gas pressure.

5. In its most common form, the internal combustion engine involves four strokes per cycle as follows:

(*a*) Intake—explosive mixture enters through intake port on first outstroke—only intake valve is open.

(*b*) Compression—explosive mixture is compressed, and so heated, on next instroke—both valves are closed.

(*c*) Ignition-Power—spark plug fires the explosive mixture, piston moves out under power—both valves still closed.

(*d*) Exhaust—burnt gas is pushed out through exhaust port on second instroke, completing cycle—only exhaust valve is open.

The four strokes of the Diesel engine are similar except that only air is taken in and compressed and fuel oil injection replaces the firing of the spark plug.

6. The pressure of an expanding gas may be applied to the blades of a continuously rotating turbine, instead of being used to push a piston back and forth in a cylinder.

Turbine engines are more efficient than reciprocating engines in that inertia always aids the desired action during continuous operation.

Generalizations

A heat engine is a machine for transforming heat energy into mechanical energy.

In the external combustion type, fuel is burned outside the cylinder to provide heat to vaporize and subsequently further expand a liquid, generally water, the force of which expansion pushes a piston in a cylinder or rotates turbine blades.

In the internal combustion type, an explosion takes place within the cylinder, the force of which directly pushes a piston or other essential moving parts.

Problems

Group A

1. What is a steam engine?
2. What are the two main types of heat engine?
3. What do you know of the history of the steam engine?
4. Can you explain the action of the reciprocating engine?
5. What is a steam turbine? What are its advantages?
6. Explain what happens in each of the four strokes of the four cycle gas engine.
7. What is the powerful advantage of the Diesel type of internal combustion engine over the usual automobile engine?
8. What do you think of steam engines as a source of power for automobiles? Have they ever been tried? With what success?
9. Illustrate by labeled diagrams what happens in the cylinder of an automobile engine in each of the four strokes of the piston.
10. One pound of water at the top of Niagara Falls can fall 165 feet. If the energy of falling could all be converted into heat, find the B.T.U. developed. (Suggestion: The mechanical energy available is equal to the potential energy at the top of the fall.) .212 B.T.U.
11. The force required to move a piece of sand paper over a board is 5 lb. Find the rate of developing heat if the sand paper is moved 5 ft. per second. (Assume that all of the mechanical energy expended goes into heat. Is this true?) .0321 B.T.U. per sec.

Group B

1. If the kinetic energy of a car is all converted into heat in stopping, find the heat developed when a 2600-lb. car traveling 70 ft. per sec. is brought to rest. 255.8 B.T.U.

2. If the car in the above problem is stopped in 20 sec. find the rate at which heat is developed. 12.79 B.T.U. per sec.

3. Why does a dull drill heat metal more than a sharp drill?

4. Heat energy is converted into mechanical energy in a certain engine at the rate of 500 B.T.U. per minute. Compute the mechanical power output in foot lb. per second and in horse power. 11.8 H.P.

5. Coal that gives 13,000 B.T.U. per lb. on burning is used to heat water and the steam produced operates a steam engine. Find the power developed by the engine if 10 lb. of coal are burned per minute and if the process is 8 per cent efficient. 245 H.P.

6. What are the relative advantages of the four and the two-stroke per cycle internal combustion engines?

7. For what purposes are two stroke per cycle internal combustion engines especially adapted? Why?

8. There has been relatively less economic necessity for the extension of the use of the Diesel engine in the U.S.A. than in other countries. Can you explain why?

Experimental Problems

1. If toy or other small model steam engines are available, operate them slowly with a view to following their operation in detail.

2. Visit a service garage for the purpose of examining an automobile engine which has been taken apart:

Note particularly

a. How the explosive mixture is fed into the cylinders.
b. How the firing of the plugs is timed—and their firing order.
c. How the valve action is regulated.
d. How the force of the explosion is transmitted to the crankshaft.
e. How spent gas is disposed of.

Chapter 20

ADDITIONAL MOLECULAR PHENOMENA

When a compressed gas expands through small openings it tends to cool itself. Practical applications of this effect are found in household refrigerators and also in methods for the liquefaction of gases such, for instance, as air.

When holes in the wall of a container holding gas are made so small as to be comparable to the size of molecules themselves, a different effect takes place. It is called osmosis and some of its principal applications are in the fields of biology.

These two phenomena can best be explained by means of the molecular theory of matter so often referred to in the subject of heat, and the success of the theory here is added proof of its soundness.

The third and last part of the chapter deals with a group of closely related effects called by such names as "adhesion," "cohesion," "surface tension," and "capillarity." They *depend on forces of attraction among the molecules of single* substances or between the molecules of one substance and those of another when the two are in contact.

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1. The Cooling of Gas on Expansion

In the preceding chapter we saw that when a gas expands, as in the case of pushing out a piston, it can do work against outside forces and the energy for this work is taken from the heat of the gas so that the temperature of the gas drops. However, a gas may be allowed to expand by flowing out of the cylinder through a small hole, and if the small hole opens into a chamber which is evacuated, the expanding gas cannot do work against outside forces. Such an arrangement is indicated in Figure 148. However, experiment shows that the molecules escaping into chamber *B* form a gas that is at a lower temperature than that in chamber *A*.

This effect is interesting in several ways. It supplies the principle on which liquid air is produced and on which most refrigerators operate. It furnishes an additional case where a belief in the molecular theory of matter gives the best explanation.

On the *A* side of the small hole in the partition of Figure 143 there is gas pressure tending to force a small stream of gas through the hole. When a molecule is just in the opening, moving outwards, it is attracted backwards by the molecular attraction of the molecules in *A*. Since the molecules in chamber *B* are relatively scarce there is no similar force pulling it forward. The result is that the molecule slows down a bit as it travels into section *B*.

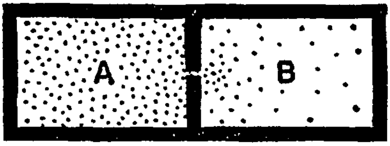


FIG. 143.—Compressed gas in chamber A expanding through a small opening into chamber B.

The result of this action on each molecule as it passes through the opening is that the average motion of the molecules in *B* is less than the average motion of those in *A*. In other words, the temperature of the gas in *B* will be lower than that in *A* if our theory is right. Experiment shows that this is the case. The fact that the molecular theory of matter furnishes a reasonable explanation for this effect is additional proof for the general correctness of the theory.

This basic discovery in pure science was made by Joule and Thompson and the effect is named after them. Some ingenuity was required to apply the discovery to the making of refrigerating devices.

2. Refrigeration

When gas is compressed into the cylinder *A*, Figure 143, by a pump it is heated in the process as we have seen in the preceding chapter. If this compressed gas is allowed to remain in the cylinder for a time it will give up its heat to the surroundings. Then when it expands into *B* it will drop to a temperature below that of the surroundings.

This process can be speeded up by having the compressed gas go through a coil as shown in Figure 144. Air circulates around the coil either by convection or with the aid of a fan in small refrigerators such as are used in houses. In large installations cold water may be sprayed over the coils.

The cooled compressed gas is then allowed to expand and it drops to a temperature considerably below that of the surroundings.

This cold gas may be circulated directly in a refrigerator or it may be used to cool brine which can then be circulated in coils that are placed in the refrigerators.

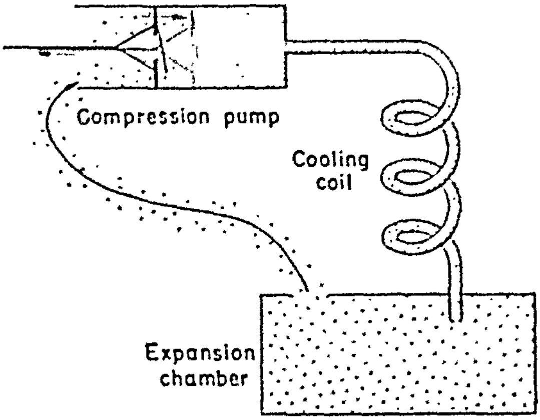


FIG. 144.—When gas is compressed its temperature rises. It may be cooled in a coil system to its original temperature while still under pressure. If it now expands it will drop to a lower temperature. The arrangement is called a refrigerating system.

Instead of using a substance such as air, which remains gaseous throughout this process, some gas such as ammonia which will liquefy at the pressures and temperatures readily produced by this process may be used. The hot compressed gas is cooled by air or water as described above and becomes a liquid. The liquid then expands through the small opening and becomes a gas at the reduced pressure in an expansion chamber. This change of state from the liquid to the gaseous condition requires heat energy which must be taken from the substance itself. This means that the temperature of the substance must drop as it goes from the liquid to the gas state.

Ammonia is commonly used in large refrigeration plants.

Various other substances, such for example as sulfur dioxide and ethyl chloride are often used in small refrigerators.

3. Liquefaction of Air

In Figure 145 is shown an extension of the refrigeration system described above. It is used for cooling gases such as air

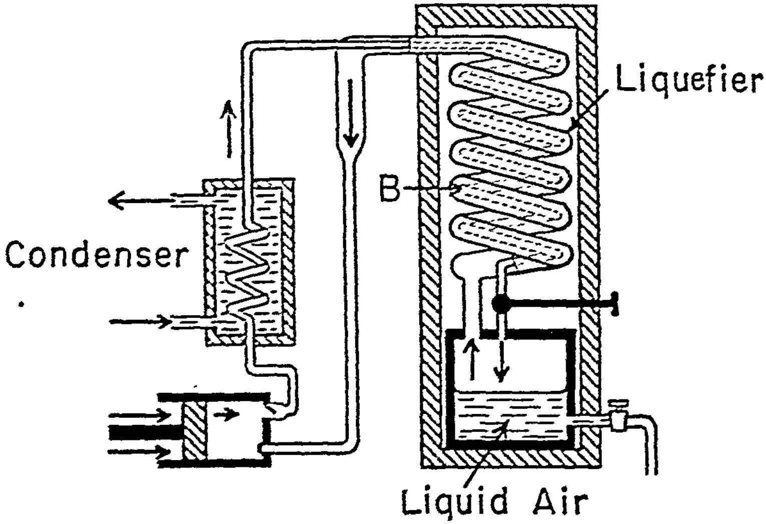


FIG. 145.—Diagram of the apparatus for liquefying air.

to the point where they will liquefy. The pump and initial cooling coil are essentially like those used for ordinary refrigeration.

The second coil *B* is arranged so that the gas expanding through the small hole at the end of this coil will pass back over the coil. In this manner the escaping gas cools the compressed gas coming down through this second coil.

Hence the gas on expanding is still colder than that which first escaped. The process is continuous and the cooling effect builds up until the gas in the second coil begins to liquefy and drip out of the hole at the bottom of the tube along with some expanding gas.

Liquid air so produced contains oxygen, nitrogen, helium,

neon, argon—the latter three in small quantities. Carbon dioxide and water vapor may be removed from air before it is liquefied. The temperature of liquid air that is open to the atmosphere is about −191° C. when first produced. The nitrogen boils off much more rapidly than the oxygen and the temperature of the latter is about −184° C.

Liquid air enables one to perform many spectacular experiments. Mercury placed in it becomes a hard substance. A piece of rubber hose becomes brittle. Raw meat can be broken like glass.

4. Passage of Molecules Through Extremely Small Openings—Osmosis

In Figure 146 we have a drawing similar to that of Figure 143 except that the opening in the new drawing is very small as compared to that in the earlier figure. In fact this new opening is about the same size as that of a molecule itself. In this case there will be no stream of gas flowing through the hole and the only time that any molecule will get through will be on the rare occasion when, in its random motions, it happens to glide straight into and through the opening.

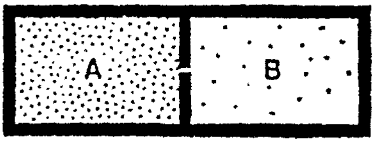


FIG. 146.—The passage of molecules through very small openings—osmosis.

Under these conditions it will make practically no difference whether there is or is not gas in the chamber *B*. In fact the gas in the second chamber may even be at a higher pressure than that in the first. The passage through the opening is now purely an individual chance occurrence for any molecule.

Suppose that chamber *A* is filled with oxygen at high pressure and chamber *B* with nitrogen at low pressure. Occasionally an oxygen molecule will pass into *B* and a nitrogen molecule into *A*. Once in a while one of these molecules may reenter the passage and return to the original chamber. After a long time we would expect to find the density of oxygen molecules in *B* to be the same as in *A* and

the density of nitrogen molecules in A to be the same as that in B. Experiment bears out these predictions.

Next we may imagine employing two gases such that the molecules of one are much larger than those of the other. We may also suppose the passage way to be large enough to let the small molecules through, but not large enough to let the larger molecules pass.

For example, we may place helium (He) in chamber A and carbon dioxide (CO_2) in chamber B. If the size of the passage is just right, helium molecules can pass into B until the number in B is so great that the rate of returning helium atoms equals that of the new ones coming in from A. In the meantime the carbon dioxide molecules have to stay in B. The gas pressure in A goes down as a result of a loss of gas and the pressure in B goes up since pressure from the helium adds to that already existing due to the carbon dioxide.

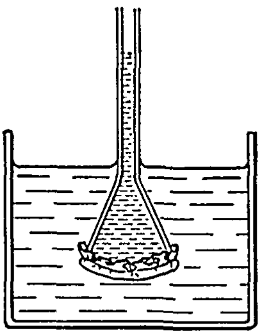


FIG. 147.—Osmosis in liquids.

The processes described above are called *osmosis*. Osmosis is not restricted to gases, but takes place in solutions also. Membranes with enormous numbers of tiny holes are used, for the amount of transfer of molecules through a single hole would be very small in any reasonable length of time.

One of the common examples to show in a laboratory is the case of a water solution of sugar. The water-sugar solution is placed in a container such as the inverted funnel shown in Figure 147 and is immersed in pure water. The end of the funnel is covered with a porous membrane.

The membrane is chosen so that sugar molecules cannot pass through it but water molecules can do so readily. The sugar solution may be thought of as a solution of water diluted by the sugar, whereas the pure water in the outside container may be thought of as relatively concentrated water. It is to

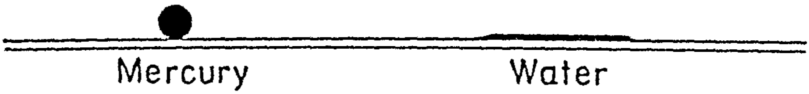


FIG. 148.—Behavior of a drop of mercury and a drop of water after being placed on a glass surface.

be expected then that more water molecules will pass into the solution with the sugar than will pass out of it.

If the level of solution in the funnel is the same as that of the water outside at the beginning of the experiment, it follows that the level in the funnel will rise as the funnel gains water through the membrane. The increased level in the funnel is a measure of the pressure created by the process and is called *osmotic pressure.*

The process is extensively involved in the distribution of food and liquids in all forms of plant and animal life and so is of particular interest to the biologist.

5. Adhesion and Cohesion

If a small drop of mercury is placed on a glass plate it takes on an almost spherical shape, whereas a drop of water on the same plate tends to spread over the plate. We say that water "wets" the glass and that the mercury does not wet it. (See Figure 148.)

Again we can resort to the molecular theory of matter for an explanation of these facts. All molecules have an attraction for one another. In the case of mercury the attractions among the mercury molecules are greater than the attraction between mercury and glass molecules. In the case of water the attractions between water molecules and glass molecules are greater than those among the water molecules themselves.

Intermolecular attractions among molecules of the same kind are called cohesive forces and those between molecules of unlike kinds are called adhesive forces.

We can say that a liquid is wet with respect to a solid if the adhesive forces are greater than the cohesive forces in the liquid. Water is wet with respect to glass but is not wet with respect to paraffin. Similarly mercury is not wet with r

to glass but is wet with respect to some metals, such as lead and copper, for example.

6. Surface Tension

We have just seen that the behavior of a drop of mercury on a glass plate may be explained by assuming forces of attraction to exist among the molecules of the mercury. The surface of the drop seems to behave like an elastic film to hold the drop in the shape that we find.

This elastic film effect is found on all liquid surfaces. For example, a light sewing needle may be placed on the surface of water in a beaker. If it is not "wet" by water it will depress the surface slightly and be supported much as if the water were actually covered by a film. Similarly, if water is placed in a container that it does not wet, it is possible to "heap" the container. Sometimes this effect can be produced even with a glass if the surface of the glass is contaminated so that it does not wet readily.

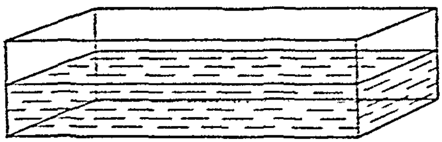


FIG. 149.—Water behaves as if its surface were covered with an elastic film.

Let us imagine a long rectangular container as shown in Figure 149. If it is filled with water and if the liquid surface really were covered with an elastic film, we could determine the breaking strength of this film by simply measuring the pull that would be required to break it. Although there is no actual film over the water, we can measure the corresponding property of a water or other liquid surface in the following manner. In Figure 150 a short piece of wire is held in a horizontal position below the surface of the water. The wire is wet by the water. The vertical thread holding the wire is fastened to a delicate weighing balance so that as the wire is pulled into the position shown in Figure 151 the pull may be measured.

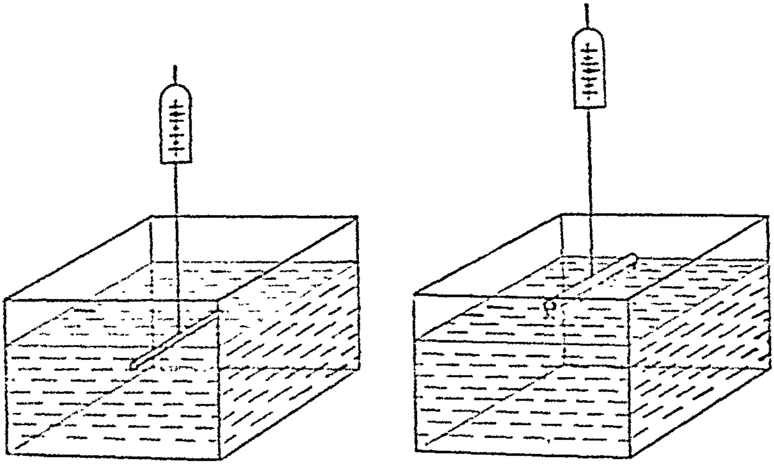


FIG. 150.—(*Left*) A piece of wire beneath the surface of water.
FIG. 151.—(*Right*) Water films suspended from raised wire.

The amount of pull required to break the wire loose from the film of water can be easily found in this manner.

Here we see that the film of water surface extends downward on two sides of the wire and allowance for this fact mus- be made in computing the force required to break one centit meter width of the film. The force required to break one centimeter width of the film is a measure of the surface tension of the particular liquid in question. The following table gives the surface tensions (against air) in dynes per centimeter for a few common liquids (at 20° C.)

Alcohol (ethyl)	22.3
Benzene	28.9
Water	72.8
Mercury	465

7. Capillarity

An interesting application of the subject of surface tension and the related effect of adhesion is found in the behavior of liquids in small tubes. In Figure 152 is shown such a glass tube in a beaker of water. Water climbs up inside the tube as shown, and the smaller the diameter of the bore the higher

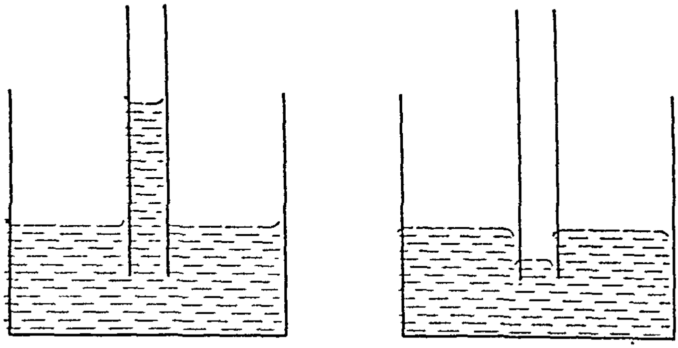


FIG. 152.—(*Left*) Capillary rise of water in a glass tube.
FIG. 153.—(*Right*) Capillary depression of mercury in a glass tube.

the water goes. On the other hand, mercury is depressed under similar circumstances as shown in Figure 153.

The surface of the liquid as shown in Figure 152 behaves as a film which is attached at its outer edge to the inside of the glass tube. This film can supply a force depending on the strength of the film and the distance around its outer edge. If T is the strength of the surface tension and r the radius of the tube the force is

$$F = T2\pi r \tag{1}$$

Liquid will rise in the tube until the column of liquid weighs this amount. The weight of the column of liquid will be the volume of liquid multiplied by its density, d, that is

$$W = \pi r^2 hdg \tag{2}$$

where h is the height to which the column rises above the level in the container. Multiplying by g gives the weight in dynes.

Equating (1) and (2) gives

$$\pi r^2 hdg = T2r\pi$$

From which
$$h = \frac{2T}{rdg}$$

If the surface tension is known, this formula can be used to predict the height of capillary rise for a tube of known size.

On the other hand, unknown surface tensions may be measured by observing the rise of the liquid in a tube of known radius of bore.

Some Important Facts

1. When gas molecules escape through a small opening from a high pressure space into a low pressure space, the average kinetic energy of the molecules and hence temperature of the escaped gas decreases.

2. The latent heat of vaporization, as well as the cooling effect of gaseous expansion, is utilized in refrigeration.

3. In liquefying air, the cooling effect due to rapid expansion of some of the air is used to cool a smaller quantity of air to a temperature below the boiling points of its constituent gases.

4. Porous membranes may be found such that small molecules will pass through but larger ones are stopped. Such a membrane may be used to separate two unlike fluids. Some of the fluid with the smaller molecules may then pass into the region of the other and build up additional pressure there. The process is called osmosis.

5. All molecules mutually attract each other. If the molecules are alike, this attraction is called cohesion; if unlike, it is called adhesion.

6. *Cohesion packs molecules on the surface of a liquid more closely than those below the surface, causing liquid surfaces to behave somewhat as stretched membranes. This phenomenon is called surface tension.*

7. If the adhesion of liquid molecules for the sides of a container is stronger than their cohesion for each other, the combined action of such adhesion and surface tension causes the liquid to rise in tubes and spaces of hairlike or capillary size. This is called capillary action, or capillarity.

If the molecules of a liquid cohere to each other sufficiently more strongly than they adhere to the molecules of the side walls, the liquid sinks in capillary spaces.

Generalizations

Molecules have two basic, mutually opposing, behavior tendencies: (1) Very energetic motion at any temperature above absolute zero. (2) Mutual attraction for each other.

Heat changes accompanying volume changes in cases, as well as changes of state, are essentially changes in kinetic energy of molecular motion.

Such molecular properties of fluids as diffusion and osmosis are evidences of energetic molecular motion.

Such molecular properties as cohesion, surface tension and capillarity are evidences of molecular attraction.

Questions and Problems

Group A

1. What is a molecule?

2. What two basic characteristics do all molecules have?

3. Distinguish cohesion and adhesion and give a few common examples of each.

4. Define and give some common examples of surface tension, capillarity, diffusion, and osmosis.

5. In terms of molecular properties, distinguish "winter" from "summer" crankcase-oil.

6. Mention some uses of diffusion, osmosis, surface tension and capillarity.

7. How is artificial refrigeration accomplished? What are some desirable physical and chemical properties of a refrigerant?

Group B

1. Give an explanation for the cooling of a gas as it expands in a cylinder in such a manner as to push a piston and hence do external work. (See the preceding chapter.)

2. Give an explanation for the cooling of a gas as it expands through a small opening.

3. Since a gas is heated on compression and cooled on expansion, what addition to the process must be made to make it a practical method for obtaining refrigeration?

4. For ordinary refrigeration temperatures, what difference will be found if ammonia vapor is used in one case and air in another?

5. Describe in detail the extension of the refrigeration process to the procedure employed for liquefying air.

6. Hydrogen can be liquefied only at a considerably lower temperature than air ($-234°$ C.). Suggest a plan for using liquid air to aid in the process of liquefying hydrogen. Make a diagram of the apparatus.

7. Describe the process of osmosis:

   (*a*) Giving the molecular reasons for the action.
   (*b*) Showing the effects with gases.
   (*c*) Showing the effects with solutions.

8. Is water always wet?

9. Describe surface effects on liquids due to the intermolecular attractions within the liquids.

10. Describe an experiment for measuring the strength of the surface tension of a liquid.

11. How is surface tension involved in capillarity?

12. Tubes with a radius of bore of 0.05 cm. are used to determine the surface tensions of a number of liquids. From the table given in this chapter estimate the heights to which you would expect alcohol, benzene, and water to rise in the tubes. 1.15 cm. 1.31 cm. 2.97 cm.

Experimental Problems

1. *As another evidence of surface tension,* carefully lay a clean, double-edged safety razor blade on a smooth water surface.

2. To demonstrate capillarity, insert several capillary tubes of different size, first in water; then in mercury. Blow a little very thin oil through the larger tubes and again try them in water.

Chapter 21

VIBRATORY MOTIONS—ELASTICITY

This chapter opens with a review of to and fro motions that are common to our everyday experience. Objects able to sustain such motions are seen to have mass (so that they may gain momentum) and elasticity (so that each may tend to return to its initial position.)

An object may have volume elasticity or form elasticity.

The amount of force required to stretch or compress a spring unit distance is called its force constant.

The period of a to and fro motion of an object depends on its elastic strength and on its massiveness. The greater the mass the slower the motion, the greater the elastic strength, the faster the motion will be.

The simplest form of to and fro motion is called "simple harmonic motion."

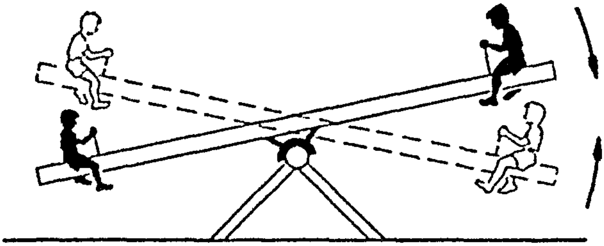
The number of complete motions that a vibrating object makes each second is called its frequency.

An object may be set in vibratory motion either by giving it a single push or pull or by giving it a number of relatively small impulses timed to coincide with the natural frequency of vibration of the object. The latter method results in what is called resonance. Resonance phenomena are found in musical instruments, in the action of automobiles in going over rough roads and in many other everyday experiences.

1. To and Fro Motions

To and fro motions are almost as common in our daily experience as motions in any one direction or in a curved path. Teeter totters, lawn swings, and rope swings give us a first-hand acquaintance with such motions in our early childhood. An even earlier familiarity with the idea may have been gained from bouncing up and down in a baby carriage.

The behavior of the prongs of a tuning fork or the antics of a loose rod on an automobile are examples of vibratory motions that reverse in direction much more rapidly. Sometimes the to and fro motion repeats itself with great regularity, as is the case with the tuning fork; sometimes it is a rather irregular affair as often happens with some loose part on the automobile.



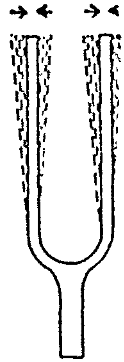


FIG. 154.—(*Left*) Early impressions of a to and fro motion are gained from experience on a teeter totter.

FIG. 155.—(*Right*) The prongs of a tuning fork vibrate to and fro at a rate that is usually high in comparison to the teeter totter.

An examination of such motions of objects shows that the object must have two properties. The first of these is a tendency to return to some fixed position, usually at the center of the motion. The second requirement is that the object must have mass; that is, inertia. Inertia is necessary so that the momentum (mass × velocity) of the object can carry it past the natural rest position in spite of the force tending to return the object to this position.

Since all real objects have mass, the requirement of inertia for a natural vibratory motion is easily met, and we can devote our attention to the forces available to make the object return towards its natural fixed position.

In the case of a swing, this force is supplied by gravity. In the case of the tuning fork it is supplied by the elastic property of the steel of which the fork is made.

2. Elasticity

Elasticity is the name applied to the tendency of any body to return to its initial size or shape after being deformed by some external force. The return of a rubber band to its initial size after being stretched, the similar action of a coiled spring, the return to spherical shape of a gas-filled rubber

balloon after being squeezed—these are all familiar examples of action due to elastic properties.

A body which returns, after being distorted, to the same shape which it had initially is said to have *form elasticity.* This property is called *rigidity.* The tuning fork is a good example of an object having rigidity; so also is any springy piece of metal.

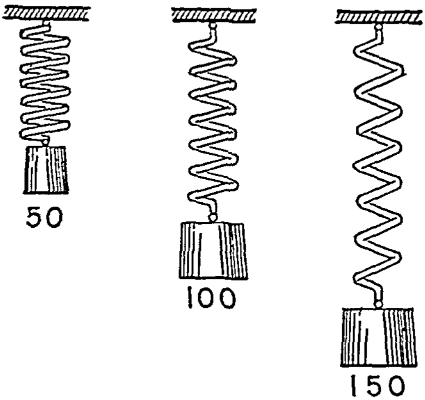


FIG. 156.—The stretch of a spring is proportional to the stretching force.

Water, on the other hand, has no rigidity at all but it does resist changes in its volume when it undergoes pressure. This property is called *volume elasticity.*

Any object is said to be perfectly elastic if it experiences distortions that are proportional to the forces applied and if it returns to its initial state when the forces are removed. For example, the coiled spring of Figure 156 might stretch 2 cm. when a weight of 50 g. is hung from its lower end. A second 50-g. weight (making 100 g. altogether) would stretch the end of the spring another 2 cm. (making 4 cm. altogether). If a sufficient number of 50-g. weights were added a point would be reached where another 50-g. weight would not stretch the spring another 2 cm. We would say that the elastic limit had been approached. If so much weight has been added that the elastic limit has been reached or exceeded it is probable that when all the weights are removed the spring will not return to its original state.

The fact that equal forces produce equal distortions of an elastic object, or, as a mathematician would say, that the ratio of force to displacement for an elastic body is a constant, is often called "Hooke's Law."

The amount of force required to stretch the spring unit distance is called the *force constant* of the spring. In the

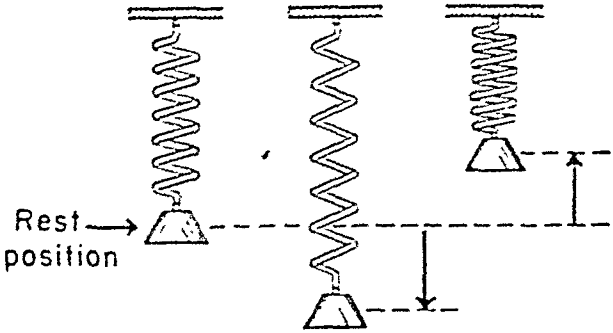


FIG. 157.—End and rest positions of a bouncing mass attached to the end of a spring.

above example we see at once that the force constant is 25 g. per cm. In many cases this force must be expressed in absolute units, dynes per cm. in the metric system or poundals per ft. in the English system. For the above spring we would have

$$25 \times 980 = 24{,}500 \text{ dynes per cm.}$$

3. Motion of a Vibrating Mass

If any mass is hung on the end of the spring in the above example, the spring will stretch due to the weight of the mass. If the mass is pulled below the natural rest point and then released it will start up. Its momentum will carry it through the rest point, but its weight will tend to stop it when the spring is no longer stretched sufficiently to support it. It then starts to fall. Its momentum carries it downward through the rest point. It is finally stopped due to the increased pull in the spring, and then starts up again.

An attempt to show the rest point and the two end positions of this bouncing mass is made in Figure 157, but the action must be familiar to everyone, and in any case it is an

is, the number of to and fro excursions in a given time will be greater.

These results are what one would expect and that they are correct is easily found by laboratory experiment.

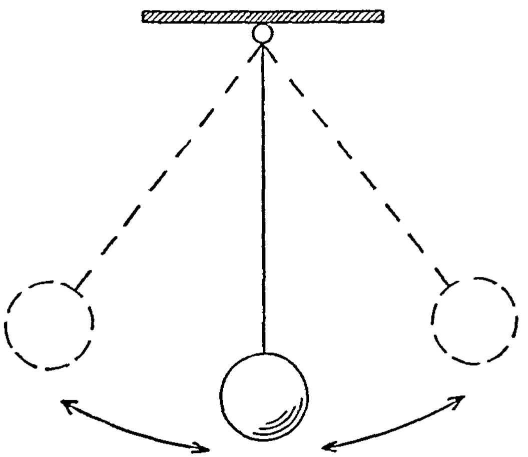
The exact relation between the time T of one complete to and fro motion, the mass of the bob M and the force constant of the spring F_c is

$$T = 2\pi \sqrt{\frac{M}{F_c}}$$

A check of this formula may be made experimentally by determining the force constant of a spring in the laboratory and then timing its period when a bob of known mass is suspended from the end of the spring.

4. Simple Harmonic Motion

The motion of the bob on the spring in the preceding example is an example of the simplest type of vibratory motion and is called *simple harmonic motion.* The prongs of a tuning fork also perform simple harmonic motion. The motion of the pendulum of a clock is approximately simple harmonic if the bob does not swing through too large an angle. The to



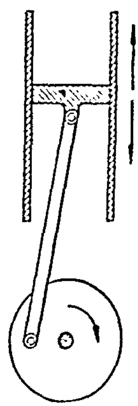


FIG. 158.—(*Left*) A pendulum swings to and fro with a motion that is nearly simple harmonic.

FIG. 159.—(*Right*) The to and fro motion of a piston in a cylinder is almost simple harmonic.

and fro motion of the piston in the cylinder of an engine is almost, but not quite, simple harmonic.

Simple harmonic motion is defined as a motion in which the acceleration is always towards the same fixed point and where the amount of the acceleration is always proportional to the displacement from that fixed point.

5. Period and Frequency in Simple Harmonic Motion

The time for a complete to and fro motion is called the period of the motion, and it can be calculated as shown above from the formulae given there. The periods of such motions vary enormously even in motions with which we are familiar. The period of a moderately long clock pendulum may be of the order of two seconds, the period of the prongs of a small tuning fork may be a few ten-thousandths of a second.

It is often convenient to talk about the number of complete vibrations that will occur in one second and this number is called *frequency*. Obviously the frequency is found by taking the reciprocal of the period; for example, if the period of a tuning fork is one five-hundredth of a second the number of to and fro motions per second must be 500.

6. Swings and Pendulums

A swing or pendulum is a good example of a common periodic motion, which, although not quite simple harmonic, is near enough to it to be treated in the same manner. No elastic property of material is involved, for the force tending to move the swing or pendulum back to its rest position is provided by gravity; that is, it depends on the weight of the object. The further the latter is moved from the rest position, the greater the fraction of its weight that can act to return it.

If a swing or pendulum consists of a bob that is small in comparison to the mass of the bob, the period for the motion may be expressed as

$$T = 2\pi\sqrt{\frac{L}{g}}$$

where L is the length of the pendulum measured from the

point of support to the center of the bob and where g is the value of gravity.

This formula may be used to find the length of a pendulum which will have a specified period. On the other hand, it may be used as a convenient and accurate method for measuring the value of g. It is only necessary to measure the length of the pendulum and to time its period over a large enough number of vibrations to get an accurate value for T. The experiment may easily be repeated for a number of values of L, since a small sphere of lead or other metal hung by means of a light thread makes a quite satisfactory pendulum.

It is interesting to notice that the value of the mass of the bob does not affect the period of the pendulum, although it did affect the period in the case of the vibrating bob on the spring. Change of mass does change the inertia of the pendulum just as it does that of the spring system, but the strength of the spring remains constant no matter what the value of the mass may be, while in the case of the pendulum a new value of mass also means a new value of force, for the force depends on the weight of the object. The two effects cancel each other in the case of the pendulum.

7. Resonance

From the above discussions we may conclude that any object having the properties of elasticity and inertia may have a natural vibration if it is free to move. Such a body may be set vibrating by distorting it and then setting it free. The vibratory motion will die down in amplitude and finally stop as the energy originally put into it becomes lost through friction with the air or internally in the material.

The body may also be set into vibration by giving it a succession of small pulls or pushes. If these are timed just right, the object will vibrate violently. The most efficient timing is a number of pulses per second just equal to the natural frequency of the object. For example, if a series of little pushes is applied to a pendulum that has a natural period of two seconds, the pushes should be delivered to the pendulum one every two seconds.

In practice this is what one does in pushing a rope swing. A push on the back of the person every time he starts to swing away *from* you will soon build up the amplitude of the swinging motion.

The fact that a body with a natural vibrational period will *develop largest amplitudes of motion when a series of impulses* is timed to coincide with the natural frequency of the body is called *resonance*. Resonance plays an important part, sometimes favorably, sometimes unfavorably, in many everyday activities.

If one is driving a car across a number of tracks on a railroad crossing, the maximum bouncing of the car occurs if the speed of the car is such that the next rail is hit just as the car rebounds from the effects of striking the first rail. For any given car there will be some speed for crossing that is more uncomfortable than either higher or lower speeds.

A loose fender may rattle violently at some car speeds and scarcely at all at others. The original vibratory impulses may come from the engine or from the gears or other parts of the mechanism of the car having to do with the speed at which it is being operated.

A clock pendulum is kept moving by an arrangement within the clock which gives a small impulse to the pendulum at the same position on each swing.

If a vibrating tuning fork is held near a second identical fork, the second one will begin to vibrate.

The object receiving the energy is said to have "sympathetic" vibrations, but the whole process of receiving energy pulses at the proper rate to build up a maximum vibratory motion is called *resonance*.

If a body is perfectly elastic, displacement is proportional to the force causing it, and when the force is removed the body completely recovers.

The force causing unit displacement is called the force constant of a body.

3. The time of one vibratory cycle increases with the amount of mass moved but decreases as the force constant is increased.

4. Simple to and fro motions are generally called periodic motions and also simple harmonic motions.

5. The full period of a simple harmonic motion is:

$$T = 2\pi \sqrt{\frac{M}{F_c}}$$

6. It can be shown that for swinging pendulums the ratios M/F_c and L/g are equal, where L is the length of the pendulum and g is the local acceleration due to gravity, so the full period may be expressed as:

$$T = 2\pi \sqrt{\frac{L}{g}}$$

7. A body is most easily set in vibration at its natural frequency.

When a body is caused to vibrate at its natural frequency by another vibrating body of the same natural frequency, it is said to experience sympathetic vibrations and the whole process is called resonance.

Generalizations

Free bodies having elasticity as well as mass, vibrate readily with a period of vibration that is greater for greater masses and less for greater elastic forces.

Questions and Problems

Group A

1. Why does an object that can sustain vibrations have to possess mass?

2. In what way must the force acting in a simple harmonic motion always be directed?

3. What part of the mass of the steel in a tuning fork is most important in determining the rate of vibration?

4. In what parts of a tuning fork are the important elastic forces located?

5. What is meant by the frequency of a vibrating system? Give examples.

6. What is meant by resonance? Give examples other than those in this text.

7. If a pendulum clock loses time, should the pendulum be lengthened or shortened? Why?

8. How could a pendulum be used to determine the local value of g? Approximately, what is the value of g in English units? In metric units?

9. When Galileo discovered the laws of the pendulum, he used his own pulse rate as a time measuring device. Comment on the accuracy of this procedure. What precautions might he take to reduce error?

10. If g is 32 ft. per sec. per sec., find the length of a pendulum whose period is one second. 0.8106 ft.

11. If a pendulum whose full period is one second has a length of 24.8 cm., what is the local value of g? 979.1

Group B

1. If a little steel were removed from a tuning fork near the base of the prongs, what would happen to the natural period? Why?

2. A weight of 100 g. is found to stretch the end of a spring 4 cm. What is the force constant of the spring? 24,500 dynes per cm.

3. What is the natural period of the spring in problem 2 when a mass of 300 g. is suspended from it? 0.695 sec.

4. How much mass would you hang on this spring (problem 2) to get a period of 1.5 sec.? 1396 g.

5. The body of an automobile settles on its springs 1.2 in. when five people totaling 800 lb. get into it. Find the force constant of the springs. (Suggestion: The displacement must be converted to feet, and the force must be expressed in poundals.) 257,600 lbal. per ft.

6. Find the up and down period of the above automobile with its passengers when the car hits a bump. The car alone weighs 3200 lb. 0.782 sec.

7. As a man bounces up and down in a pitching automobile does he appear to weigh more or less than normal at the top of his bounce? Why?

8. A simple pendulum with a length of 99.0 cm. is found to have a natural period of 2.0 sec. What is the value of gravity at the location where the experiment is performed? 977.1

9. What length simple pendulum would be used in a place where g has a value of 980 cm. per sec.^{2} if the period is to be 1 second? (Notice that the period is the time of a complete to and fro motion. The time when the pendulum goes from one end of its swing to the other, or from the center position to the end and back is only one half the whole period.) 24.82 cm.

10. Find the length of a simple pendulum to have a period of 0.5 sec. in a place where g is 980 cm. per sec.^{2} 6.205 cm.

11. On a sufficiently ancient car one learns to estimate his speed by various rattles. Explain why this method is reasonable.

12. How might a dog set a suspension bridge in vibration?

13. A delicate meter is to be used in a machine shop where there is a great amount of vibration. The meter is placed on a little platform

suspended by springs. When a certain machine in the shop goes into action the meter is jarred more than if it were simply placed on a table. What is the difficulty? What would you suggest as a remedy?

Experimental Problems

1. Suspend a weight of convenient size from a coiled spring. Set the arrangement in oscillation by pulling the weight down a little and letting go of it. Count the cycles for two minutes and calculate the period and frequency.

As the weight comes to rest does it slow down in its rate of vibration, that is, does the frequency decrease?

From your data, calculate F_c, the restoring force constant of the spring.

Repeat the above procedure, using both heavier and lighter weights. How do T and F_c vary with the weight used?

Repeat again with springs of different degrees of stiffness. How do T and F_c vary with the stiffness of the spring? What is stiffness in terms of F_c?

2. Suspend a 500-gram weight by a light thread whose length can be varied between about 2 ft. and 3 ft. Vary the length at six-inch intervals between these limits, in all cases measuring the length L from the point of suspension to the center of the weight. For each length let the weight swing through an arc of about 15°, counting the swings for two minutes. Repeat for arcs of 30°, 45° and 60°. In all cases, calculate T and F_c.

Arrange all data in an orderly table, and determine how period and frequency are affected by: (*a*) length, (*b*) weight, (*c*) amplitude of arc.

Using the pendulum formula, calculate the local value of g and try to account for any deviations from the accepted value.

WAVE MOTION—SOUND

The preceding chapter dealt with the vibrations of objects and in particular with those that performed vibratory motion in a simple and regular fashion.

The present chapter is concerned with transferring energy from a vibrating body to the medium in which the body is located. Such energy in the form of waves then travels through the medium and may be picked up at a distant point.

Among the more familiar types of wave motions in everyday life are water waves, waves on strings, and waves in air. These three types of waves are good examples of the transmission of energy from one point to another by wave motion. Such transmission of energy is called energy transfer by radiations.

Waves in air are particularly interesting because they form the basis of the important phenomenon of sound.

The chapter also shows how strings of fixed length and at a fixed tension, and how air in tubes of definite length, may themselves behave as vibrating bodies with natural periods of their own. Most musical instruments with which we are familiar (for example, piano, violin, flute, saxophone) are developed from one or the other of these types of vibrators.

1. Water Waves

In the preceding chapter we saw that an object possessing mass and elasticity may have a natural frequency of vibration, and when once set in motion it will vibrate at this frequency until all the energy of its motion is dissipated.

Some of this energy of motion is used up in moving the air or other medium in which the object is vibrating. For example, we may consider a block of wood hung on a spring. If the block of wood is pulled down and released, it will bob up and down as we have seen. The motion will gradually decrease in amplitude and the block will finally come to rest.

Suppose now that the spring is mounted over a body of water so that the block dips into the water. (See Figure 160.) When the block bobs up and down water waves will spread

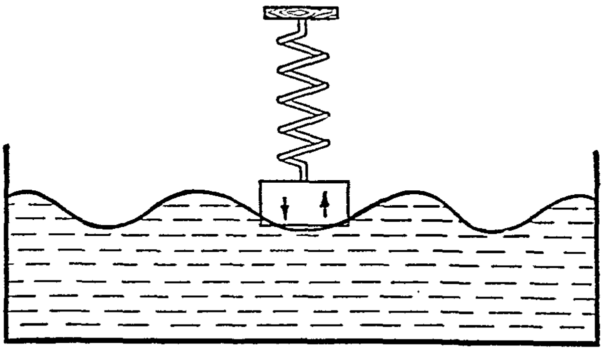


FIG. 160.—A block of wood bounces up and down and sends out waves on water.

out in all directions and the block will come to rest much sooner than if it had been bouncing in air. The energy of motion has been transferred to the water and has been radiated away from the block and spring.

That the waves carry energy may easily be seen if we place a second block of wood on the water at some distance from the first block. (See Figure 161.) When the waves come along this block will be carried up and down, and if it is connected with a crankshaft it may be able to do useful work.

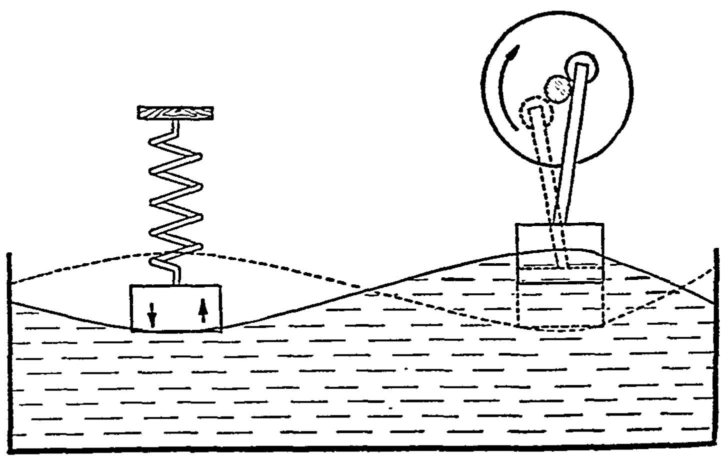


FIG. 161.—The block at the left sends out water waves. The waves cause the second block to rise and fall. If this block is properly connected to a wheel it can operate as an engine to deliver useful work.

Wave motion is obviously a method for getting energy from one point to another without direct contact as in the case of applying forces to objects, and without carrying the energy directly, as would be the case with convection currents of heated air for example. The transfer of energy by wave motion of a medium is called *radiation.*

It is interesting to notice that in water waves, the water located near the bouncing block does not move over to the second block about which we have been talking, but it is the motion of the water that is transmitted. A small piece of floating cork on water is observed to remain in nearly the same location as wave after wave goes by. Such a particle rides up and down as the crests and troughs of the waves pass and this is the more obvious motion. Actually it moves slightly to and fro horizontally also. However, the general impression that one gets of water waves is that the water appears to move up and down, and this motion, obviously, is at right angles to the horizontal direction in which the waves are traveling.

Waves which move at right angles to the direction of travel are called transverse waves. Particles in a water wave have a somewhat more involved motion, as pointed out above, but the general appearance is that of a transverse wave.

2. Waves on a Stretched String

The motion of waves may also be studied with the aid of a long rope as indicated in Figure 162. The rope is fastened at one end and is held with the hand at the other. A kink upward is pulled in the rope and it is then released. The kink will travel down the rope at a speed depending on the tension in the rope and on how heavy the rope is.

For a heavy rope the rate of travel will be slower than for a light rope. The rate of travel will be higher for a well-stretched rope than for one which is held loosely.

This is a case of the particles of the medium (in this case the rope) undergoing vibration sidewise while the motion is propagated along the rope, and this is a true case of transverse

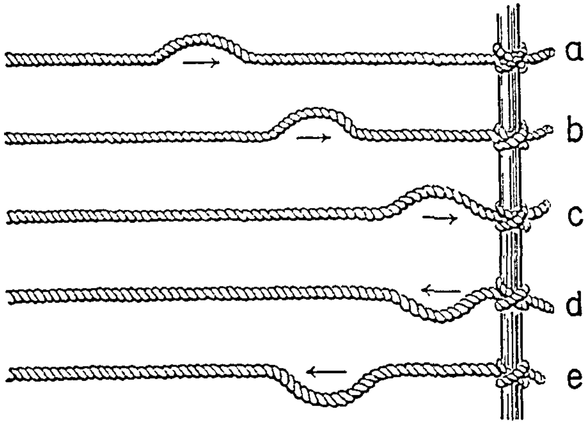


FIG. 162.—Successive pictures of a single kink traveling on a rope and showing the reflection of the kink at the fastened end of the rope.

vibrations. A series of kinks may be put into the rope at one end and so a whole train of waves will pass along the rope. A person with sufficient imagination might arrange a device to be driven by the motion of the rope at its far end and then again we would be able to demonstrate the transmission of energy through a medium by means of a wave motion in the medium—in this case, along the rope.

When one tries the experiment with the rope, he should choose a rather long one, and even then the simple phenomenon just described will be somewhat hard to observe clearly; for when the kink reaches the tightly fastened end of the rope it will reverse itself and start back up the rope just as a rubber ball will rebound from a concrete sidewalk. The reflected train of waves will then appear to be mixed up with the train we are sending along the rope. This state of affairs is important in itself and we will discuss it in Section 6 of this chapter.

3. Waves in Air

We will now return to the case of the bouncing block on the spring, but this time we will suppose the block to be in the air instead of in water. As it moves downward, the bottom of the block must strike the molecules of air below it and put them into motion much as a swinging bat puts a baseball in

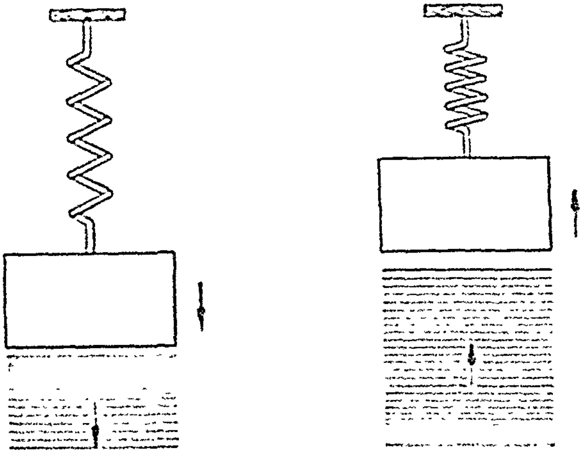


FIG. 163.—As the block moves downward in air it compresses the air beneath it. As it bounces upward it leaves a rarefied region. These regions of compression and rarefaction travel away from the point of origin and constitute sound waves.

motion. These molecules bump into those ahead of them, imparting their motion to this second group of molecules. These in turn bombard more molecules in front of them and so on, so that even at a considerable distance away from the block we might expect any object in the air to be bombarded by the air molecules as a result of the downward motion of the block.

When the block starts up it leaves a partially evacuated place beneath it and air molecules rush in to fill this vacuum. However, this action produces a partial vacuum in the region from which the first molecules moved and hence molecules from further away move into this region. So this disturbance is propagated until at the location of the object mentioned above where there was first a bombardment of molecules we now find the same molecules jumping back to fill the partially evacuated regions.

The result on the object is an increase of air pressure when the air molecules bombard it and a reduction in pressure when they start back. So it would be within the realm of imagination to suppose that the object could be arranged to move

back and forth as a result of these pressure changes and if it were properly connected it might do useful work. The amount of energy that can be readily sent through air per second in this manner is small, but it does represent another example of sending energy through a medium by means of a wave motion.

There is one striking difference between the case of air waves and those in water or on a stretched rope. The air molecules move back and forth along the same direction as that in which the wave motion is moving, as distinguished from the case of the particles moving at right angles to the direction of travel. Waves of the air type are called *longitudinal.* They are also called *compressional* waves.

As in the case of water waves, the student should notice that it is the motion of the air molecules not the air molecules themselves that is transmitted from the vibrating object to some distant point. The molecules that bombard the distant object are the same ones that were originally in the vicinity of that object.

In spite of the fact that the amount of power transferred by air waves is small, it happens to be one of the more important physical occurrences in everyday life. The human ear has a membrane stretched in it in such a manner that it can move to and fro when air molecules bombard it and then bounce away. A sensation results which we call sound. The human ear is not very sensitive to pulses unless they come in close sequence. So nothing would be heard from the waves produced by the bouncing block described above.

However, the prongs of a tuning fork move to and fro much more rapidly and produce waves that give a better response in the human ear. The ear interprets a succession of waves as a continuous tone if there are as many as approximately 20 per second. If the number per second is increased the sensitivity of the ear improves up to about 1000 waves per second. Beyond this frequency the sensitivity decreases until at about 20,000 waves per second the average person does not hear the sound at all.

The transmitter of an ordinary telephone also uses a stretched diaphragm. Here sound energy is used to control electrical energy, somewhat as sound energy is transferred into physiological effects in the case of hearing.

4. Characteristics of Medium

The student should notice that the presence of a medium is required for the transmitting of energy by wave motions. In other words, there must be something to do the waving. Notice that this medium must be elastic (as in the case of the molecules of air rebounding from one another or from any object which they strike) and notice also that the medium must have mass. The air molecules could not gain momentum on impact and move on to give up this momentum to other molecules unless they had mass.

These necessary qualities of the medium, mass and elasticity, are the same two that we found essential in the vibrating object itself.

That sound waves cannot be transmitted without a material medium may be shown by suspending an electric bell inside a large glass jar from which the air may be exhausted. The bell is set ringing and then the air pump is started. The sound of the bell grows fainter and fainter as the air is removed from the jar.

5. Velocity of Travel of Wave Motions

In the experiment with ropes described above we saw that the rate at which waves moved along the rope depended on the massiveness of the rope and on the tension in the rope. In the case of waves in a gas, such as air, the velocity of travel depends on the massiveness of the molecules (and hence on the density of a gas) and on the pressure, for the elastic properties of a gas may be measured by its pressure.

In air at normal pressure and temperature (76 cm. mercury and $T = 0°$ C.) the velocity of propagation is 1087.5 ft. per sec. At normal pressure, but at 20° C., the velocity is 1127 ft. per sec. For many problems it is sufficiently accurate to take the velocity of sound as 1100 ft. per sec.

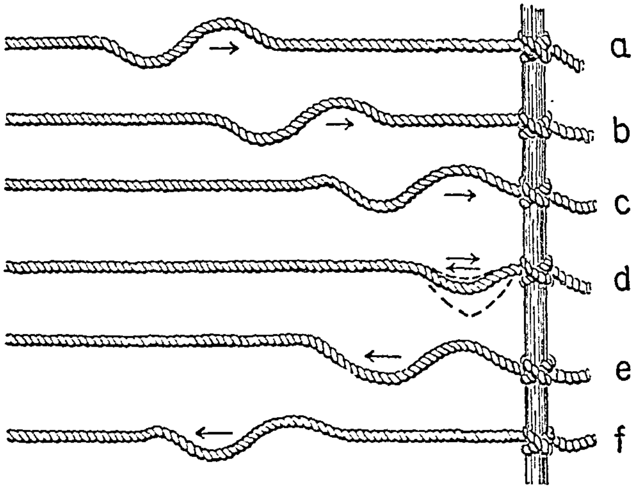


Fig. 164.—Successive pictures of a single complete wave on a rope showing how the first section of the wave, after reflection, aids the incoming section in moving the rope to one side.

In an atmosphere of hydrogen one would find the velocity of propagation to be considerably greater, because the density of hydrogen is much less than that of air under equal conditions of pressure and temperature, whereas the elasticity is the same.

The velocity of propagation is evidently fixed by the medium, and the frequency of vibration is fixed by the vibrating source. So it is easy to compute the wave length, which is the distance between corresponding points on successive waves. For example, if 1,000 waves are emitted in one second, the first wave will have progressed a distance of 1127 ft. in air at normal pressure and 20° C. by the time the one thousandth wave is sent out. Obviously the distance from one wave to another will be this distance, 1127 ft., divided by the number of waves filling the space; that is, the wave length, L, is given by

$$L = \frac{1127}{1000} = 1.127 \text{ ft.}$$

It is evident that the frequency, f, multiplied by the wave

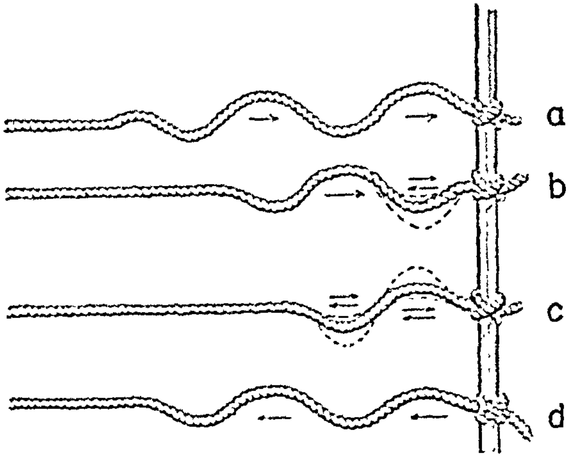


FIG. 165.—Successive pictures similar to those in Fig. 164 but with two whole waves on the rope.

length, L, is equal to the velocity, V. In symbols this may be given thus

$$fL = V$$

This equation is very useful in the study of wave motions. It should be both understood and memorized.

6. Standing Waves on Strings

In Section 2 above we mentioned that with the experiment of sending waves along a rope tied at one end, some difficulty in watching the original train of waves is experienced due to the reflected waves that come back over the rope. If a single kink can be sent down the rope, we may picture the kink in successive positions as shown in Figure 162. The experiment can easily be performed with a rope or better still with a piece of rubber hose about 12 ft. long. You should notice in (c) and (d) that the wave reverses itself at the fastened end of the rope.

Figure 164 shows one full wave instead of a single kink. When a full wave is sent down the rope, the first section of the wave will be found on reflection to be aiding the second section of the incoming wave in moving the rope to one side. In (c)

the first section of the wave is just reaching the wall. It is reversed on reflection and starts back over the rope just as the second section of the wave arrives. This state of affairs is shown in (*d*) where the returning reflected wave is shown by the dotted line. Notice that both the incoming second section of the wave and the reflected first section both tend to move the rope downward. Figures (*e*) and (*f*) show later stages as the full wave travels back over the rope.

Suppose now that two whole waves are sent along the rope as indicated in Figure 165. Part (*b*) shows the state of affairs when the first section of the first wave has been reflected. It aids the second section in moving the rope to one side just as in Figure 164(*d*). Part (*c*) shows the condition when a full wave has been reflected. Reflected section 1 of the first wave is aiding incoming section 4 (2d part of 2d wave) to move the rope down. Reflected section 2 of the first wave is aiding incoming section 3 (1st part of 2d wave) to move the rope up. Part (*d*) shows a later view of the travel of these two waves.

In performing this experiment it is found difficult to send a single full wave or just two full waves along the rope. On the other hand, by shaking the end of the rope it is easy to put a whole series of waves on the rope. In this case there will be a train of waves continually going in each direction over the rope.

At one instant the rope may present the appearance of the dashed line in Figure 166(*a*) and an instant later it may appear as in Figure 166(*b*). At the points marked by the vertical dotted lines incoming and reflected waves always oppose each other. The result is that here the rope stands still. At the places between these points, the incoming and the reflected waves unite to make the rope move from side to side. The regions of motion are called loops and the still spots on the rope are called nodes. The whole effect is known as standing waves.

Experiment with a rope or hose will show that the standing waves can be well produced only when the end is moving at certain frequencies. It will be found that these correspond to wave lengths on the rope of such size that a whole number

of half wave lengths make up the length of the rope. The distance between adjacent nodes is one-half of the wave length on the string as may be seen from a study of the figures in this section.

If the rope is moved up and down quite slowly, it will be found possible to get a single half wave on it. The frequency

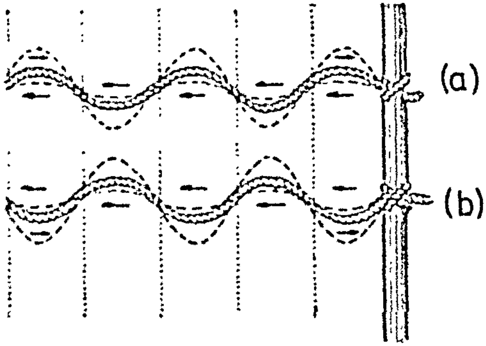


Fig. 166.—Behavior of a stretched rope when waves are sent along it continuously. The rope appears to stand still in the positions indicated by the dotted red lines. Between these lines the rope swings to and fro between the extreme positions shown in (a) and (b).

of this up and down motion is called the fundamental for the rope of that particular length and mass and tension. The fundamental is the lowest frequency at which the standing wave effect can be produced. It is also the lowest frequency at which the rope will tend to keep moving on its own account. In other words, the rope may be thought of as a vibrating body, with mass and elasticity, and hence with a natural period of vibration as well as thinking of it as a medium along which waves may be transmitted.

If the frequency of moving the rope is doubled, two half waves will be found on it. The motion is called the second harmonic of the vibration. A frequency of three times the fundamental frequency will result in three half waves on the rope. This motion is called the third harmonic.

The violin, harp, banjo, piano, etc., are examples of the

practical application of waves on strings to the construction of musical instruments. Friction between the bow and the strings of the violin, plucking the strings of the harp or the banjo, striking the strings of the piano, starts them in vibration. They usually vibrate not only at the fundamental frequency but also at some of their harmonic frequencies at the same time. Different relative intensities of the harmonics give quality or timbre to the sound of the tones and enables us to tell what type of instrument is producing the vibrations.

7. Standing Waves in Air

A convenient source of sound waves in air is a tuning fork. The prongs of the fork strike the molecules of air and drive them forward until they meet other molecules and rebound. On the side of the prong from which it is moving a partial vacuum is created and molecules rush in to fill this region. This process was described in detail in Section 3 of this chapter.

If these air waves strike a hard flat wall, they will rebound and start back in somewhat the same fashion as waves on strings reverse and return from the fastened end. But in the case of the waves on strings, they were confined to the string; while in the case of air waves in the open the energy of the waves spreads out indefinitely into the surrounding atmosphere. So the reflected waves from a flat wall in the open will not be as strong as the incoming waves, for they have spread out to cover more space.

8. Standing Waves in a Closed Tube

This state of affairs may be remedied by starting waves in air into a tube which is closed at one end. The waves on reflection at the closed end can only return inside the tube and so the reflected train may be almost as strong as the original train. These two trains of sound waves moving in opposite directions may interfere with each other in much the same fashion as the trains of waves on a string were seen to interfere. The result will be standing waves in air.

At the closed end of the tube the average velocity of the

molecules will always be zero because the end wall stops them completely and the rebounding velocity is just equal to the velocity with which they strike. The closed end of the pipe must be the location of a "velocity" node, the same as was the tied end of the string.

At the open end of the pipe the molecules will have no difficulty moving in or out, and so a standing wave will occur in the air column when the pipe is of such length that there is

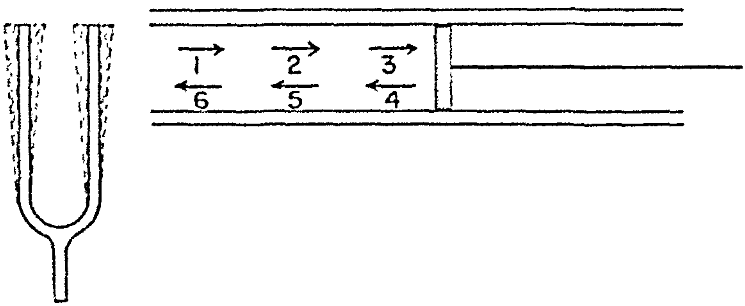


FIG. 167.—When the prong of the tuning fork moves toward the tube, molecules of air are given a motion as indicated by (1). This forward motion is imparted to other molecules until those at (3) strike the end of the tube and rebound. If the length of the tube is just right, the return motion reaches (6) just as the prong of the fork springs away.

a velocity node at the closed end and a velocity loop at the open end of the pipe. This is a case of a standing wave with only a quarter of a wave present, as compared with a minimum of a half wave on a string.

It is easy to perform an experiment with a tuning fork and a pipe to show this effect. The fork and pipe may be arranged as shown in Figure 167. When the prong of the fork moves toward the tube the molecules near the mouth are given a motion into the tube. These molecules hit others and the motion of molecules in the tube might first be represented by (1) and a little later by (2) and (3). These molecules rebound from the end as indicated by (4) and soon those near the open end of the tube move out as shown at (6). If the molecules at (6) move out just as the prong of the fork springs away from the tube and the molecules start moving in the same

direction to fill the partial vacuum left by the prong, the motion at (6) will be greater than it would be due to either effect alone. To get this effect timed right, it is necessary to have the correct length of pipe for a tuning fork of given frequency.

If a long tube with a movable plunger for an end is used, one may move this plunger to and fro until the best length is found. This position is indicated by the fact that the sound of the tuning fork appears to be amplified by the standing wave action in the tube.

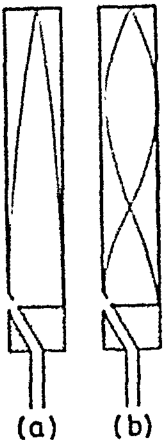
This phenomenon may be looked on as an example of resonance; for the column of air has a natural frequency of its own depending on the length of tube, and the tuning fork applies energy at a definite frequency.

If one blows across the open end of the tube he can cause the column of air to vibrate at its natural frequency. This simple experiment becomes the basis of construction for many musical instruments including the flute, piccolo, and some pipes of a pipe organ.

In performing the experiment with the tube and plunger one finds that resonance for a given frequency can be obtained for more than one length of the pipe. In fact resonance will be found for any length of pipe that meets the condition that a velocity node exists at the closed end and a loop at the open end. Figure 168 (*b*) shows that a pipe closed at one end is resonant to a sound wave when the tube is three-fourths the length of the sound wave. The student may make diagrams to show that this condition is met not only for one-fourth of a wave in the tube and for three-fourths but also for five-fourths, seven-fourths, etc.

9. Standing Waves in an Open Tube

The experiment described above may be repeated for the case of a pipe with both ends open. The condition that motion of molecules at the open end of a pipe is to be expected tells us that we must have a velocity loop at each end of the open pipe. In the simplest case this can be true if there is a node in the middle of the pipe. See Figure 169 (*a*).



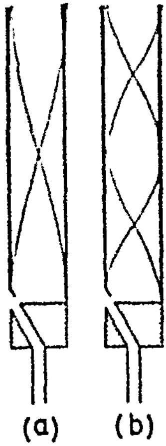


Fig. 168.—(*Left*) When resonance occurs in a closed pipe, a velocity node is at the closed end and a velocity loop at the open end. In (a) fundamental resonance is shown; in (b) the first overtone—third harmonic.

Fig. 169.—(*Right*) Open pipes produce standing waves and therefore resonance when there is a velocity loop at both ends. In (a) fundamental resonance is diagramed, so as to show that the pipe length is a half wave length. In (b) the first overtone—second harmonic—is diagramed, so as to show that the pipe length is two halves or a whole wave length.

Here we see that this condition gives one-half of a full wave as the simplest standing wave condition in a pipe open at both ends. Figure 169 (*b*) shows that the end conditions for the open pipe can also be met with one full wave in the pipe. The student may make further diagrams to show that resonance will also be obtained when the pipe is one and one-half, two, two and one-half waves, etc., times the sound wave. These experiments may easily be tried out in the laboratory.

10. Air Columns as Sound Producers

Air columns may be used as sources of sound either by simply blowing air over an open end as suggested in Section 8, or by means of something like a reed as in the case of a saxophone. Usually the air column vibrates in some of its harmonic frequencies as well as on the fundamental. This condition gives quality to the sound of the air waves and enables one to distinguish between the tones of various air column instruments.

Some Important Facts

1. Wave motion is a means of transmitting energy through a vibrating medium.

2. In transverse waves, the particles of the medium oscillate back and forth across the line of travel of the wave motion. Waves on the surface of water and vibrating strings are examples.

3. In longitudinal waves, the particles of the medium oscillate back and forth in the same general direction along which the wave motion is proceeding. Such waves consist of alternate high and low pressure regions or alternate condensations and rarefactions of the medium. Sound waves in air are good examples.

4. By wave length is meant the distance between similar parts, or phases, of adjacent waves.

The velocity of any wave motion equals the product of the frequency of the waves and the wave length.

5. When a wave train is reflected on itself so that the incoming and *reflected waves result in a fixed vibration pattern in the* medium, that pattern is called standing waves. Regions of minimum vibration are called nodes and regions of maximum vibration are called loops.

6. When a string fastened at both ends vibrates with a node at each end and a loop in the middle—a single half standing wave—it is vibrating at its fundamental frequency. This frequency is the first harmonic.

When the string vibrates with a full standing wave pattern it is producing its first overtone. The frequency is twice that of the fundamental and is known as the second harmonic. Three half waves on a string fastened at both ends produce the second overtone. The frequency is three times that of the fundamental and the tone is called the third harmonic of the fundamental.

7. *a.* Fundamental resonance is produced in closed air columns whose length is one-fourth of a fundamental wave length. Such air columns can also vibrate at a frequency that places three-fourths of a wave in the column, or five-fourths, etc.

b. Fundamental resonance is produced in open air columns whose length is one-half of a fundamental wave length. Overtones are produced when the air column is any whole number of halves of a fundamental wave length.

Generalization

Any substance possessing mass and elasticity may be used as a medium through which energy may be transmitted by means of wave motion.

Questions and Problems

Group A

1. In what way does the transmission of energy by wave motion differ from transmission by other methods?

Experimental Problems

1. Measure the air line distance from a locomotive whistling to some convenient observation point at least a half mile distant.

By means of a stop watch take the time interval between seeing the steam issue from the whistle and hearing the resulting sound, and consider this as the time the sound traveled to reach your ear. From this data calculate the speed of sound in air and compare this value with the accepted one for the atmospheric temperature at which the experiment was conducted. What are the principal sources of probable error?

2. Carefully determine the time interval between an original sound, such as a sharp whistle, and the return of its echo from a distant cliff.

Take the temperature of the air, and calculate the speed of sound at that temperature.

From the above data calculate the distance to the cliff and check by actually measuring the distance.

3. On a sonometer—or any strong 6-ft. board—stretch two steel wires of equal length, one having twice the diameter of the other. Attach one end of each wire firmly and pass the other end of each wire over a pulley so that weights may be hung from it. The wires should be about 3 in. from the board, so that they may vibrate clear of it.

The total weight hung on each wire may reasonably be varied from 4 to 36 pounds.

The effective vibrating length of each string may be varied by sliding the edge of a ruler along it as the string is plucked. To facilitate the accurate measurement of the vibrating length, mark off the board like a meter or yard stick.

Using both strings at full length, vary their tensions together and note the pitches. Then vary their lengths together, keeping both at the same tension, and compare their pitches. What qualitative conclusions are justified?

In order to reach a quantitative conclusion repeat the above procedure using only pitches having the major chord interval, possibly only those an octave apart. If you have a fairly good ear for tuning, the quantitative results will be quite satisfactory.

4. Use a glass tube, one or two inches in diameter, about two feet long and open at both ends as a container of the resonant air column. To close one end of the tube, immerse it in water, which should be as deep as the tube is long. Hold the tube and a meter stick vertically in the water. Then sound a tuning fork and hold it over the open end of the tube so that the flat side of the prongs are in a horizontal position and as close to the edge of the tube as possible without touching it. Slide the whole arrangement—tube, meter stick and vibrating fork—up and down until maximum resonance is heard. Then read the length of the air

column. Continue this process to see if a length of secondary resonance may be found.

Repeat with one or more differently pitched forks.

From the length of the resonant air column determine the wave length of the frequency resonated and determine the speed of sound for the conditions of the experiment—repeating for each fork used. How do your several determinations of sound compare? How does an average of these values compare with the accepted values for the room temperature at the time of the experiment?

Chapter 23

MUSIC, SPEECH, AND HEARING

The chapter opens with a discussion of the response of the ear to sounds. The sensitivity of the ear is seen to vary with the loudness of the sounds and with their frequencies.

Sound resulting from waves that come in long trains of identical nature is interpreted as tones, and for the most part is pleasing to the ear. Sounds consisting of miscellaneous assortments of waves that are not repeated are usually disagreeable and are called noise.

Examination of sound waves has been greatly aided by the development of microphones and recording instruments called oscillographs which enable one to take pictures that represent the sound wave.

Harmonic frequencies of sound waves add quality to the sound effect, and in general make it more pleasing. Similarly, playing various tones together may give a pleasing effect provided the tones to be played at one time have relations among their frequencies properly chosen. Otherwise the effect is said to be a discord.

The combination of tones that are pleasing is a matter of arbitrary selection on the part of any group of people. From these choices by the people in our civilization an accepted sequence of tones called a musical scale has been developed.

The human voice is a type of musical instrument employing the vocal chords as reeds to start tones, and using the various cavities of the head and chest to give resonance to the tones and overtones. By moving the jaws, lips, tongue, and soft palate these resonance regions can be changed. In this manner the tones can be modulated into the various vowel and consonant sounds required to produce intelligible speech.

The chapter closes with a brief description of two special items; first, the fact that two tones of different pitch played simultaneously give rise to a beat note; and second, that the apparent pitch of a source of sound changes if the source is moving toward or away from the listener.

1. Sensitivity of the Ear

We have already seen that a succession of waves in the air produces a sensation called sound when the waves strike the ear. Sometimes the waves themselves are called sound; so that sound may be looked on either as the physical energy

itself or as the sensation experienced by a person or animal. We say, for example, that a piano produces sound, and we may talk about the sound energy produced per second by a large band or by a radio loudspeaker.

The human ear is a very sensitive instrument—giving a response to amounts of power that are very small in comparison to power as we ordinarily think of it. However, the ear's sensitivity varies greatly both with the actual loudness of a sound and with its frequency. For example, if two identical whistles are sounded so that one actually produces twice as much energy per second as the other, it will sound only a little louder to the human ear.

If two whistles are arranged to produce the same actual volume, but one at a very low frequency and the other at a *medium high frequency*—say 100 cycles per second for the low one and 1000 cycles per second for the high one—the high one will sound very much louder to a person than the low frequency whistle. In fact, if the volume of the two whistles is uniformly reduced, a point will be reached where the lower frequency whistle is no longer heard at all while the high frequency whistle is still clearly audible.

If quite high frequencies are tried—say 10,000 cycles per second or more, the sensitivity of the ear will also be found to be low in comparison to that for tones of medium high frequency.

If a tone is made very intense, a person gets a sensation of pain in addition to hearing the sound. At very low and at very high frequencies there is not much variation in loudness between the point where we begin to hear and the point where we get a sensation of pain. In the medium frequency region, wide variations of loudness are possible between the minimum intensity to hear and the amount that is painful.

Figure 170 is a rough graph of sound intensities at various frequencies plotted to show the minimum amount of sound that can be heard by the average person and also the amount required to produce a pain sensation.

Very loud sounds seem to affect the nervous system in such

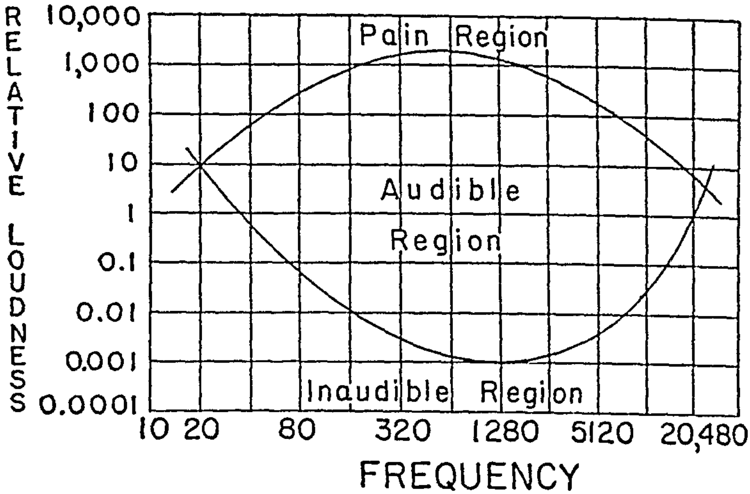


FIG. 170.—Sensitivity of the ear. The lower curve represents the loudness necessary to make tones just audible at various frequencies. The upper curve shows the loudness at which pain sensations begin.

a manner as to produce digestive upsets similar to seasickness. Headaches and general loss of vitality may result from prolonged exposure to extremely loud sounds.

The term *pitch* is used to describe the characteristic of a tone that depends chiefly on its frequency. A tone is said to have a high or a low pitch depending on whether or not the frequency is high or low.

2. Tone and Noise

Some emphasis has been put on the fact that the ear interprets a *sequence* of waves as a continuous sound and not as a series of pulses, providing the number reaching the ear per second is greater than about 20. If the waves are simple and of the same wavelength, such as would be sent out by a tuning fork, the ear interprets the sound as a pure tone of definite pitch. The sensation of pitch is directly associated with the frequency of the vibrating device as described above. Faster rates of vibration give a sensation of higher pitch and vice versa. For example, the note of highest pitch on the average piano has a natural frequency of about 4180 and the lowest note a frequency of about 27 cycles per second.

The sensation of definite pitch is somewhat lost if the sound waves die out rapidly so that only a few reach the ear. However some effect is made on the ear so that there is a sense of sound although the idea of definite pitch is lost. The sensation is now interpreted by the ear as *noise.*

Noise is the sensation produced on the ear by sound waves which do not exist in long enough trains of waves to produce a sensation of tone. Of course, in practically all cases of noise, sound waves of a great many different frequencies are started. Some start as others die out. None of the individual trains of waves is very long and there is no simple relation between the frequency of one short train of waves and another in a noise.

Sometimes noise is defined in a more general way as being any kind of sound that is disagreeable to a person. A pure tone of sufficient loudness is often classed as noise. In other cases this general definition of noise may be hard to apply. For example, the splashing of waves might be a very agreeable sound to a romantic couple walking on a beach on a moonlight night, but it might be most disagreeable to shipwrecked sailors whose boat had just grounded on a rocky reef. Hence the somewhat more prosaic distinction between tone and noise given above is more workable.

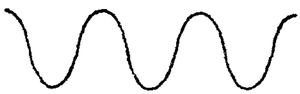
Many noises consist largely of high frequency sounds. This is true for the case of crumpling a piece of paper, for jingling keys, for rubbing together two pieces of sandpaper. When a book is dropped on a table there is a mixture of low frequency and high frequency waves all of which die out quickly. The continuous noise from the average automobile engine is also a mixture of low and high frequencies. Sometimes a part of some continuous noise such as this may be almost in the nature of a pure tone mixed with miscellaneous frequencies.

3. Recording Sound Waves

A small part of the energy of sound waves may be converted into electrical energy with the aid of a microphone such

as is used for ordinary telephone purposes or with a better one such as is used for radio broadcasting. Electrical currents are created in this manner such that they move to and fro exactly in the same manner as the vibratory motion of the air molecules in the original sound wave.

With the proper electrical equipment these tiny currents can be made to control the motion of a very small mirror which is used to reflect a beam of light. The mirror turns in such a



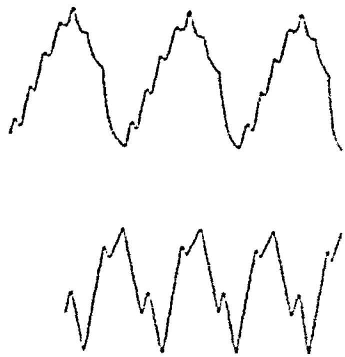


FIG. 171.—(*Top, left*) Oscillogram of a pure tone.

FIG. 172.—(*Right*) Oscillograms of a tone from a violin (*top*) and from a flute (*bottom*).

FIG. 173.—(*Bottom, left*) Oscillogram of noise from an automobile engine with no muffler.

manner that the reflected beam of light moves to and fro. If the light falls on a strip of photographic film which is moving, it will trace a wavy path. If the sound is a pure tone, the film will show a trace such as is indicated in Figure 171. If the tone has some harmonics with it, as would be the case with musical instruments, it might look like Figure 172 for a violin tone and for a flute tone. Figure 173 is a noise picture obtained from an automobile engine with no muffler attached.

Such pictures are called oscillograms. This method of recording sound waves has proved very useful in the study of noise, in the study of musical instruments, and in the study of voice sounds both for talking and singing.

4. Harmonics—Overtones—Quality

A succession of exactly similar waves containing no frequencies except the fundamental gives rise to a pure tone. The presence of harmonics (multiples of the fundamental frequency such as twice, three times, four times, etc.) changes the quality or timbre of the tone. If one of the harmonics is louder than the fundamental, it may predominate, and the fundamental may not be distinguished by the listener.

Fundamental pitch of frequency f

First overtone of frequency 2 f

Third overtone of frequency 4 f

Tuning forks come closer to giving pure tones than most other types of musical instruments. A flute when played softly may give almost pure tones, but when it is played loudly the harmonics become very noticeable.

the fundamental. On the other hand overtones are named in the order of their production in a musical instrument.

The first overtone is the first higher frequency above the fundamental that is possible in the particular instrument being discussed. For the case of a closed organ pipe the first overtone has a frequency three times that of the fundamental. However, for an open organ pipe, the first overtone has a frequency of only twice the fundamental.

5. Pleasing Combinations of Tones

From the fact that small amounts of harmonics such as the second, third and higher seem to add to the pleasing quality of a tone, one might jump to the conclusion that frequencies related to each other as two to three or three to four, etc., might be pleasing even though the fundamental is entirely absent. Experience shows that such tone combinations are pleasing so long as the frequencies are related to one another by small numbers. For example, two tones with frequencies of 200 and 300 cycles per second would give a pleasing combination as would also tones with frequencies of 300 and 400, and 400 and 500. When the frequencies can be related to each other only by larger numbers, say 17 and 18 as would be the case with notes of frequency 170 and 180, the effect is displeasing and is said to be "inharmonious." The effect is called a *discord*. There is no sharp dividing line between pleasing and displeasing combinations, since the effect is one of personal interpretation.

A point to notice is that the sound effect of a combination of tones is characteristic of that combination no matter what the actual frequencies may be. For example, a person with only a small amount of musical training would recognize the three-four relation with tones of frequencies of 150 and 200 just as readily as with 300 and 400, or 600 and 800.

6. Musical Scales

The relative frequencies of the notes in a musical scale are determined from an experimental knowledge of what is pleas-

ing to the human ear, after the manner of the discussion in the above section.

A sequence of four tones having the particular frequency relations of 4, 5, 6, and 8 is called the *major chord.* For example, four notes having frequencies of 400, 500, 600, and 800, played together would produce a very pleasing combination of tones. The same characteristic sound, but at a lower pitch, would be obtained with frequencies of 256, 320, 384, and 512, or 200, 250, 300, and 400, etc.

In all of these examples the last note in the group has a frequency twice that of the first. This higher note is said to be the *octave* of the lower. The *frequency range* between the lowest and highest notes where the high one has twice the frequency of the lower one is also called an octave.

Numerous other frequency combinations are in common use in music. One using ratios of 10, 12, 15 and 20 is perhaps the next best known to that just described. This latter combination is called a *minor chord.* The general impression is a rather pensive sensation as compared to a grand or joyous effect as produced by the major chord.

To produce a musical instrument capable of having notes with all the frequencies that might be desired for music is an almost impossible task and the accepted scales on modern musical instruments represent a compromise. Each note is adjusted to have a definite percentage increase in frequency of vibration as compared to the preceding note. This percentage is chosen so that the twelfth note from any position will have just twice the frequency of the one at the starting point.

For convenience we may examine a small section of piano keyboard. (See Figure 175.) The keys of the piano are connected to felt covered hammers in such a manner that a hammer hits a stretched string when the key is struck. Although some of the keys are black and inset with respect to the white lettered ones in the diagram, consecutive keys play strings that are successively tuned as described above. Each suc-

cessive string has a frequency that is $\sqrt[12]{2}$ (1.0595 approx.) times that of the preceding one.

A scale employing notes tuned in this sequence is called a *tempered scale* as distinguished from one having the precise frequencies called for by an application of the rules for combinations given for the major, minor, or other chords. Scales having the precisely correct frequencies for these chords are called *true scales* as distinguished from the tempered scale.

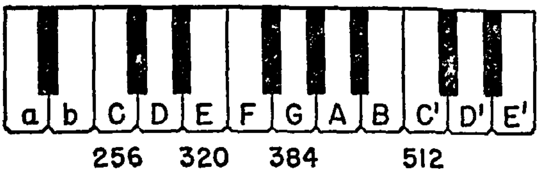


FIG. 175.—Section of a piano keyboard.

The tempered scale is used on such instruments as pianos, fretted stringed instruments (such as guitars), keyed instruments such as cornets, flutes, etc. True scales can be played on instruments such as the violin where the exact frequency played is at the will of the performer.

7. Speech and Singing

Vocal tones are produced something after the manner of tones from a wind column instrument with a reed to start the vibrations. The vocal chords function as reeds and the tension under which they can be held muscularly permits the person to vary the fundamental pitch of the sounds. The cavities of the mouth and head provide resonance for the fundamental frequency and for many harmonics. Varying the extent to which the mouth is opened, the position of the tongue, the soft palate, and the lips, enables the person to change the frequencies to which the various cavities resonate.

Most of the energy of a spoken or sung word is contained in the vowels. The consonants are simply devices for starting or shutting off the flow of the vowel tones. Closing the lips while blowing through them gives rise to the sounds charac-

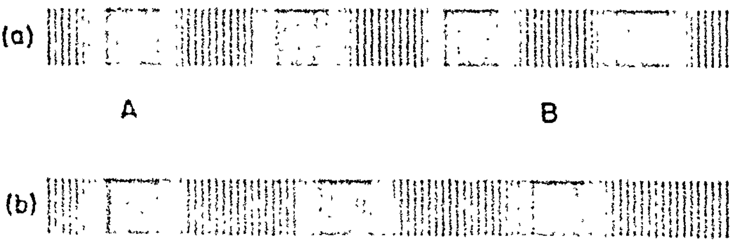
teristic of *m*, *p*, or *b*, depending on the exact manner in which it is done. Placing the tongue against the roof of the mouth may result in the sounds of *d*, *n*, *s*, or *t*. In similar fashion, production of the other consonant sounds may be accounted for. Much of the intelligibility of speech depends on clearly enunciating the consonants.

Any vowel may be spoken or sung on any fundamental pitch within the easy range of the average set of vocal chords. By changing the shape of the mouth cavities as described above, resonance to different high frequencies (which are harmonics of the fundamental) are given for the different vowels. In this manner the quality of sound produced varies enough from one vowel to another to enable the ear to distinguish among them.

8. Beat Notes

Figure 176 shows the compressions and rarefactions of two trains of sound waves traveling from left to right. The drawing shows that the wave represented at the top has the shorter waves and that at the bottom the longer ones. At position *A* we see a condensation in each wave train. At position *B* there is a rarefaction in the upper wave and a compression in the lower wave.

If these two trains of waves actually travel through the same space instead of being side by side as shown in this figure, we can see that at *A* the two waves reinforce one another while at *B* they tend to cancel each other.



The whole action of the two trains of waves on one another is called *interference*. At places where the waves tend to neutralize one another the interference is said to be destructive. These regions of destructive interference and of reinforcement move along through the atmosphere so that at one instant relatively strong sound waves appear to strike a listening ear. A moment later the destructive interference region has arrived at the ear, and the intensity of the combined tones fades. So the ear appears to hear a rise and fall in intensity of a single tone. These repeated rises and falls of intensity are called *beat notes*. The number of beat notes heard per second is found by experiment to be equal to the difference in frequency of the two original notes.

For example, if two tuning forks of 320 and 322 cycles per second respectively are struck, a single tone will seem to rise and fall twice each second.

This fact furnishes a means for comparing the frequency of an unknown tuning fork, for example, with that of a standard. The experiment may easily be carried out in any laboratory. The number of beats per second tells the departure from the frequency of the standard fork, but does not tell whether the unknown has a frequency that is too high or too low. A little wax may be stuck on a prong of the unknown fork. This will give it more mass and make it vibrate more slowly. From the frequency of the new beat note, the student may now find out whether the original frequency of the unknown fork was greater or less than that of the standard.

If the two notes being compared differ from one another by an amount of the order of ten or twelve cycles per second, the beat frequency is so rapid that it is hard to count. Also the general effect is distinctly disagreeable, since the tone simply seems to be interrupted in a choppy manner.

If the two notes differ from one another by a large number of cycles per second, each note is heard and the beat frequency itself also sounds to the ear like a musical tone. Hence two notes sounded at one time may sound like three notes. This fact is used in the tuning of stringed instruments, especially

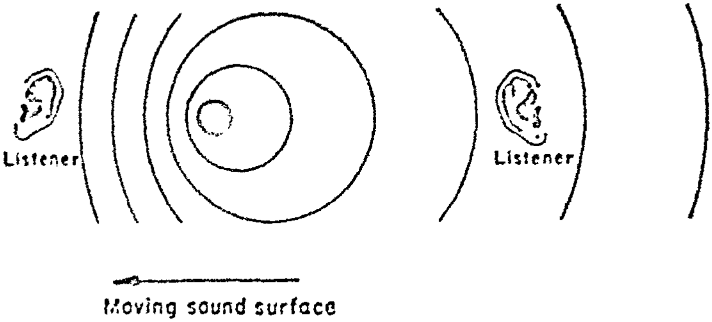


FIG. 177.—A moving sound source moves toward its waves on one side and away from those on the other side. In the first direction the waves are shorter, and in the opposite direction longer, than normal.

violins. For example, if the "D" string is tuned to a frequency of 288, the "G" string should be tuned to 192. The beat note has a frequency of 288 − 192 = 96 cycles per second. This beat frequency is just half the frequency of the "G" string, and is therefore in harmony with it. The whole combination of three tones is readily recognized by the experienced violinist.

9. Changes in Pitch Due to Motion of the Sound Source

A common experience is to have the sound from the horn on a passing automobile change from a moderately high pitch as the car approaches to a relatively low one as the car moves away. This effect is named the "Doppler" effect after the man who observed a similar change in light waves.

The actual velocity of sound in air is not affected by motion of the source of the sound. On the other hand, the distance between waves is altered. See Figure 177. As the source of sound approaches the listener the successive waves are closer together than if the source were standing still. If the source is moving away from the listener, extra distance is added between successive waves. So far as the listener is concerned the relation between frequency, wave length, and velocity of propagation still holds; that is,

$$\text{Frequency} \times \text{Wave length} = \text{Velocity}$$
$$FL = V$$

If the velocity V stays the same, then when L is decreased the frequency F will have to increase, and vice-versa. This change in frequency is easily observed by any listener if the source of sound is moving at a moderately high rate of speed.

Some Important Facts

1. The human ear interprets as tones those waves whose frequencies lie between approximately 20 and 20,000 cycles per second. The sensitivity of the human ear is greatest in the middle tone region (about 300 to 1200 cycles per second) and decreases towards both the low and the high frequency ends.

2. Musical sounds have a definite uniform frequency, as distinct from noises which have irregular or variable frequencies. Musical tones may or may not be pleasing to any individual ear.

3. Musical tones may have three characteristics; pitch, loudness, and quality.

Pitch depends on frequency, the greater the frequency the higher the pitch. Loudness depends on energy per second of the sound. Quality depends on the number and relative prominence of harmonics blended with the fundamental tone.

4. Harmony is the simultaneous sounding of pitch intervals generally found pleasing. The opposite of harmony is discord.

5. Musical scales were developed prior to the physics of sound and so contained many odd and inconvenient pitch intervals. Modern even-tempered scales attempt to iron out the worst of these irregularities. For example, each piano string has a frequency $\sqrt[12]{2}$ or 1.0595 times the next lower one.

6. The human voice is a combined stringed and wind instrument. The energy of a voice tone is mainly in the vowel sounds; consonants serve to vary the method of starting and shutting off vowel tones.

7. When two tones of different pitch are sounded together, they reinforce each other as many times per second as the difference of their frequencies. These rhythmic recurrences of reinforcement are called beats. If more than about 20 beats occur per second, the effect on the ear is that of a new tone with frequency equal to the beat.

8. If a source of sound and an ear hearing the sound approach each other rapidly, the effect is to shorten the wave length and hence raise the apparent pitch. If the sound source and ear are receding from each other, the effect is to lower the apparent pitch.

low frequencies with that at medium frequencies. What limits the experiment at the low and the high frequency regions with the apparatus?

2. Compare tones of the same fundamental frequency from various musical instruments such as the flute, clarinet, violin, trumpet.

3. Compare combinations of tones by playing on a piano a major third (e.g. C and E), a minor third (e.g. C and E♭), a major chord (e.g. C E G C′), a minor chord (e.g. C E♭ G C′). Try also two notes close in frequency such as C D and C D♭.

4. Compare the frequency of an unknown tuning fork with that of a standard tuning fork by the method suggested in Section 8.

5. Estimate the pitch and the change in pitch of the whistle of a passing locomotive. From this data calculate the speed of the locomotive and decide if your answer is reasonable.

Chapter 24

SOME PROBLEMS IN SOUND

In about 1920 sound was generally considered an interesting but old subject in physics. Not much new was expected of it. Then the needs of long-distance telephony, trans-atlantic telephony, the newborn art of radio broadcasting, attempts to perfect talking moving pictures, all gave impetus to a new order of research in this old field. The public became interested. The tone quality of the simplest radio sets seemed better than that of the better record-playing machines. The acoustics of one auditorium seemed to differ greatly from those of another. The effects of noise were studied and made public. Anti-noise campaigns were organized. Architects employed physicists to aid them in designing new auditoriums. New and improved record-playing equipment was developed.

An old subject has come to life and has been playing an important role in our every day lives since the early 1920's.

This chapter discusses a few of these sound problems that in one way or another touch on our welfare and happiness.

Out-of-doors in the open, this difficulty cannot occur, for the sound travels away instead of being reflected back. On the other hand, speakers do not like talking out-of-doors, because, since the sound does not come back, they do not hear themselves well and feel that they are not talking loud enough. Also the sound goes out in all directions from the person and so is not as loud in front of him as would be the case if he were talking in a room with a wall back of him.

A fairly ideal arrangement would consist of a solid wall behind the speaker and extending for a short distance on each side of him and over him. These walls would direct most of the energy towards an audience in front of the speaker and would give the speaker a feeling of support. With open space on the sides, rear, and overhead in the place where the listeners are located no disturbing echoes would come back to them.

Conditions approaching this arrangement can be made indoors by using smooth flat walls in the part of the room where the speaker is to be located, and by placing absorbing material on the walls, ceilings and floor of the main part of the auditorium so that the sound waves will be absorbed instead of reflected where the audience is located.

If material could be found that would absorb all of the sound and reflect none of it, the condition would be identical with the out-of-doors setup described above. Special kinds of porous plaster absorb about one-third to one-half of the energy of a sound wave whenever it strikes the wall. A good grade of carpet absorbs about one-fifth of the sound energy striking it. Various kinds of porous tiles, sheet cork, wall boards made out of sugar cane, felts and other kinds of building materials have various abilities to absorb sound, some of them running up to three-fourths or more of the energy reaching the surface.

Although this absorption is not perfect, it is found to be very satisfactory for use in auditoriums. In fact complete absorption makes a room sound *dead* as contrasted to the too *live* effect in a room where all the walls are good reflectors.

If too much absorbent material is used on the walls of a room many people do not like this dead effect, and they

actually prefer a room that is more live, or as some people describe it, more brilliant. This is especially true if the room is to be used for music. However, the objection to a rather quiet room is partly due to our being accustomed to the other kind. People soon learn to like fairly dead rooms after they become used to the change.

If the walls and ceilings are covered with material having an absorption of about one-fourth, and if no carpet is used on the floor, a fairly satisfactory compromise is produced in the case of small auditoriums. For large auditoriums a more absorbent material on the walls is desirable.

Nearly all radio broadcasting studios have all their walls and ceilings treated in the manner just described so that most of the sound that goes into the microphone comes directly from the performers and not from echoes in the room.

In many instances, office buildings have their walls similarly covered with absorbent material. Reduction of the noise due to typewriters, computing machines, people's walking and talking and other causes is thought to be good for the nerves of the people working and to make them more efficient.

effective in preventing the creation of sound when people walk on them, or when chairs are moved about on them.

Stairwells going through several floors of a building act as conduits to carry noise created on one floor of a building to others. Extensive use of absorbing plasters on the walls of such stairways will reduce this objectionable characteristic.

Students interested in the acoustical treatment of buildings, or in the problem of suppressing noise are referred to such books as "Acoustics of Buildings" by F. R. Watson, "Acoustics and Architecture" by Paul E. Sabine, and "Architectural Acoustics" by Vern O. Knudsen.

3. Mufflers for Automobile and Airplane Engines

When the mixture of gas and air in a cylinder of a gasoline engine explodes, the valves of the cylinder are closed and very little of the noise of the explosion is to be heard outside. However, when the exhaust valve opens, the burned gases escape with explosive violence and an objectionably large amount of noise is produced.

With a six or eight cylinder engine, the number of these explosions varies from as low as about 15 per second when the motor is idling to as many as 300 to 400 per second when it is running at full speed. The bursts of gas involved in these explosions flow from the engine through a pipe called the exhaust pipe to a muffler located under the body of the car and from the muffler the gas is carried to the rear of the car through what has come to be called a tail pipe.

One purpose of this set of conduits is to get the burned gases away from the occupants of the car. The arrangement also provides space for the muffler whose function it is to smooth out the bursts of gas and deliver this burned gas to the atmosphere in a smooth flow. If the muffler can accom-

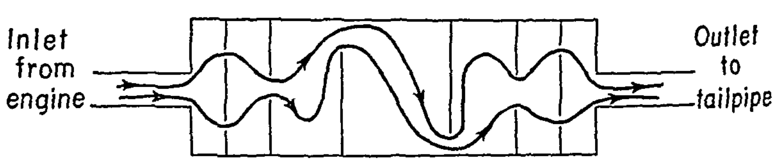


FIG. 178.—*Cross section of a baffle type muffler. The curved lines indicate the principal direction of flow of the exhaust gas.*

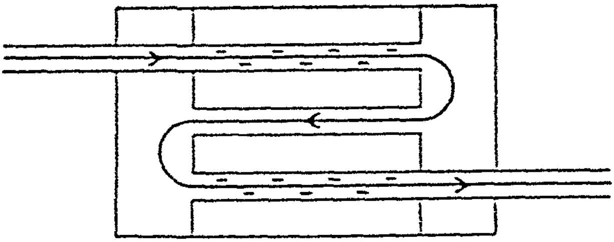


FIG. 179.—Cross section of a reverse flow type muffler.

plish these results little if any noise will be heard as a result of the exhaust of the burned gases after each explosion.

One way to build a muffler is to use a system of labyrinths through which the gas must wander. A cross section of such a muffler is shown in Figure 178. Many modifications of the details of such a labyrinth are possible, but the principle of operation is the same in all. The results in silencing the exhaust with such an arrangement can be fairly satisfactory, but usually such a muffler offers considerable resistance to the flow of the gas. This resistance causes a building up of gas pressure between the muffler and the engine. This extra pressure (known as back pressure) tends to cut down the efficiency of the engine slightly, for the burned gases are not carried out of the cylinders as efficiently as if there were less back pressure. Usually the effects of back pressure are not noticeable at ordinary speeds, but as much as two to three miles per hour loss at top speed may result if the muffler is especially bad from a pressure point of view.

Recently a muffler has been developed which has a straight through passage as large as the diameter of the tail pipe itself. A cross section of such a muffler is shown in Figure 180. This

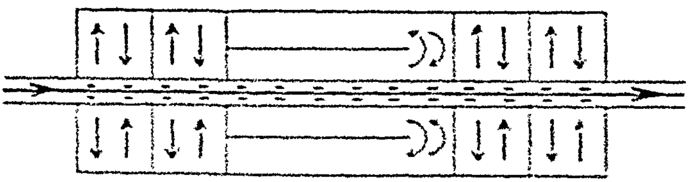


FIG. 180.—Cross section of a straight through type muffler.

muffler contributes little if any more back pressure than would the same length and size of straight pipe.

Part of the gas in each burst tends to pass sideways through the holes in the inner conduit near the front end of the muffler. The space into which this gas may flow acts as an expansion reservoir, storing part of the gas until the pressure in the inner conduit begins to recede and then returning the gas to this passage. In a somewhat similar manner the smaller cavities tend to equalize the pressure and rarefaction regions in the gas that correspond to sound waves.

In this manner excellent silencing can be obtained without appreciably adding to the back pressure.

The advantages of such design are, as indicated above, of chief importance in automobiles only when they are driven at moderately high speeds. They are, however, of great importance whenever it becomes necessary to muffle the motors in airplanes, for here the motors are always driven at fairly high speeds, frequently at top speed. Also the loss of a small fraction of the available power in an airplane is more important from the point of view of pay load than in the case of the automobile.

To date almost no airplanes use mufflers. Aviation engineers, thinking in terms of labyrinth type mufflers, sometimes say that they cannot use mufflers on account of back pressure. Of course this objection has been removed by the development of the straight through type. There are, however, other objections. One is that mufflers large enough to throttle the noise of 1,000 horsepower motors are both large and heavy. The weight subtracts from the pay load of the plane. The bulk means added air resistance unless the muffler can be built into the plane's structure.

So the use of mufflers on airplane motors seems to await the demands of customers or the legislation of communities where citizens grow tired of the roar of planes overhead while they try to sleep.

4. Public Address Systems

In the early 1920's, when the radio art was in its infancy, radio equipment in the form of microphones, amplifiers and

loud speakers was used to enable the voice of an individual to be carried to a large crowd. Such combinations of equipment were called public address systems. Since these early days the systems have improved in quality and their uses have been multiplied.

We find the barkers in midways using them instead of depending on their own vocal powers. Police safety cars use them to correct erring motorists and pedestrians. Announcers at sporting events easily talk to audiences of many thousands. Record-playing machines can take the place of the microphones and produce music over the same amplifier-loudspeaker systems.

These public address systems are sometimes used in auditoriums, and with varying degrees of success. Small auditoriums seating up to 500 people seldom need such aids unless they are of unusual shape. Auditoriums seating 1,000 people or more almost always need such a device. Naturally the need depends somewhat on the volume of sound which the speaker himself normally produces. Few people, however, talk loudly enough to be heard easily by all of the people in an auditorium seating more than 1,000, and many speakers need the added volume of a public address system for audiences smaller than this number.

In many cases, however, the chief trouble in an auditorium lies in having too little absorbing materials on the walls and ceiling. Excessive echoes make it difficult to understand speech, and in such a case, making the speech louder only adds to the reverberations. So, for good results in large auditoriums, attention must be paid to acoustical treatment as well as to providing public address equipment.

6. Some Electrical Aids to Music

a. Recorded music

inherent in the making of the records as they were in the reproducing machines, perhaps more so.

With the development of microphones and amplifiers it became possible to obtain more power to cut the records. Moreover the uniformity of microphones and amplifiers from low to middle to high tones was markedly good. The result, of course was greatly improved recordings.

Electrical pick-up devices with amplifiers and loudspeakers were then developed for playing the records. So reproduced music today, although not perfect, is near enough like the original to give a high degree of satisfaction to the listener.

There is still some difficulty in getting sufficient bass response, and in getting sufficient volume in the high tone region without getting too much record scratch. These points will be improved as time goes on, but the present fidelity of reproduction is surprisingly good.

b. Musical instruments

Electrical equipment largely from the radio art has invaded the field of musical instruments in several ways, one, for example, as an aid to existing instruments and another in the invention of new musical instruments.

One example of the former is the use of a microphone attached directly to an instrument as is done with the guitar in some dance orchestras. An amplifier and loudspeaker complete the equipment. The increased volume of the guitar gives it an entirely new importance in such an orchestra.

A more spectacular, but somewhat similar arrangement is found in a new violin where the entire sounding body has been removed. The vibrations of each string are picked up individually by tiny microphones. The total volume obtained makes it easy for the single violin to stand out as a solo instrument no matter how large the orchestra may be.

In the field of entirely new instruments one form of the electric organ might be mentioned. The body of the organ is filled with tiny electric generators each one producing a different frequency of alternating current. The keys and stops of the organ enable the player to connect the currents of the frequencies he desires to amplify. Then loudspeakers

convert the electric currents into sound waves. Many of the tones of a regular type pipe organ can be imitated with great success and many new tone combinations are possible.

c. Electronic devices aid in the study of sound

In still another manner the skills developed in connection with the art of broadcasting have been used to aid in the development of musical instruments. Instruments for photographing the wave forms of sounds and for studying such wave forms enable us to study the actual performance of any musical instrument. An experimenter can change the construction of an instrument and then observe the changes in wave form which result. In this manner greater progress can be made in improving older types of instruments than would result by the use of pure cut and try methods. So far the piano and the violin have been the chief objects of study. Already some progress has been made, and doubtless more will follow.

Some Important Facts

1. When sound waves encounter a surface such as a wall, part of the energy is reflected. Some of the sound energy may be absorbed by the wall and some may be transmitted through the wall.

2. From surfaces near the source of sound, reflection may occur in time to add energy to the original sound and so reinforce it—particularly if the surface is shaped like a parabolic reflector with the sound source near its focus.

3. For a room to have good acoustical properties, the sound source should be backed by a reinforcing reflector. Other surfaces should be made sufficiently absorbent to prevent interference by reflected sound.

4. The three principal ways of combating noise are:

   1. To prevent it—as in the case of noiseless typewriters.
   2. To throttle it near its source—as in the case of automobile mufflers.
   3. To take it out of circulation by absorbent surfaces as in the case of acoustical plaster and tile such as are used in broadcasting studios.

Generalizations

Three significant modern problems in sound are:

1. The improvement of acoustical properties in auditoriums.
2. Noise suppression.
3. The development of electronic devices for recording and amplifying sound, and as aids in musical instruments.

Questions and Problems

Group A

1. When a sound wave strikes a surface, what may happen?

2. When may reflected sound waves reinforce the original sound?

3. When may reflected sound waves interfere with the original sound?

4. What are three general methods of combating noise? Mention an example of each.

5. In what ways have electronic devices, such as used in radio, been found useful in acoustics?

Group B

1. What do musicians mean by saying that an auditorium is dead?

2. What are some of the considerations that determine the desirability of using sound-absorbent materials in auditoriums?

3. Can you explain the acoustics of whispering galleries such as the rotunda of the Capitol Building in Washington, D.C.?

4. Why are not mufflers used on airplane engines as well as on automobile engines?

5. How are oscilloscopes and oscillographs used in improving the tone qualities of musical instruments, public address systems and radio loudspeakers.

Experimental Problems

1. Using a hack saw, make cross and longitudinal sections of a few discarded automobile mufflers, comparing them as to structure, type of engine and car on which used, weight, and other features.

2. Suspend a ticking watch at slightly more than one—but less than two—focal distances from a parabolic reflector. As the ear is gradually moved away from the watch, along the principal axis of the reflector, note carefully any change in loudness of the ticking.

3. Find a large room such as a gymnasium where no acoustical material has been applied to the walls and where there is no carpet on the floor. Clap your hands or whistle and attempt to time with a stop watch the length of time that you can hear the sound in the room.

4. If you can find a room of nearly the same size that has acoustical treatment on the walls, carpets on the floor, and upholstered furniture (an empty moving picture theatre may do) try to repeat these measurements.

Part II

ELECTRICITY, OPTICS AND NUCLEAR PHYSICS

Chapter 1

THE DISCOVERY AND NATURE OF ELECTRICITY

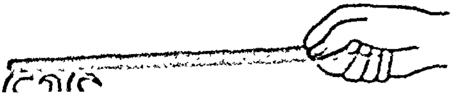
This chapter is an historical and descriptive introduction to the subjects of electricity, magnetism and light. It gives a brief account of some of the first discoveries, partly to give the student an *appreciation for the work of early scientists*, but chiefly to give him a feeling of familiarity with the nature of electricity and electrical things.

Many of the subjects mentioned briefly in this chapter for the sake of giving background will be considered more fully in later chapters. But there the discussion will be purely from the modern point of view even though the thing being discussed is old.

To allow a modern viewpoint to be used throughout the chapters on electricity, magnetism, and light, a brief explanation of the electrical nature of atoms is given at the end of this first chapter.

1.1. Electricity by Friction

In about 600 B.C. it was discovered by the Greeks that a piece of amber which had been rubbed with silk had the property of attracting light objects. This was the first



sleeve, or to pass a rubber comb through one's hair. Enough electric charge is easily obtained to enable one to pick up small pieces of paper.

An ordinary wooden pencil may be rubbed over a sheet of paper as it lies on a desk. The pencil will acquire enough charge to pick up small pieces of paper, and the large sheet of paper on which the pencil was rubbed will be found to have enough charge so that it will stick to the wall of a room or to the side of a desk.

This experiment with the pencil and sheet of paper shows that when electricity is produced by friction between two objects, each one becomes charged, and we shall later see that the charges on the two objects are of opposite kind, the one called positive, and the other called negative.

The amount of electricity produced by friction varies a great deal with the substances used—the paper-wood combination mentioned above being rather poor; fur and hard rubber, or fur and some types of wax being rather good. The latter combination is often used in laboratories for demonstration purposes.

2.1. Electricity by Induction

Suppose that a flat sheet of metal is brought near a piece of wax that has first been rubbed with fur. (See Figure 2.) Even if it is placed on the wax, it will touch only in a few spots, because of the roughness of the metal and wax surfaces. If the plate is now touched by the person performing the experiment and then picked up by a wooden handle, it will be found to be charged. In fact a good spark will jump from the edge of the plate to the person if the edge is brought near to any part of the body. The fact that the plate is charged may also be shown by picking up bits of paper with it.

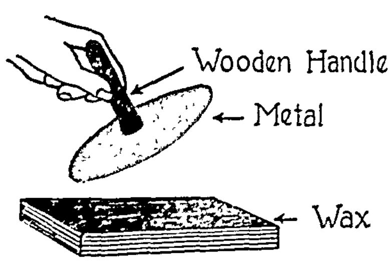


FIG. 2.—Apparatus for obtaining electricity by induction.

When a spark jumps between the plate and a person, the charge of electricity is lost from the plate. However, the plate can be charged again by placing it back on the wax and touching it.

The charging process can be repeated many times without having to rub the wax with the fur a second time.

This method of getting charge on the plate is known as electrification by induction.

3.1. The Motion of Electricity

The experiment just described shows that electricity must move in and out of metals easily. Apparently this is not true for wood, since the charge does not escape to the person through the wooden handle; and it is not true for the wax, since the charge seems to remain there.

Things that conduct electricity readily we call conductors, and things which appear not to conduct it well we call insulators. All metals are good conductors. Glass, dry wood, rubber, cloth and many other things are insulators. Some liquids, such as oils, are insulators while others, such as dilute acids, are good conductors. Pure water might almost be called an insulator, but slight amounts of impurities give it the ability to conduct electricity.

4.1. Methods for Producing Electricity

Charging bodies by friction with one another indicates that electricity must already have been present in the bodies. Later we shall see that the atoms of all bodies are made up of positive and negative charges of electricity and that if these are present in equal numbers, they neutralize one another. In the frictional process, electrons, the negative particles of electricity, are more attracted to one of the objects than the other. So the object that attracts the electrons becomes negatively charged and the other is left positively charged by the loss of some of its electrons. (See Figure 3.)

Evidently the process of friction does not create electricity but merely separates negative from positive charges that already exist in ordinary material. The induction method

described above, and discussed in more detail in the next chapter, is an improvement on the simple frictional method. Either of those methods may be further improved so that they can be carried out by turning a crank instead of having all of the operations done separately. Such machines are

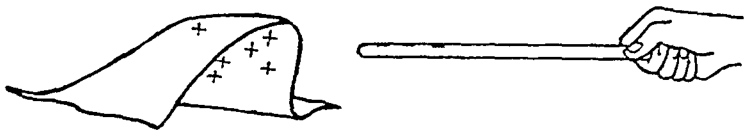


FIG. 3.—The piece of fur at the left has been rubbed on the hard rubber rod at the right. The fur lost electrons to the rod making the rod negatively charged and leaving the previously neutral fur with a positive charge.

called by various names such as "static" machines and "induction" machines. But even these machines do not separate rapidly any very large amounts of positive and negative electricity.

Electrical progress took a new leap forward in the year 1800 when the battery was developed to supply steadier and

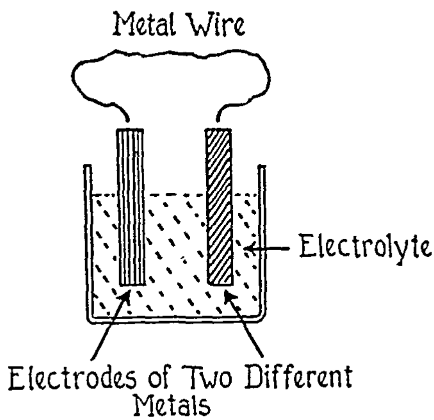


FIG. 4.—A simple battery.

larger currents of electricity. Galvani and Volta, Italian scientists, discovered that if two different metals were placed in certain solutions, especially acids, the one metal would become positively charged and the other negatively charged with respect to the solution. A metal wire connecting the two pieces of immersed metals would carry elec-

tricity from one to the other. (See Figure 4.) At first it was supposed that it was the positive charges that moved along the wire, but we now know that it is the electrons that move in a metallic conductor.

Chemical action takes place between the solution, which is called the electrolyte, and the metals, which are called the electrodes. Energy freed by this chemical change supplies *the work necessary to separate the electrical charges on the* two electrodes.

5.1. Motion of Electricity in Conductors

The motion of electricity through a conductor is called an electric current. The amount of electric current is measured by the quantity of electricity that flows by any point in the

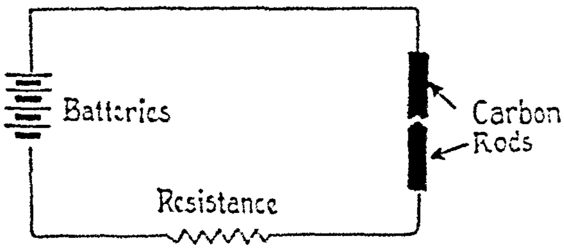


FIG. 5.—A simple arc lamp.

The electrical art developed rapidly after the discovery of batteries, for experiments were possible that could not be done with the feeble currents delivered by static machines.

One effect early noticed was that the flow of electricity through a metallic conductor heated the conductor. This fact is used nowadays in electric toasters, flat irons, the filaments of electric lamp bulbs, and many other household appliances.

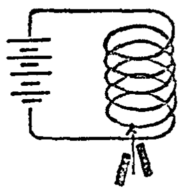


FIG. 6.—Electric current in a coil attracts pieces of iron.

It was also early discovered that if the wire connecting the terminals of a large bank of batteries was cut and the ends touched to one another, sparking resulted and the point of contact of the wires became very hot. If carbon rods are connected to the ends of the wires, (see Figure 5) electricity may continue to flow across the gap as the carbons are pulled a short distance apart. The ends of the carbons become white hot and the space between them is filled with electrically conducting vapor. This device is

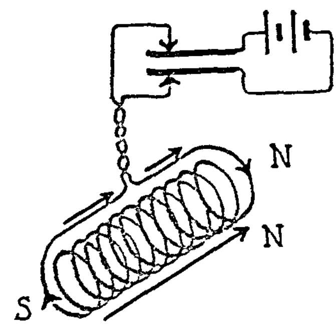


FIG. 7.—A coil suspended horizontally and free to turn will point its axis in a north-south direction when there is current in it.

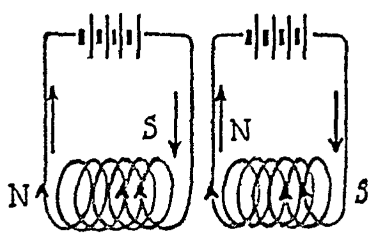


FIG. 8.—Two coils carrying electric current may attract or repel one another depending on the current directions.

called an arc lamp. It is used in search lights, in some street lamps, and sometimes in moving picture projectors.

Another effect of electric currents may be seen if the wire connecting a bank of batteries is coiled as shown in Figure 6. The arrangement will attract bits of iron or steel. If the

coil is suspended so that it can turn as shown in Figure 7, it will point with its axis north and south. If two coils are arranged as shown in Figure 8, the coils will be attracted to one another.

These are called "magnetic" effects of electricity. They were discovered between the years 1819 and 1835. They are used today in such simple things as door bells and telegraph sounders and also in more complicated appearing apparatus such as electric motors.

6.1. The Electric Power Industry

The great electric generators that are driven by steam turbines and by water wheels to separate electricity in our big

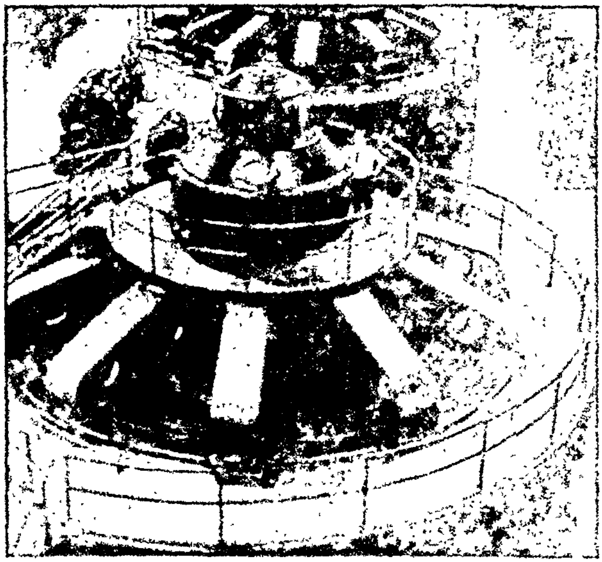


FIG. 9.—A large modern electric power generator.

power plants today are also based on the magnetic effects of electricity.

This particular development was due to experiments performed by Michael Faraday in 1831. So we see that most of the electrical power industry as we know it today came about in the last hundred years.

7.1. Wireless and the Nature of Light

Michael Faraday was one of the greatest experimenters the world has known; and he was followed in England by Clerk Maxwell, who was one of the best thinkers in the field of science that we have had. Faraday did a great number of electrical experiments and carefully wrote up his observations. Then came Clerk Maxwell, who read Faraday's notes and those of other experimenters and then spent his time trying to figure out theories to account for all the experiments.

One of Maxwell's predictions was that ordinary visible light was of an electrical nature and therefore it ought to be possible to send radiant energy from an electrical system through space to be picked up at a distant point just as light travels through space and is picked up at remote points. This prediction was made in the early 1860's. It was read by many scientists all over the world. Some of them scoffed at the idea, some took it seriously. Those who took it seriously may be divided into three groups.

One group centered about Helmholtz in Germany, another about Sir Oliver Lodge in England, and another about the Italian physics teacher, Righi. In the first of these schools a pupil, Heinrich Hertz, invented wireless telegraphy in 1888. A short time later, Marconi, pupil of Righi, added the large out-of-door antenna and started wireless on its way to its present state of development. In the meantime Sir Oliver Lodge's group discovered wired wireless.

It is almost a shock to people who think wireless began with organized broadcasting in the fall of 1920 to learn that both ordinary wireless and wired wireless were discovered more than thirty years earlier. Several basic patents on improvements in radio transmitters were held by Sir Oliver Lodge and by Marconi before 1900.

8.1. Electric Discharge Tubes.

The period from 1880 to 1900 was one of great activity in many fields of science and especially in the electrical art. While the groups mentioned above were working on wireless,

many other people were developing the type of gas filled tube now so extensively used for advertising signs. Often they are called neon tubes, because the brilliant red light from the gas, neon, has been used so much for this purpose.

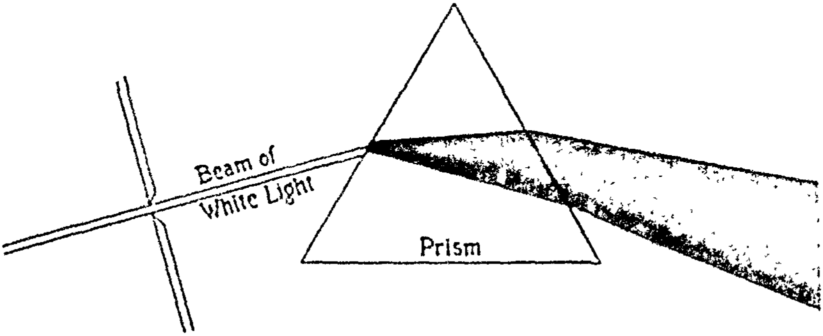


Fig. 10.—A beam of white light (daylight or light from a hot filament type lamp) is spread out by a prism into a continuous rainbow of colors ranging from violet to red.

Previous to this period scientists thought, as they do now, that matter may exist in three states, solid, liquid and gaseous. But for a time, the behavior of gases in these elec-

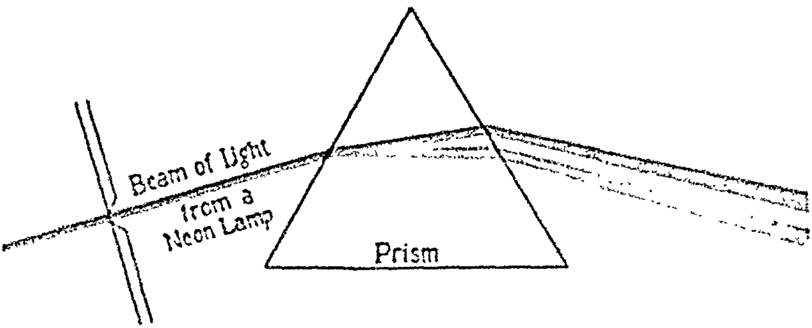


Fig. 11.—A beam of light from an electric gas discharge tube (such as Neon) shows only certain definite colors when spread out by a prism.

tric signs seemed to suggest that there might be a fourth state, the "electrical" state of matter. The electric discharge in these tubes is now explained quite easily on the modern

belief that atoms of matter contain electrons and protons, (negative and positive particles of electricity); but actually the beginnings of this new knowledge about matter came from a study of the light produced by these tubes.

When a narrow beam of white light from a filament type electric lamp, or from the sun, passes through a prism, as shown in Figure 10, it is separated into all the colors of the rainbow. But when a beam of light from a gas filled electric discharge tube is substituted for the white light only a limited number of definite colors are sorted out. (See Figure 11.) These colors differ in a systematic manner with the gases used, and this effect gave the first good clue to the electrical nature of atoms themselves. (See Figure 12.)

9.1. X-rays

When the amount of gas left in a discharge tube gets too small the light given off becomes fainter, and larger electrical voltages are needed to make the discharge visible at all. In 1895, in the course of experiments on tubes, Roentgen, a German physicist, was operating one under these conditions. On the same table where the tube was placed was a piece of the ore, willemite. Roentgen noticed that this piece of ore became fluorescent; that is, it gave off visible light, whenever the tube was operating near it.

It was soon discovered that this invisible something from the tube that could make the willemite glow could also affect a photographic plate. It could pass through black paper and cardboard such as are used to protect the plate from ordinary light. The rays from the tube were given the name X-RAYS to indicate our lack of knowledge concerning them, and ever since that time they have gone either by this name or by the name, Roentgen rays.

The use of these rays for making shadow pictures of broken bones was early recognized, and the x-ray became a

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FIG. 12.—Colored plate of line spectra of light from various sources. The dark lines in the spectra for sunlight are due to absorption in the atmospheres of the earth and the sun.

valuable aid to the surgeon. (See Figure 13.) At the same time, the production of these rays indicated to the scientist that, as in the case of wireless, the x-rays were radiant energy of electrical nature. The experiments with the x-rays gave further information about the nature of the atoms of matter, and this knowledge was added to what was being learned from the colors of light from the ordinary discharge tubes.

Fig. 13.—An x-ray shadow picture showing broken bones in a leg. (*Fedzy.*)

10.1. Radium

The experiments with x-rays stirred up a great deal of interest in materials that could fluoresce or phosphoresce. (Fluorescent substances give off light only while they are being stimulated by x-rays, ultra-violet, sunlight, or other radiant energy; phosphorescent substances are able to store up energy and to glow for some time after the stimulating source has been removed.)

Among the scientists interested in this problem was Professor Henri Becquerel of Paris. He discovered that some kind of penetrating rays were given off from a compound of uranium with which he experimented. These rays would go through cardboard and black paper and would affect a photographic plate so that it would appear to be light struck.

At first Becquerel thought that the uranium compound l
to be exposed to sunlight before it could emit these rays
other words, he thought the process was one of phospho
cence. However, he discovered that the ability of
uranium compound to emit these penetrating rays was
affected by keeping the material in the dark for many da
So he concluded that the peculiar radiation, which in so
respects resembled x-rays, was a natural property of
uranium compound.

Marie Curie, a graduate student in physics, was attrac
to this problem and began an examination of all the tl
known elements to learn what other substances might ha
this property of emitting penetrating radiation. She exten
her study to ores, and particularly ores containing urani
In 1898, in one of these ores, she found two new elements
minute quantities, but each showed much greater ability
radiate than did uranium. She named the substan
radium and polonium.

Later experiments have shown that the atoms of all
these "radioactive" substances are transmuted by nat
from atoms of one element to those of another at the tin
when they give off the radiations. In these process
electrons and helium ions are thrown out of the insides of t
atoms with great violence; and in some cases gamma ra
which are extremely penetrating x-rays, are emitted.

11.1. Modern Electrical Progress

The electrical world as we know it today may be said
date from the discovery of batteries in 1800—not much mo
than one century back. In all the centuries from the d
covery of frictional electricity in 600 B.C. to 1800 A.D.
discoveries of any importance were made in the field of ele
tricity, and even the improvements on static machines we
elementary.

For the twenty years after the first development of ba
teries, electricity was something of a scientific toy. But
1819 a Danish physicist, Oersted, discovered that electr

currents had magnetic properties. During the 1820's a great increase in knowledge about the flow of electricity in ordinary wire circuits came about, and in the 1830's Faraday and other experimenters studied the flow of electricity through solutions and also made further progress on relations between electric and magnetic effects. The first of these experiments led to electroplating and also to some of the processes of ore refining used in the great physical chemistry plants that cluster around Niagara Falls and other places where there is cheap electrical power. These electro-magnetic experiments led, as we have already seen, to the development of motors and generators which make possible the production and use of electrical power as we know it today.

So all of the practical side as well as most of the pure science side of electricity is the work of the last hundred years. In comparison to the lack of progress in the long period before, it all may be called "modern," but there has been some tendency in recent years to date modern physics from the discovery of x-rays in 1895.

This point in science is used since it represents the time when experiments began to tell us something about the nature of the atoms of which matter is made. However, the year 1895 probably does not seem very much like modern times to the students who read this book; and the writers of the book feel that it may be well to forget the word "modern" and simply include the knowledge gained from x-rays, radium, wireless and the more recent discoveries wherever they logically come in a study of electricity.

12.1. The Electrical Nature of Atoms

Beginning with the discoveries mentioned above, our knowledge about atoms (the basic particles of the elements of matter), has grown rapidly for a period of 40 years. The more we have learned, the better our position has been to look for still more new things. The details of our guesses about atoms have changed many times in this period and are still changing, but the underlying belief in all of the

theories has been that electrical charges, negative and positive, are the principal parts, if not all of the parts, of every atom.

All of these theories about atoms are founded on what a court of law would call circumstantial evidence; for no one has ever seen an atom and there is no reason to think that any one ever will. Atoms are smaller than the wave-lengths of light to which the human eye responds, so the use of microscopes does not help.

In the following paragraphs a simple account is given of the nature of atoms according to present beliefs. It is the atomic picture on which much progress both in electricity and in chemistry has been made in recent years, and it is given here so that it may be used throughout the remaining chapters in this book to provide explanations for many electrical and optical things.

a. Hydrogen

The simplest atom that exists is that of the element hydrogen. It is believed to consist of one positive charge called a proton and one negative charge called an electron. (See Figure 14.) Experiments have shown that this positive charge, called the proton, has the same amount of positive electricity as the electron has of negative; but the proton weighs about 1800 times as much as the electron. Curiously enough, however, the proton seems to be smaller in physical size than the electron.

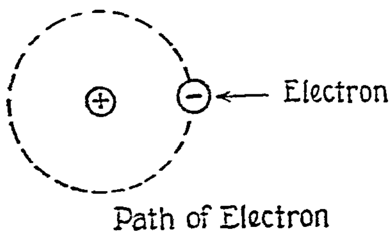


FIG. 14.—An imaginary picture of a hydrogen atom along the lines of the Bohr theory. The light electron revolves around the relatively heavy nucleus somewhat like a planet around the sun.

According to the Bohr theory the electron moves around the proton in a nearly circular path something after the fashion of our earth moving around the sun.

This planetary-solar system makes a very easy-to-understand picture of an atom. It may not be entirely

right, for we cannot be sure that electrons and protons are little particles as we have assumed them to be. So while the picture is useful to us in getting some practical understanding of the nature of atoms, we must always accept it with some reservations so that we will not be too shocked if and when we discover that some other picture of an atom may be nearer the true one.

Hydrogen exists as a gas at ordinary temperatures and pressures. Under these conditions two hydrogen atoms always attach themselves to one another, and so hydrogen is said to be "diatomic."

b. Helium

In the periodic table of the elements we find helium following hydrogen. Experiment has suggested that the central section of the atom, called the nucleus, contains two protons and two neutrons as shown in Figure 15. Two electrons, to make the atom as a whole neutral, move about in orbits that are far removed from the nucleus in comparison to the size of the nucleus. The neutrons in the nucleus are little particles of matter each weighing about the same as a proton but having no electric charge.

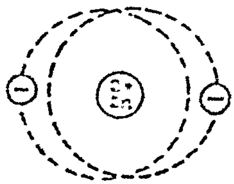


FIG. 15.—Structure of a helium atom showing two electrons revolving about a central nucleus consisting of two protons and two neutrons.

Helium, like hydrogen, exists as a gas at ordinary temperatures and pressures; but unlike hydrogen, its atoms stay independent of one another. It is called "monatomic."

c. Lithium

The third element in the periodic table is lithium. It is found to have 3 protons in the nucleus and either 3 or 4 neutrons. The three positive charges in the nucleus are balanced by three electrons in external orbits. (See Figure 16.)

Lithium is a metal existing as a solid at ordinary temperatures and pressures.

The fact that one atom of lithium may have 7 protons and neutrons and another 6 protons and neutrons in it means that there are really two kinds of atoms of this element, one weighing appreciably more than the other.

The chemical nature of the two atoms as well as most of their physical properties are the same, so they are considered to be atoms of the same element even though their masses are different. They are called *isotopes*. Many elements in addition to lithium are found to have isotopes, some of them

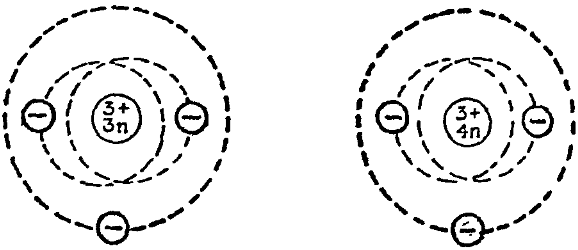


FIG. 16.—Structures for lithium atoms showing three electrons in orbits and three protons with either three or four neutrons in a central nucleus.

having several. Lithium as found in nature consists chiefly of atoms with seven protons and neutrons with a small percentage of atoms with six protons and neutrons mixed in.

d. Atomic Weight and Atomic Number

If we were to continue a study of elements through the periodic table we would discover that their atoms have increasing numbers of protons and electrons in such a manner that each successive one has one more positive charge (proton) in the nucleus than the preceding one. For example, the nucleus of a hydrogen atom consists of one positive charge; that of helium has two positive charges; lithium has three, beryllium four, boron five, carbon six and so on up to curium which has 96 positive charges in the nucleus of each of its atoms.

It is evident that the number of protons and neutrons in each atom is the chief weight determining factor, while the number of protons in the nucleus classifies the element chemi-

THE ELEMENTS

Their names, symbols, atomic numbers and atomic weights

Name	Symbol	At. No.	At. Wt.	Name	Symbol	At. No.	At. Wt.
Hydrogen	H	1	1.008	Tin	Sn	50	118.70
Helium	He	2	4.003	Antimony	Sb	51	121.76
Lithium	Li	3	6.94	Tellurium	Te	52	127.61
Beryllium	Be	4	9.02	Iodine	I	53	126.92
Boron	B	5	10.82	Xenon	Xe	54	131.3
Carbon	C	6	12.01	Cesium	Cs	55	132.91
Nitrogen	N	7	14.008	Barium	Ba	56	137.36
Oxygen	O	8	16.000	Lanthanum	La	57	138.92
Fluorine	F	9	19.00	Cerium	Ce	58	140.13
Neon	Ne	10	20.18	Praseodymium	Pr	59	140.92
Sodium	Na	11	23.00	Neodymium	Nd	60	144.27
Magnesium	Mg	12	24.32	Illinium	Il	61	146 *ca.*
Aluminum	Al	13	26.97	Samarium	Sm	62	150.43
Silicon	Si	14	28.06	Europium	Eu	63	152.0
Phosphorus	P	15	30.98	Gadolinium	Gd	64	156.9
Sulfur	S	16	32.06	Terbium	Tb	65	159.2
Chlorine	Cl	17	35.46	Dysprosium	Dy	66	162.46
Argon	A	18	39.94	Holmium	Ho	67	164.9
Potassium	K	19	39.10	Erbium	Er	68	167.2
Calcium	Ca	20	40.08	Thulium	Tm	69	169.4
Scandium	Sc	21	45.10	Ytterbium	Yb	70	173.04
Titanium	Ti	22	47.90	Lutecium	Lu	71	174.99
Vanadium	V	23	50.95	Hafnium	Hf	72	178.6
Chromium	Cr	24	52.01	Tantalum	Ta	73	180.9
Manganese	Mn	25	54.93	Tungsten	W	74	183.9
Iron	Fe	26	55.85	Rhenium	Re	75	186.31
Cobalt	Co	27	58.94	Osmium	Os	76	190.2
Nickel	Ni	28	58.69	Iridium	Ir	77	193.1
Copper	Cu	29	63.57	Platinum	Pt	78	195.23
Zinc	Zn	30	65.38	Gold	Au	79	197.2
Gallium	Ga	31	69.72	Mercury	Hg	80	200.61
Germanium	Ge	32	72.60	Thallium	Tl	81	204.39
Arsenic	As	33	74.91	Lead	Pb	82	207.21
Selenium	Se	34	78.96	Bismuth	Bi	83	209.0
Bromine	Br	35	79.92	Polonium	Po	84	210 *ca.*
Krypton	Kr	36	83.7	Astatine	At	85	211
Rubidium	Rb	37	85.48	Radon	Rn	86	222
Strontium	Sr	38	87.63	Francium	Fr	87	210
Yttrium	Y	39	88.92	Radium	Ra	88	226.05
Zirconium	Zr	40	91.22	Actinium	Ac	89	227 *ca.*
Columbium	Cb	41	92.91	Thorium	Th	90	232.12
Molybdenum	Mo	42	95.95	Protactinium	Pa	91	231
Technetium	Tc	43	..	Uranium	U	92	238.07
Ruthenium	Ru	44	101.7	Neptunium	Np	93	239 *ca.*
Rhodium	Rh	45	102.91	Plutonium	Pu	94	239 *ca.*
Palladium	Pd	46	106.7	Americium	Am	95	..
Silver	Ag	47	107.88	Curium	Cm	96	..
Cadmium	Cd	48	112.41	Berkelium	Bk	97	.
Indium	In	49	114.76				

cally. The number showing the positive charge in the nucleus is called the *atomic number* of the element.

The combined number of positive charges and neutrons in the nucleus is approximately the *atomic weight*. The reason that it is not exactly the atomic weight is that protons have the property of being able to change their masses slightly depending on how they are packed together. So, for example, a proton in a lithium atom weighs slightly less than a proton in a helium atom.

The atom of oxygen, which contains 8 protons and 8 neutrons in its nucleus, has been accepted as a standard of weight for comparing other atoms. It is said to have an atomic weight of 16. Other atoms have atomic weights approximately, but not exactly, equal to the total number of protons and neutrons in them.

The student must notice that atomic weights are not weights in grams, but weights with the proton or neutron instead of the gram, as the unit. Tables of such weights are frequently useful for the comparison of the weight of the atoms of one element with those of another. In tables of atomic weights, the average values as found in nature are given. So for example, the value of lithium, 6.94, shows that lithium as ordinarily found is a mixture of sevens and sixes with sevens predominating.

13.1. Newer Beliefs Concerning Atoms—Neutrons

Previous to 1932 the picture of the atom differed somewhat from that given in the above section. Neutrons had not yet been discovered, and it was believed that atoms had more protons in their nuclei than we now think. The extra protons were thought to be neutralized by electrons in each nucleus. This earlier picture of the atom is often referred to as the "Bohr" theory after Niels Bohr, a Danish physicist. In 1932 experiments were performed which showed that atoms contained little particles of matter equal in mass to that of the proton but without electric charge. These were called neutrons and Bohr's picture of the atom was modified to that given in the above section.

The new picture is simpler than the old and comes nearer to explaining many known facts about atoms. It is also easy to remember that the atomic number of an element is equal to the number of protons in the nucleus and that the atomic weight is approximately equal to the number of protons and neutrons together.

Still another new belief concerning the nature of matter is that protons and electrons sometimes seem to behave as if they were groups of waves instead of being little particles. Another theory is that electrons and protons may spin about axes of their own just as the earth spins about its axis as well as travelling about the sun. These new points of view need not concern us greatly at this stage in our study of physics; but it is obvious that there is still much to be learned about atoms.

Some Important Facts

1. Since about 600 B.C. it has been known that objects can be electrified by friction.
2. Electrification of an object can be obtained by a process called induction from a second object already charged with electricity.
3. Substances in which electricity moves readily are called conductors. Substances in which electricity moves with difficulty are called insulators.
4. Since about 1800 we have known how to obtain a fairly large and continuous flow of electricity by chemical action.
5. Heating, lighting and magnetic effects are important results of the flow of electricity through conductors.
6. Most of our modern electric power development is based on the magnetic effects of electric currents.
7. Wireless waves and visible light are electrical in their origin.
8. When electricity passes through a rarefied gas, radiations are produced some of which are visible.
9. While studying electric discharges in gas, Roentgen discovered ultra high frequency radiations in 1895 and they were called x-rays.
10. In 1898 Madame Curie isolated radium, a substance which naturally produces penetrating radiations.
11. Modern electrical science, pure as well as applied, is about the same age as the U.S.A.
12. All atoms may be regarded (on the basis of the Bohr theory) as made up of electrical particles arranged as minute solar systems.

a. The "sun," or nucleus, of the system consists of plus particles called protons closely united with particles of equal weight called neutrons.

b. Distributed around this nucleus are enough "planetary" electrons to make the whole atom electrically neutral.

c. The proton, though smaller, weighs about 1800 times as much as the electron.

d. The number of protons in the nucleus (which is the same as the number of planetary electrons) is called the atomic number for the particular element.

e. The weight of any atom is due almost entirely to the weight of all the protons and neutrons in the atom. The atomic weight is approximately equal to the number of protons and neutrons combined.

Generalization

Electric charges, electric currents, magnetism, wireless, light, x-rays, gamma radiations and the structure of atoms are largely explainable in terms of unit particles of positive and negative electricity which are called protons and electrons.

Problems

Group A

1. Make a list in historical order of four ways of producing electricity.

2. Is electricity created by the devices you have just mentioned? Where does the electricity come from? Why does it require work to get it?

3. What proof can you give to show that electricity travels readily in some substances and not at all in others?

4. Obtaining heat from electricity is more expensive than obtaining it from coal or gas. In spite of this fact we use electricity for a number of heating appliances around a house. Make a list of these appliances and show why electricity is used.

5. Make a list of all the electrical ways that you know about for obtaining light.

6. For what purposes other than making pictures of broken bones do doctors of medicine use x-rays?

7. In what ways do we believe that an atom of one element differs from that of another?

8. Account for the fact that a charged piece of paper shows attraction with an electrically neutral object like a wall or the side of a desk.

Group B

1. Look up in an encyclopedia the interesting biological experiments of Galvani on the stimulation of muscles with electric currents.

Chapter 2

SOME SIMPLE CHARACTERISTICS OF ELECTRICITY

This chapter opens with a number of suggestions on doing experiments with electricity obtained by frictional methods. It then attempts to explain how negative charges of electricity are separated from positive charges by this method. The chapter continues with an explanation of electrification by induction and then proceeds to a discussion of electrostatic machines,—particularly a new one which is expected to develop ten million volts.

Some simple experiments are then described to show the forces that electric charges exert on one another and to show also that there are two kinds of electricity, which we have called positive and negative.

A commonly used unit for measuring electricity is called the coulomb. It contains 6,285,000,000,000,000,000 electrons or protons. But since one electron or proton is such a very tiny amount of electricity the coulomb is only a moderate amount of electricity.

The question of finding anything that we cannot detect with our senses comes up when we ask how one would know if an electric charge were present at a given spot. The answer is that a second electric charge would be brought near the place where the first one is supposed to be and if a force acts on the second one, we will know that there must be another charge present.

1.2. Frictional Electricity

In the preceding chapter we learned that some substances become electrified when rubbed against one another and that this fact was probably first discovered by the Greeks about 600 B.C. Examples of frictional electricity are numerous.

If a person shuffles along on a carpet he may succeed in charging his own body to such an extent that a spark large enough to be easily seen and felt will fly from a finger as it approaches a metal object such as a door lock, or stove. Or one may give both himself and another person a shock by shuffling up to him with an extended finger.

Stroking a cat will often develop enough frictional electricity to make a crackling sound, and in a darkened room sparks may sometimes be seen.

An air-filled rubber balloon can be made to stay against the ceiling of a room just like a hydrogen-filled one if the balloon is first rubbed over one's coat sleeve.

These experiments work better on cold days than on warm moist ones; for on the latter type of day the electricity may leak off of the object almost as rapidly as it is developed.

If we accept the description of atoms of matter given in the latter part of the preceding chapter it is easy to give a simple explanation for the method by which electricity is produced in the frictional experiments.

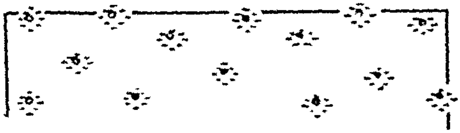


FIG. 17.—An imaginary and greatly enlarged picture of a wax surface drawn to show the electrons surrounding each atomic nucleus.

According to that description, every atom of matter is made up of protons, neutrons and electrons, with all of the protons and neutrons in a central part called the nucleus and the electrons at some distance from the nucleus, possibly going around it in orbits like planets going around the sun. If this is true, the outside surface of anything at all must really be a rough surface of electrons belonging to the nuclei of atoms that lie beneath.

According to this description our imagination would give us a picture something like the greatly enlarged section of a piece of wax shown in Figure 17. No attempt is made in this drawing to show the orbits of the electrons or even to give the right number of electrons for the various atoms that may be found near the surface of the wax.

A very similar picture may be drawn for any other surface, such as the surface of a single hair, or a piece of silk, or anything else. Of course the exact number of electrons on each

atom would vary since the atoms would be those of different elements in the different cases. It is also fairly reasonable to suppose that the electrons might be easier to pull off of the atoms of some substances than others.

If two surfaces of unlike materials are rubbed against one another it seems very likely that the attraction of one for electrons may be enough greater than that of the other so that some electrons will be dragged away on the one substance

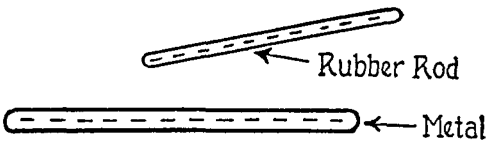


FIG. 18.—When a negatively charged hard rubber rod is rubbed on a piece of metal, some of the electrons are transferred from the rubber to the metal.

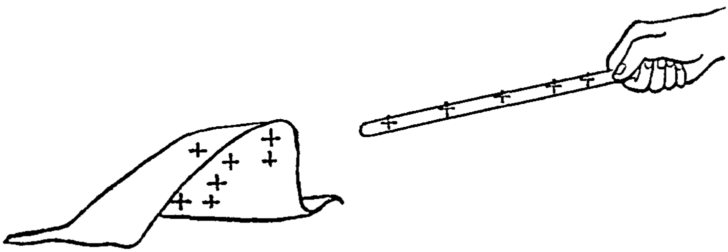


FIG. 19.—When a positively charged piece of fur is rubbed on a piece of metal some electrons go from the metal to the fur and so leave a positive charge on the previously neutral metal.

leaving a scarcity of electrons on the other. Since both substances would have been neutral to begin with, the one would now be negatively charged and the other positively charged.

If either charged substance is placed in contact with a neutral body that is a conductor, there seems to be a tendency to share the charge. In one case electrons from the negatively charged body go into the neutral body. In the other case, electrons from the neutral body go into the positively charged one. (See Figures 18 and 19.)

2.2. Conductors and Insulators

The above explanation is based on the belief that in a solid conducting body it is only electrons that are free to move.

For the case of solids this seems to be true. The positive charges exist only in the nuclei of the atoms and the atoms in a solid are pretty well held in place by one another. Electrons in a conducting substance, however, seem to be able to leave the external orbits of atoms now and then and to be able to move about among the atoms. This is especially true of materials that conduct electricity well—such as metals. In the case of things that do not conduct electricity well, called insulators, it is supposed that the electrons usually stay with the atoms to which they belong unless some outside agency removes them (as in the case of friction).

When a piece of fur is rubbed over a piece of wax, we can say that the wax takes electrons from the fur, so becoming negatively charged and leaving the fur positively charged. Neither fur nor wax are good conductors of electricity, so the *surplus electrons on the wax stay about where they are left in* the frictional process. To remove these electrons, one would have to place a piece of metal, or some other conductor like the hand or a damp cloth all over the wax. If the charged object had been a conductor instead of an insulator, the charge could all run off from one point as was described in the section on electrification by induction in the preceding chapter.

3.2. Induced Electric Charges

When a metal plate (see Figure 2 on page 362) is placed on a charged wax surface we see from the above discussion that most of the charge will remain on the wax, since the surfaces make contact at only a few points. The method of charging the plate can be easily explained as follows.

FIG. 20.—The electrons in a conductor, *B*, are attracted by a neighboring positive charge, *A*.

In Figure 20, a positively charged body, *A*, and a neutral body, *B*, are shown. If the neutral object is a conductor, the electrons in it will be attracted toward *A* and those that are free to move (electrons that have broken loose from their parent atoms) will move to the end of *B* near *A* as shown.

This action will leave the other end of *B* positively charged, since it was neutral when the electrons in *B* were distributed uniformly.

Now a person is a large reservoir of positive and negative electricity in comparison to the amount present in the atoms of a small object like the one we are discussing. Hence when a person touches *B*, electrons from his body are drawn into *B* just as the free electrons already there were drawn from one end to the other. These tend to neutralize the positive charge on the far end of *B*. (See Figure 21.) Of course if you now take your hand away from *B* and then remove *B* from the vicinity of *A*, you will discover that *B* is negatively charged. (See Figure 22.) Your body is slightly posi-

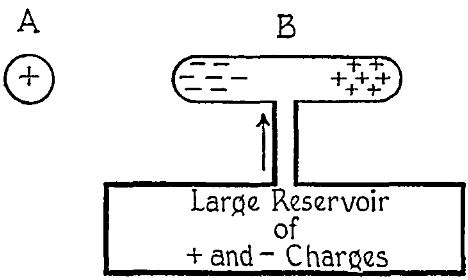


FIG. 21.—From a large object, such as the earth, or even a person's body, electrons can flow into the object, *B*.

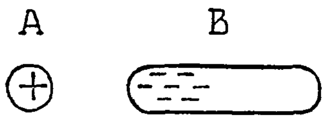


FIG. 22.—When contact with the large object is removed from *B*, it is easy to discover that *B* has acquired a negative charge.

tively charged by the loss of electrons and when you bring the object near you a spark results as electrons fly through the air from the object to your body.

This method of electrification is said to occur by "induction."

4.2. Electrostatic Machines

It is quite simple to arrange a rotating machine for obtaining electricity by friction or induction. For the former a piece of wax or hard rubber may be mounted on an axle to which a crank is attached. (See Figure 23.) A piece of fur may be held against it on one side and a metal brush may make contact on the other side as shown. The metal brush and anything connected to it then becomes negatively charged.

Such a machine is not very efficient. The fur becomes so positively charged with loss of electrons to the rubber that it cannot lose more, and also the operation of the machine is very dependent on whether dry or damp weather occurs when one wishes to use it.

The induction principle lends itself to the construction of a more successful type of static machine. The details of adapting the induction principle to a practical rotating machine may be studied in more advanced text books if a student is particularly interested. An examination of such a machine in your laboratory will also be helpful as well as entertaining.

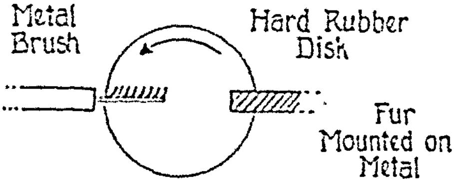


Fig. 23.—A friction type electrostatic machine.

Static machines of both types mentioned above have been known for a long time, but little was ever accomplished with them because they did not develop electricity at a fast enough rate to be very useful. The electricity was deposited on metal spheres or other arrangements of metal (called condensers) until finally a spark would flash from one collector to the other. The distance that the spark would jump was a measure of what is known as electrical potential, which we shall discuss in the next chapter. Potential is measured in "volts." The number of volts that can be developed by one of these machines depends chiefly on how well the metal collectors can be insulated to keep the electricity from leaking off. Sparks a great many inches in length are possible, indicating potentials of the order of several hundred thousand volts. However, as soon as a spark occurs, the collectors are discharged and the potential falls to zero. Then the action of the machine starts charging the collectors again.

A commonly used type of electrical collector for these machines was an arrangement of metal plates and glass called a Leyden jar. A simple form of Leyden jar consists of any glass jar with metal foil on the inside surface and a second piece of metal foil on the outer surface. More elaborate forms consist of two metal cups and a glass jar made to nest into one another with the glass between the two pieces of metal. (See Figure 24.) Such a collector is connected to a static machine so that one plate collects positive and the other plate negative electricity. The Leyden jar is simply one form of electrical collector and the general use of such devices will be discussed in a later chapter.

FIG. 24.—A Leyden Jar.

No further improvements of any real value occurred in static machine construction until 1930, when R. J. Van de Graaf got the idea of using two large spheres for electrodes and actually carrying electricity into these spheres on belts as shown roughly in Figure 25. These belts are driven by an

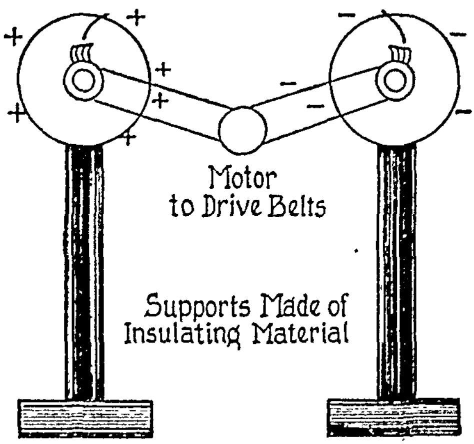


FIG. 25.—Van de Graaf's electrostatic generator to develop millions of volts.

electric motor. Electricity may be put onto the belts by induction or by means of batteries. Inside the spheres the electricity is transferred from the belt to the sphere by means of a metal brush that touches the belt and that is connected to the sphere.

This arrangement is capable of building up potentials between the spheres of the order of millions of volts, and the electricity may be carried into the spheres on the belts at a much faster rate than was possible with the older machines. One of the early models of these machines was located in an old balloon hangar at Round Hill, Massachusetts. The spheres were 20 feet in diameter and laboratory rooms were built inside the spheres. In fact, the inside of a sphere is the only safe place in the vicinity of this machine when it is operating. It is designed to develop ten million volts, but as this value is approached great flashes of lightning-like sparks crash between the spheres and the steel girders of the building.

It is expected that machines of this type may have value in some experiments on the nature of atoms, and in particular on transmutation of atoms from those of one element to those of another.

5.2. Simple Experiments with Electric Charges

In all of the explanations given above we have taken for granted the electron-proton-neutron nature of atoms, but actually the fact that there are two kinds of electricity was fairly well established long before anything definite was known about the structure of atoms, or even that there were such things as atoms.

Suppose that we hang a light ball with a string as shown in Figure 26. The ball may be made of pith or other light substance but should be covered with tinfoil to make its surface conducting.

If a charged object *A* is brought near the pith ball, the latter should develop a distribution of charge as shown for object *B* in Figure 20. What one observes is that the light ball is attracted toward the charged object. However, as soon as it touches the charged object, it moves away. (See Figure 26*b*.) This fact leads one to believe that when the charge on the object and that on the pith ball are the same, there is a repulsive force between the two.

This experiment may be carried on by first letting the pith ball touch a piece of hard rubber that has been rubbed with fur. It will then be repelled by the hard rubber. However, if a piece of glass that has been rubbed with silk is brought

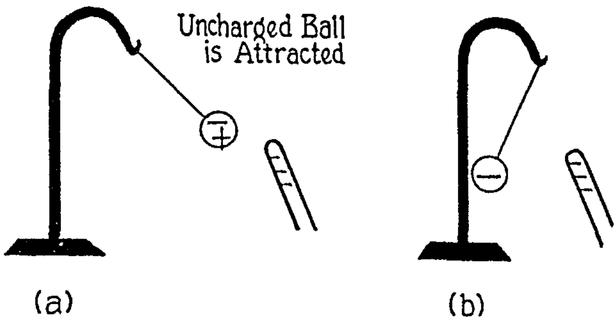


FIG. 26.—(*a*) An uncharged pith ball is attracted by the charge on a nearby rod. (*b*) If the ball touches the rod and acquires some of the charge, the ball is repelled.

near, the ball will be attracted. If it makes sufficient contact with the glass, it will be repelled.

The student should try out these experiments. They indicate quite clearly (1) that there are two kinds of electricity, (2) that forces exist between electric charges, (3) that the forces are repulsive when the charges are the same kind and (4) that the forces are attractive when the charges are of opposite kind.

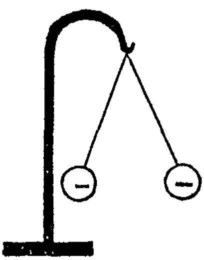


FIG. 27.—Two light pith balls each charged with the same kind of electricity as the other repel each other.

The above described experiments may be repeated in a slightly different manner by using two pith balls hung close to one another before charging. They will move into a position such as indicated in Figure 27 when they are both charged with the same kind of electricity. In fact, the amount of their separation may be used as a measure of the amounts of electricity on them.

A somewhat neater and much more sensitive arrangement to show the existence of electric charge is made by fastening a folded piece of light metal foil (such as gold leaf) to a solid

metal stem as shown in Figure 28. When electricity is placed on this system it spreads over both the stem and the foil and the latter separates as shown in Figure 29. This device is called a "Gold Leaf Electroscope." It may be charged by wiping a charged piece of rubber or other substance

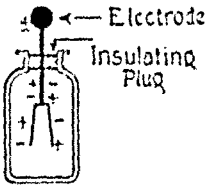


FIG. 28.—A gold leaf electroscope, uncharged.

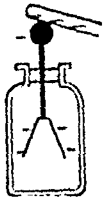


FIG. 29.—Charging a gold leaf electroscope by direct contact with a charged object.

on the electrode. Care should be taken to use only a small charge, for a large charge may cause the gold leaf to tear.

It may also be charged by induction. In this case one brings a charged object near the electrode, then touches the

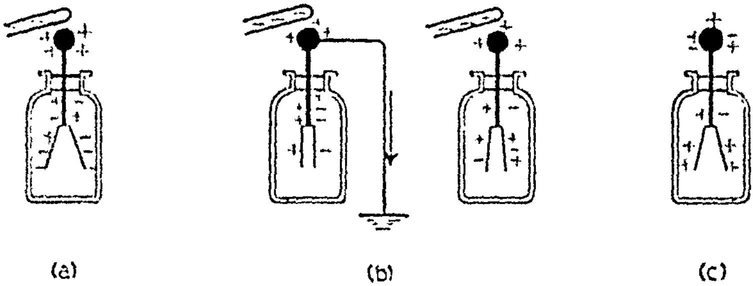


FIG. 30.—Charging a gold leaf electroscope by induction. (*a*) A negatively charged body is brought near the electroscope. (*b*) The electroscope is momentarily touched by the finger or by a wire connected to the ground and electrons run out of the electroscope. (*c*) When the negatively charged body is removed the positive charge distributes itself over the electroscope and the gold leaves separate.

electrode with a finger, and after removing the finger, the charged object is also removed. This method is illustrated in Figure 30.

A simple electroscope of this type may be made by using a wide mouthed bottle for the container, a rubber cork for the

insulating plug and a wire for the electrode and gold foil support.

6.2. Forces between Electric Charges

The experiments described above show clearly that electric charges exert forces on one another. If these experiments, or others, are carefully carried out, one finds that the force of one charge on another is proportional to the size of each charge and inversely proportional to the square of the distance between charges. For example, if the force between two charges is known for a distance of 2 cm. the force would be only one-fourth as great if the distance were doubled, and only one-ninth as great if the distance of separation were multiplied by three. This description of the simple behavior of electric charges is called Coulomb's law and may be expressed in words and symbols as follows:

$$\text{Force} = \frac{\text{charge 1} \times \text{charge 2}}{\text{distance squared}}$$

$$F = \frac{Q_1 \times Q_2}{d^2}$$

7.2. Units for Measuring Electric Charge

We have always found it convenient to have units for measuring quantities in physics. For example, we have the foot and the centimeter for measuring distance, seconds for measuring time, pounds and grams for measuring masses. The sizes of these units have been chosen quite arbitrarily in many cases, and we are familiar with using different sizes of units to measure the same kind of thing.

For convenience in electrical measurements, we arbitrarily pick out a certain amount of electricity for a unit quantity and give it a name. It is called the *coulomb*. In terms of electrons, it is quite a lot of electricity; for it takes 6,285 thousand million million (6,285,000,000,000,000,000) electrons or protons to make up one coulomb. But, on the other hand, one electron is not very much electricity, so the coulomb has really just a convenient value for ordinary experiments.

As you may guess, other units for measuring electricity are possible. Two systems of units, one called the electro-static and the other the electro-magnetic, are used for some scientific calculations. In this text we shall stick to one system called the "practical" system, of which the coulomb described above is the unit for quantity of electricity.

8.2. Detecting an Electrical Charge

One may raise the question as to how he could find out that he had a charge of electricity. In the experiments described above, we placed what appeared to be electricity on something like a gold leaf electroscope or a set of pith balls and observed the results of the forces that were present. Or if the electric charge was left on an object, we could tell that it was present by bringing it near a single charged pith ball as shown in Figure 26.

Finally we come to the conclusion that you cannot tell that you have an electric charge unless you have a second charge on which to observe the mutual force between the two.

Some Important Facts

1. When two nonconductors are rubbed together, one takes electrons from the other. The first substance then has a negative charge; the second has a positive charge.

2. In a conductor, electrons are freely exchanged; in an insulator, all electrons stay with the atoms to which they belong.

3. To charge a neutral, insulated conductor by induction: (1) Bring it near a charged body, (2) ground the conductor, (3) remove the ground. The charge on the conductor is then opposite to that of the charging body.

4. Common electrostatic induction machines may build up a potential difference of several hundred thousand volts, but involve very small amounts of electricity. The Van de Graaf generator may develop a potential difference of ten million volts, and is intended for use in "atom smashing."

5. Like electrostatic charges mutually repel each other; unlike ones mutually attract each other.

6. The mutual repulsion or attraction varies inversely as the square of the distance between the charges.

7. A coulomb is the amount of electricity in 6.285×10^{18} electrons or protons.

8. The presence of an electric charge is detected by noting its mutual action with a known charge.

Generalization

A negative electric charge is a concentration of electrons; a positive charge is obtained by removing electrons from normal atoms. Observed electrostatic behavior is the result of the mutual attraction of unlike, and repulsion of like, charges, according to the law of inverse squares.

Problems

Group A

1. Explain the differences between a neutral, a positively charged, and a negatively charged object.
2. What is an electroscope? Name three uses for such an instrument.
3. A negatively charged rubber rod is brought to within an inch of a charged electroscope and the leaves of the electroscope collapse slightly from their original position. What is the kind of charge on the electroscope? Explain how you arrive at this conclusion.
4. Give an explanation for the fact that when a piece of glass is rubbed with a piece of silk, the glass becomes positively charged with electricity.
5. Account for the action occurring when a piece of wax becomes negatively charged as it is rubbed with a piece of fur.
6. Explain the process of charging an object by the inductive method.
7. By the aid of a series of diagrams explain all the steps in charging an electroscope by induction with a negatively charged rubber rod.
8. By the aid of a series of diagrams explain all the steps in charging an electroscope by induction with a positively charged glass rod.
9. If the electrode of a gold leaf electroscope is rubbed with a hard rubber comb that has been drawn through one's hair, it becomes negatively charged. If the electrode is charged by induction with the same comb, care being taken not to let the comb touch the electrode, the charge is positive. Explain these effects.

Group B

1. The object B in Figure 20, page 385, is attracted toward the charged object A although B is neutral. Try to account for this force.
2. Show how the experiments with pith balls and with charges on glass and hard rubber prove that there are two kinds of electricity.
3. Why is the distance to which two pith balls will fly apart if suspended as in Figure 27, page 390, a measure of the amount of charge on them?
4. How much electricity in terms of the coulomb is in one electron?
   0.000,000,000,000,000,000,159 coulomb. (1.59×10^{-19} coulomb.)

5. Why is it necessary to have one electric charge in order to detect the presence of another one?

6. A force of 10 grams exists between two electric charges when they are separated by a distance of 8 cm. What is the force when the distance is increased to 6 cm.? To 9 cm.? To 10 cm.? When the distance is decreased to 1.5 cm.? To 1 cm.?

2.5 grams, 1.11 grams, 0.90 grams, 40 grams, 90 grams.

Experimental Problems

1. By means of an electroscope which you have charged by induction, determine the nature and relative strength of any charge that may be present on:

a. An ebonite rod that has been rubbed with fur.
b. The fur used in *a* above.
c. Glass that has been rubbed with silk.
d. The silk used in c above.
e. A rubber comb that has been passed through one's hair.
f. Inflated rubber balloons that have been rubbed with fur or wool.
g. Running tap water.

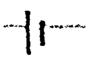
Electrical Symbols

It is now common practice to use symbols for electrical instruments and other devices of all kinds when they appear in wiring diagrams. After one becomes familiar with these symbols it is easier to make diagrams and easier to follow them than when an attempt is made to draw pictures that resemble the appearance of the apparatus.

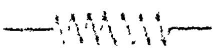
In most of the drawings in the chapters that follow, symbolic rather than graphic diagrams are used.

Whenever you have a problem to solve that involves several pieces of electrical apparatus connected together, or whenever such a set-up is to be made in the laboratory, a wiring diagram of this type should first be made.

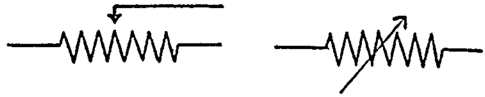
Below are given some of the more commonly used symbols. Others will be given in later sections of this text.

Battery of one cell  Battery, several cells 

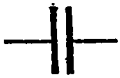
Perfect conductor ————

Conductor whose resistance must be considered, commonly called a resistor 

Conductor whose resistance is variable

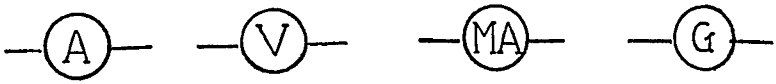


Condenser of fixed value



Condenser, variable

Meters
(A) Ammeter (V) Voltmeter (MA) Milliammeter (G) Galvanometer



Chapter 3

VOLTS, AMPERES AND OHMS

If one charge of electricity exerts a force on another we must expect that work will be required to move either charge against this force. The amount of work required to move unit charge of electricity from one point to another is called potential difference. If the unit of electricity is the coulomb and the unit of work is the joule, potential difference is measured in volts.

The amount of electricity that moves past any point in a circuit in one second is called electric current. It is measured in amperes—that is, one coulomb per second is one ampere of current.

The amount of current in a conductor is found to be proportional to the potential difference over the conductor. This is known as Ohm's Law. It is the most used relation in simple electric circuits.

The latter part of the chapter is devoted to a discussion about the resistance and conductance of electric conductors, together with some examples of conductors in series and parallel combinations.

1.3. Potential Difference

In the preceding chapter a good bit of emphasis was put on the fact that electric charges exert forces on one another. So

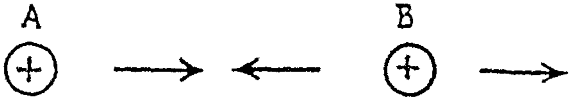


FIG. 31.—A repulsive force exists between two similar charges (such as the positive electric charges at *A* and *B*). If *A* is fastened in position, it will require work from an outside source to move *B* toward *A* against the electric force. On the other hand, if *B* can move, the repulsive electric force can do work in moving *B* from its initial position away from *A*.

it is obvious that if one moves a charge against electric forces he will have to do work in moving it. On the other hand, if the charge is allowed to move in the direction of the electric force it can do work as it moves. (See Figure 31.)

The amount of work done in moving one coulomb of electricity from one point to another is a measure of the *potential difference* between these two points and potential difference is measured in volts.

There is one volt of potential difference between two points if one joule of work is required to move a charge of one coulomb from one point to the other, or if the charge of one coulomb can do one joule of work in moving from one point to the other.

2.3. Potential Difference and Electro-motive Force

In a simple battery such as the dry cells with which we are familiar, the energy of chemical changes provides the work

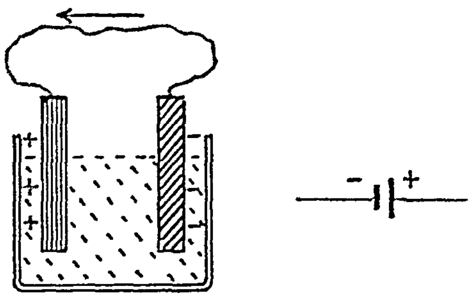


Fig. 32.—A drawing of a section of a simple battery is on the left. On the right the standard symbol for a battery is shown. The motion of electrons through a connecting wire on the drawing of the battery is indicated by the arrow.

necessary to remove electrons from one electrode and to pile up electrons on the other. The electrodes thus become positively and negatively charged respectively and in this case an electrical potential difference of about 1.5 volts is built up between the electrodes as a result of the chemical action.

In the case of an ordinary lead storage battery such as is used in an automobile, the chemical energy succeeds in doing work in piling up charges on the electrodes until a difference of potential of about 2.0 volts is reached.

If the electrodes of a battery are connected by means of a metal conductor, (see Figure 32) electrons from the negative electrode of the battery move into the wire, and electrons from the wire move into the positive electrode of the battery.

A motion of the electrons takes place all along the wire. They encounter some resistance in this motion so that work has to be done in moving them. The electrons move as a result of the forces due to the charges on the two electrodes of the battery, and in moving they are able to do the work necessary to overcome the resistance offered to their motion by the wire.

The potential difference between the electrodes of the battery is the same no matter whether one considers that he is measuring it across the battery or over the length of the wire. Within the battery, chemical energy builds up a potential difference of 1.5 or 2.0 volts for the batteries mentioned above. Over the length of the wire, electric forces do work in moving the charges, and the work per coulomb of charge moved is 1.5 or 2.0 joules for these batteries. Hence the potential difference as measured over the wire connecting the electrodes is 1.5 or 2.0 volts.

When this potential difference is considered from the point of view of what the battery can do, it is often called an *electromotive force*. When looked at from the point of view of the work done in the external circuit, it is always called a potential difference. In either case it is measured in the same unit, the volt.

If there is no connection between the electrodes of a battery, the chemical action ceases at some limiting value, such as the 1.5 and 2.0 volts in the above examples. When an external connection is made between the electrodes, and electricity starts moving as described above, the chemical action starts again in an attempt to keep enough charge on the electrodes so that their potential difference will remain the same as before. If the connecting wire is too good a conductor, the chemical action may not be able to replenish the charges on the electrodes rapidly enough, and in this case the potential difference between the electrodes drops.

The chemical actions in batteries, as well as the care and the use of batteries, will be treated at greater length in a later chapter.

3.3. Electric Current

From the above discussion and from the diagram of Figure 32 we conclude that electricity moves through an external connector from one electrode of a battery to the other as long as the chemical action within the battery can maintain the charges on the electrodes.

The amount of electricity that passes any point in a conductor each second is called the *current* through the conductor. *Current* is the *rate* of flow. It is not the velocity with which an electron goes by the point in the circuit. It is the *quantity* of electricity that goes by per second.

If one coulomb passes the point each second, the rate of flow is called an *ampere*.

4.3. Voltmeters and Ammeters

In doing experiments with batteries and conductors, it is convenient to have meters of some type with which to measure

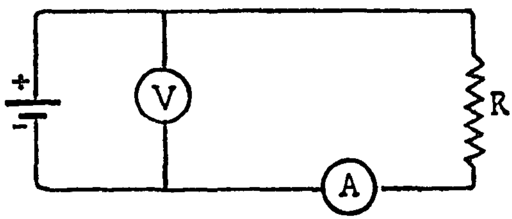


FIG. 33.—Wiring diagram showing a conductor with resistance, *R*, connected across the terminals of a battery. Notice that the ammeter is connected in series with the battery and *R*, and that the voltmeter is connected across the battery.

the potential difference between points in a circuit and with which to measure the current in any part of the circuit. These instruments are called by the descriptive names, voltmeters and ammeters. The principles on which they work and some details of their construction will be given for some types in a later chapter.

In diagrams showing the use of such meters, you should notice that an ammeter is always connected directly into the circuit where the current is to be measured, whereas the voltmeter is connected across the circuit over which the voltage is to be measured. (See Figure 33.)

5.3. Resistance and Conductance

In the diagram of Figure 33, we have a battery connected by means of perfect conductors to a conductor with resistance R. A voltmeter is connected across the terminals of the battery and an ammeter arranged to measure the current. We may suppose that in this experiment we have chosen a conductor, R, that offers enough resistance to the motion of the electrons so that it does not remove them faster than the chemical action of the battery can maintain the supply.

Suppose now that we try batteries of different electromotive force on this conductor. The experiment may easily be carried out in the laboratory where various batteries should be available. (See also Chapter 6 for connecting batteries in combinations.)

You will discover that, as might have been expected, when the voltage from the battery is increased, the current through the conductor also increases. When the voltage decreases, the current decreases. In fact, these changes are directly proportional. The proportions would read:

$$\frac{\text{Voltage 1}}{\text{Voltage 2}} = \frac{\text{Current 1}}{\text{Current 2}}$$

or in symbols

$$\frac{E_1}{E_2} = \frac{I_1}{I_2}$$

where the E's are the respective voltages and the I's the respective currents. The proportion may also be written as:

$$\frac{E_1}{I_1} = \frac{E_2}{I_2}$$

Evidently the ratio of any value of E to the corresponding value of I will always be the same. If we call this ratio R, we may write:

$$R = \frac{E}{I} \qquad (1)$$

The quantity R is called the *resistance* of the conductor. The unit of resistance is called the *ohm*, after Georg Ohm, who first performed this experiment in about 1827.

If a conductor is found such that one ampere of current will be obtained with a potential drop of one volt over it, the resistance of that conductor will be one ohm.

Equation (1) may be written as:

$$\text{Ohms} = \frac{\text{Volts}}{\text{Amperes}}$$

That current and voltage for a conductor have a constant ratio may also be written as:

$$\frac{1}{R} = \frac{I}{E} \tag{2}$$

The quantity $1/R$ is called the *conductance* of the circuit, in contrast to R which we have called the resistance.

Equation (1) may be said to measure the difficulty with which electricity gets through a circuit. Equation (2) measures the ease with which electricity passes through the circuit.

The statement of this ratio in either form (1) or (2) is named *Ohm's Law*, after the discoverer of the facts in the case. It is the most used law in elementary electrical work and must be well learned.

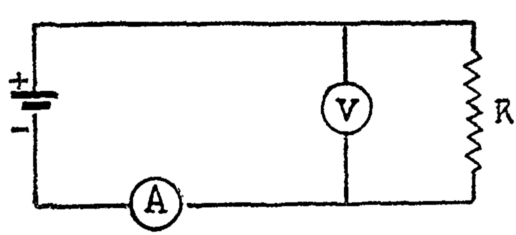


FIG. 34.—A slight variation in the method of connecting the meters in the circuit of Figure 33.

6.3. The Measurement of Resistance

Ohm's law may be used for finding the resistance or the conductance of conductors in the laboratory. The conductor

R to be measured is connected to a battery as shown in Figure 33. Slightly more accurate results may be obtained if the voltmeter is connected as shown in Figure 34, since the voltage will then be measured directly across the conductor in question. (The voltmeter must be a type that does not take much current itself, if this arrangement is used.)

The resistance or conductance of the conductor being examined is determined by simply reading the voltmeter and ammeter and taking the proper ratio (Eq. 1 or Eq. 2).

7.3. Conductors in Series

Ohm's law may also be used in determining the values of combinations of conductors from the known values of the

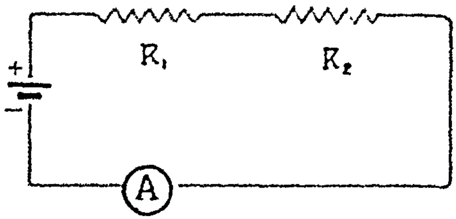


FIG. 35.—Two conductors with resistances R_1 and R_2 in series.

individual conductors. Checking the results by actual laboratory measurements makes a worth while exercise.

First we may consider the simple case of placing two conductors in series as shown in Figure 35. A *series* connection is one in which electricity must pass through one part of a circuit in order to get to another part. It is fairly obvious that since electricity must get through the second conductor as well as through the first in Figure 35, the difficulty in getting through must be equal to the sum of the difficulties presented by each part of the conductor. If R is the combined resistance and R_1 and R_2 the individual resistances, we may write

$$R = R_1 + R_2 \tag{3}$$

Similarly, the resistance of any number of conductors in series is equal to the sum of the individual resistances.

8.3. Conductors in Parallel

When two resistors are connected in parallel, it is the ease with which electricity can get through that is increased; for a parallel circuit is one in which electricity can get through one part of the circuit without having to go through another part. (See Figure 36.) The total amount of electricity going through a parallel circuit splits up, some going one way and some going the other. So it is the conductances of such a circuit that add

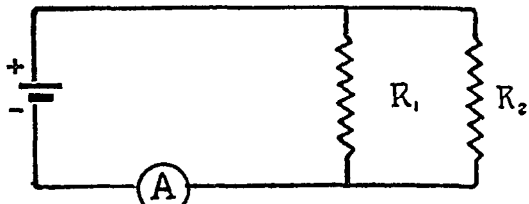


FIG. 36.—Two conductors with resistances R_1 and R_2 connected in parallel.

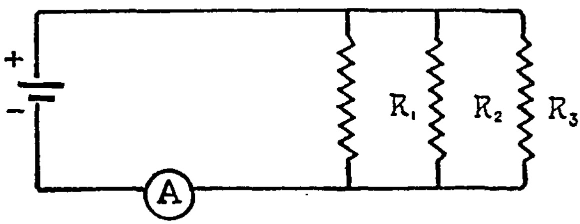


FIG. 37.—Three conductors with resistances R_1, R_2, and R_3 in parallel.

up. If $1/R$ is the conductance of the entire parallel circuit, and $1/R_1$ and $1/R_2$ the individual conductances, we may write

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \tag{4}$$

If there are three conductors as shown in Figure 37, we add all the conductances thus

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \tag{5}$$

9.3. Series and Parallel Circuits

If conductors are arranged in both series and parallel in the same circuit, the simplest solution is found by solving the

parallel section first and then the entire circuit. For example, in Figure 38, R_2 and R_3 are in parallel and this combination is in series with R_1. We may call the resistance of the parallel section R' and find it by adding conductances thus

$$\frac{1}{R'} = \frac{1}{R_2} + \frac{1}{R_3}$$

We may then redraw the circuit of Figure 38 as shown in Figure 39, where the conductor with resistance R' has replaced

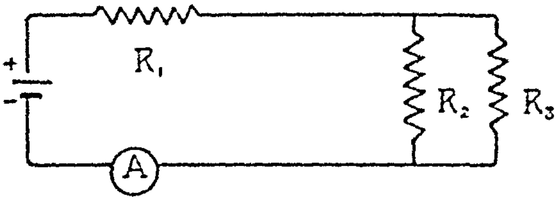


FIG. 38.—A series-parallel connection with R_1 connected in series with R_2 and R_3 in parallel.

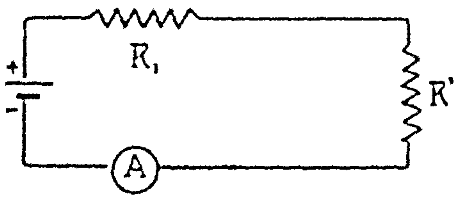


FIG. 39.—This circuit is the equivalent of that of Figure 38 if the resistance of R' is made equal to that of R_2 and R_3 in parallel.

the parallel section. The resistance of the entire circuit is simply:

$$R = R_1 + R'$$

10.3. Simple Circuits

Example 1. In the circuit shown in Figure 33, batteries of 2 and 4 volts are found to produce currents of 0.25 and 0.5 ampere respectively. What conclusions do you draw?

Since doubling the voltage doubles the current, we see that the resistance of the conductor follows Ohm's law and that the value of the resistance from equation (1) is

$$R = \frac{E}{I} = \frac{2}{0.25} = 8 \text{ ohms}$$

or

$$R = \frac{E}{I} = \frac{4}{0.5} = 8 \text{ ohms}$$

Example 2. How much current would you expect in the above conductor if a battery of 6 volts is used?
Since

$$R = \frac{E}{I}$$

it follows that

$$I = \frac{E}{R} = \frac{6}{8} = 0.75 \text{ amperes}$$

Example 3. In the circuit of Figure 35, conductors with resistance of 4 and 6 ohms respectively are used. Find the total resistance and find the current if the battery has a voltage of 1.5.

The total resistance is the sum of the individual resistances so that

$$R = R_1 + R_2 = 4 + 6 = 10 \text{ ohms}$$

The current is

$$I = \frac{E}{R} = \frac{1.5}{10} = 0.15 \text{ amperes}$$

Example 4. Conductors of 4 and 6 ohms are connected in parallel to a 1.5 volt battery as shown in Figure 36. Find the total resistance and the current.

The total conductance is the sum of the individual conductances so that

$$\begin{aligned} \frac{1}{R} &= \frac{1}{R_1} + \frac{1}{R_2} \\ &= \frac{1}{4} + \frac{1}{6} = \frac{6}{24} + \frac{4}{24} = \frac{10}{24} \end{aligned}$$

From which

$$R = \frac{24}{10} = 2.4 \text{ ohms}$$

The current is

$$I = \frac{E}{R} = \frac{1.5}{2.4} = 0.625 \text{ amperes}$$

Some Important Facts

1. Electrical potential difference between two points is the amount of work involved in moving unit amount of electricity from one of the points to the other. It is measured in volts. One volt of potential difference (P.D.) exists between two points if the passage of one coulomb of electricity, in either direction, involves one joule of work.

2. The P.D. developed between the electrodes of a battery is called the electromotive force (e.m.f.) of the battery.

3. When one coulomb of electricity passes a point in a circuit each second, the rate of flow is one ampere.

4. A voltmeter is a device for measuring volts of P.D. or e.m.f. An ammeter is a device for measuring amperes of current.

5. If a volt of e.m.f. is necessary to force an ampere of current through a given conductor, that conductor has a resistance of one ohm.

6. The reciprocal of the resistance of a conductor is called its conductance.

7. According to Ohm's Law:

$$\text{Ohms} = \frac{\text{Volts}}{\text{Amperes}} \qquad R = \frac{E}{I}$$

8. To find the combined resistance of conductors in series, we add the individual resistances.

9. To find the resistance of conductors in parallel, we add the conductances of the individual conductors.

10. To find the combined resistance of a circuit having both series and parallel conductors: (1) Find the resistance of the parallel conductors; (2) add this value to the sum of the series resistances.

Generalization

Current intensity in amperes varies directly as the volts of e.m.f causing it to flow, and inversely as the ohms of resistance overcome.

$$\text{Amperes} = \frac{\text{Volts}}{\text{Ohms}}$$

$$I = \frac{E}{R}$$

Problems

Draw wiring diagrams using symbols wherever possible before you try to solve a problem.

Group A

1. What can cause the motion of electricity?

2. Explain the similarity and the difference in the use of the terms potential difference and electromotive force.

3. In terms of present accepted theory of conductors, what is an electric current?

4. What is the conventional idea of electric current?

5. Define the ampere.

6. Define the volt.

7. Ohm's law involves volts, amperes and ohms. Express each of these quantities in terms of the other two.

8. Describe a simple method for finding the resistance of a conductor by applying Ohm's law and using an ammeter and a voltmeter.

9. Explain in words and illustrate with wiring diagrams what is meant by conductors in series? In parallel?

10. On an electric circuit in a house, when one lamp burns out, the other lamps are not affected; but in the case of some strings of lamps on Christmas trees, when one lamp burns out, all the lamps on the same string go out. Explain these cases.

11. Draw a circuit containing a battery and two resistances in series. Show how to connect a voltmeter so that it will measure the voltage across one resistor. Across both resistors.

12. Draw a circuit containing a battery which supplies current to two resistances in parallel. Show how an ammeter would be connected in the circuit to measure the current in one of the resistances. In both resistances.

13. A battery with e.m.f. of 2.0 volts is found to produce a current of 0.42 amperes in a conductor. Find the resistance of the conductor.
4.76 ohms.

14. A conductor known to have a resistance of 5 ohms is connected to a battery known to have an electromotive force of 1.5 volts. What current can be expected in the conductor? 0.30 amperes.

15. A conductor known to have a resistance of 4 ohms is connected to a battery and a current of 0.4 amperes is set up in the conductor. What is the electromotive force of the battery? 1.6 volts.

16. It is desired to obtain a current of 8 amperes in a conductor whose resistance is 3 ohms. What electromotive force will be required from a battery? 24 volts.

17. Two conductors of resistance 5 and 7 ohms are connected in series and placed across a battery of 6 volts. Find the total resistance of the two conductors. Find the current through them.
12 ohms, 0.5 amperes.

18. Find the voltage across each resistor in the circuit of problem 17.
2.5 volts, 3.5 volts.

19. Three conductors of resistance 4, 5 and 10 ohms respectively are connected in series and placed across a battery of 12 volts. Find the current through the combination.
0.632 amperes.

20. Find the voltage across each of the resistors of problem 19.
2.53, 3.16, 6.32 volts.

21. Two conductors of resistance 5 and 10 ohms are placed in parallel across a battery of 2.0 volts. Find the total resistance of the combination of conductors. Find the current supplied by the battery.
3.33 ohms, 0.6 ampere.

22. Find the current through each conductor of problem 21.
0.4, 0.2 ampere.

Group B

1. Two conductors with resistance 10 and 20 ohms are connected in parallel and the combination is connected in series with a conductor of 5 ohms resistance as shown in the circuit of Figure 38. Find the resistance of the entire circuit. Find the current supplied by a battery of 6.0 volts.
11.67 ohms, 0.514 ampere.

2. Find the current through each individual resistor of problem 1.
0.514, 0.172, 0.343 ampere.

3. Find the voltage across each individual resistor of problem 1.
2.57, 3.43, 3.43 volts.

4. You have a stock of conductors in each of the following sizes: 5, 10, 25, 50 ohms. You need a conductor with resistance 65 ohms. Pick out the proper resistors and show how you would connect them.

5. Show how you could get a combination with a resistance of 2.5 ohms out of the assortment supplied for problem 4.

6. Show how you could get a combination with a resistance of 2.0 ohms out of the assortment in problem 4.

Experimental Problems

1. Given two dry cells, a voltmeter, ammeter and resistance. Determine the relationship of volts, amperes, and ohms to each other.

2. Using a source of current, a voltmeter, ammeter and unknown resistance, determine the value of the resistance by the voltmeter-ammeter method.

ELECTRIC POWER AND ENERGY

In Chapter 3 we defined the electric potential difference between two points as the amount of work done in moving one coulomb of electricity between the two points. If several coulombs are moved between these same positions, the amount of work done will, of course, be proportionately increased. The rate at which this work is done, that is, the amount of work per second, is electric power.

Of course, electric power multiplied by the time it is used gives the total amount of work done. Meters are used in our houses and factories which automatically multiply power and time and keep a record of the total amount of energy used.

When electricity moves through conductors having any resistance at all, energy is used in heating the conductors. This fact finds useful applications in such household appliances as flat irons, toasters, electric heaters, etc.

The heating effect of electric current through a resistor is also made use of in fuses which are designed to melt easily and so protect electric lines against the passage of currents larger than those for which they are built.

1.4. Electric Power

At the beginning of the previous chapter the term potential difference was used to describe the amount of work done in moving unit quantity of electricity from one point to another either by an electric force or by some outside force working against an electric force. When the unit quantity of electricity is the coulomb and the unit of work the joule the unit of potential difference is the volt.

If several coulombs of electricity are moved from one position to another, the amount of work required will be proportionately greater than for the movement of one coulomb between the same two points. In fact it will be the product of the work per coulomb (volts) and the number of coulombs moved. If we call the number of volts between the two points E, and the number of coulombs of electricity to be moved Q, the work required will be:

$$\text{Work} = \text{Potential Difference} \times \text{Coulombs}$$

Or, in symbols

$$W = EQ \tag{1}$$

If it requires t seconds to do this work, we may write for the power, P

$$\text{Power} = \frac{\text{Work}}{\text{Time}} = \text{Potential Difference} \times \frac{\text{Coulombs}}{\text{Time}}$$

$$P = E\frac{Q}{t} \tag{2}$$

The rate at which electricity is moved, Q/t, is the electric current. If we indicate current by I we may write for equation (2)

$$P = EI \tag{3}$$

That is

$$\text{Power} = \text{Potential Difference} \times \text{Current}$$

Or, in practical units,

$$\text{Watts} = \text{Volts} \times \text{Amperes}$$

We see that when E is given in volts and I in amperes, P, power, comes out in *watts*. The watt is the unit commonly used for indicating the power that is taken by such household electrical appliances as flat irons, toasters, and even ordinary electric lamp bulbs. Power for motors is sometimes rated in watts or kilowatts and sometimes in horse power. One horse-power is the same as 746 watts.

FIG. 10.—A lamp connected in an electric house-lighting circuit.

Example. In most communities the voltage supplied by the power lines is about 115 volts. If you buy a lamp that is marked 60 watts, you can easily figure the current it will take

from the line. (See Figure 40.) From Equation (3) we have:

$$P = EI$$

and

$$60 = 115I$$

from which

$$I = \frac{60}{115} = 0.52 \text{ amperes}$$

2.4. Electric Energy (Work)

Power, as we have seen, is energy supplied or used per second. The total amount of energy used of course depends on how long an appliance is connected to the line. When we pay our electric bills we must pay for energy—not power. Where the lighting lines enter our houses meters are placed which automatically multiply together the *rate* at which we use energy and the length of time that we use it. The meter is a recording type. Pointers move around over dials and leave a record of how much energy has been used.

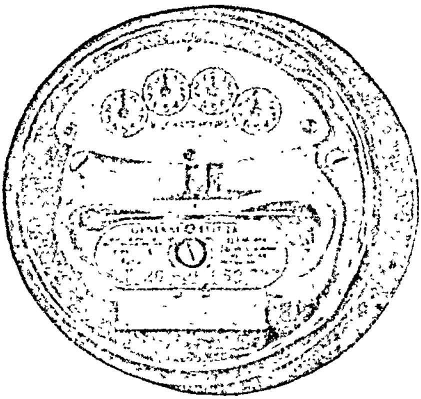


FIG. 41.—A kilowatt-hour meter such as is used to record the electrical energy supplied to homes. (*Courtesy General Electric Company.*)

Although the watt is the unit of power defined above, it is a rather small unit for such purposes as computing energy used in a house over a period of a month. Similarly, the second is too small a time unit for this purpose. So the kilowatt (1000 watts) is used for the power unit and the hour is used for the time unit in the regular meters used in most houses. (The unit of work (energy) is called the kilowatt-hour and the measuring instrument is called a kilowatt-hour meter.)

The cost of electrical energy in the United States is as high as ten or twelve cents per kilowatt hour in some residential districts, and it is as low as one-half cent or less in some indus-

trial communities located near cheap sources of power, such for example as a large natural waterfall.

Example. We may compute the cost of electrical energy as follows: Suppose that you buy a radio set which is rated as requiring 150 watts. You run this set an average of 4 hours per day 30 days per month.

150 watts is equal to 0.150 kilowatts. Hence the energy used per month in terms of kilowatt-hours is:

$$\text{Energy} = \text{kilowatts} \times \text{total hours}$$
$$\text{Energy} = 0.150 \times 4 \times 30 = 18 \text{ kilowatt hours.}$$

If electrical energy in your community costs 8 cents per kilowatt hour, the cost per month for running the radio set will be:

$$18 \times 0.08 = \$1.44$$

Example. If you forget to turn out a hall light rated at 15 watts when you go to bed and your mother reproves you for the omission, you may figure as follows:

Time light was burning—about 8 hours. The amount of energy consumed in kilowatt hours is:

$$\text{Kilowatts} \times \text{hours} = \text{Kilowatt hours}$$
$$0.015 \times 8 = 0.12 \text{ kilowatt hours}$$

If electricity costs 5 cents per kilowatt hour in your community the cost of your negligence is:

$$0.12 \times 5 = 0.6 \text{ cents}$$

Perhaps $\frac{6}{10}$ of a cent is hardly enough to rate a scolding.

3.4. What We Pay For

In settling one's bills with the power company, one pays for the work done in moving electricity through his appliances. He does not pay for electricity. It is a situation something like that of a water company. When you pay a bill to the water company you pay for energy expended in pumping the water, for investments made in the pipe lines that had to be run and for labor of operating the pumping plant. You do

not pay for the water. All of the water that comes in to the house through the faucet goes out again through the sewer or by evaporation.

Similarly when electricity comes in one end of a conductor an equal amount goes out the other end. In general, the electrons that come out are not the same ones that went in, but the number is the same. The conductor has many free electrons floating about in it and they simply drift along when an electromotive force is applied to the conductor. The work in moving them along against the resistance to their motion in the conductor is the thing for which we must pay the power company. Electric lamp filaments and heating coils are full of electrons and protons. We pay the power company for moving some of the electrons against the resistance of the conductors.

4.4. Electric Heating

In most household appliances (except motors) the energy taken from the power line is converted into heat. This is largely true even for the case of lamps, for the percentage of energy radiated as light is only a small fraction of the total amount consumed.

Heat is usually specified in terms of calories or British Thermal Units (B.T.U.). The following data show the relations between energy in the form of heat and energy in the form of work.

$$4.19 \text{ joules} = 1 \text{ calorie}$$

or

$$1 \text{ joule} = 0.24 \text{ calorie}$$

Now *one watt* is *one joule per second*. So a device rated at 4.19 watts would be able to convert electrical energy into heat energy at the rate of 1 calorie per second.

Example. From this information we can find the rate at which a coffee percolator rated at 600 watts will produce heat.

$$\frac{\text{Watts}}{4.19} = \frac{600}{4.19} = 143.2 \text{ calories per second}$$

Hence, if we know how much water is put into the percolator we can estimate the rate at which it will be heated.

5.4. Other Power Calculations

In the above calculations on power, the value of the current and the potential difference are given so that for power we write: (See page 411)

$$\text{Power} = \text{Volts} \times \text{Amperes}$$
$$P = EI \tag{3}$$

If the resistance of the conductor is known we can find the power when we know either the current or the potential difference. For this purpose Ohm's Law

$$\text{Resistance} = \frac{\text{Volts}}{\text{Amperes}}$$
$$R = \frac{E}{I} \tag{4}$$

is used to eliminate the unknown quantity E or I in equation (3).

If we eliminate E between equations (3) and (4), we obtain

$$P = I^2R \tag{5}$$

That is,

$$\text{Watts} = (\text{Amperes})^2 \times \text{Ohms}$$

If we eliminate I, we obtain

$$P = \frac{E^2}{R} \tag{6}$$
$$\text{Watts} = \frac{(\text{Volts})^2}{\text{Ohms}}$$

So we have

$$P = EI \tag{3}$$
$$P = I^2R \tag{5}$$
$$P = \frac{E^2}{R} \tag{6}$$

Example (*a*). A certain conductor having a resistance of 100 ohms will become overheated if the power in it exceeds

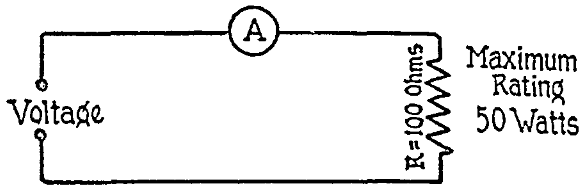


FIG. 42.—The circuit for example (*a*), section 5.4.

50 watts. Find the maximum current that may safely be allowed through this conductor. From equation (5)

$$P = I^2R$$
$$50 = I^2 \times 100$$

From which

$$I^2 = \frac{50}{100} = 0.50$$
$$I = \sqrt{.5} = 0.707 \text{ amperes}$$

Example (*b*). A lamp filament is known to have a resistance of 12 ohms when it is heated by power from a 110 volt

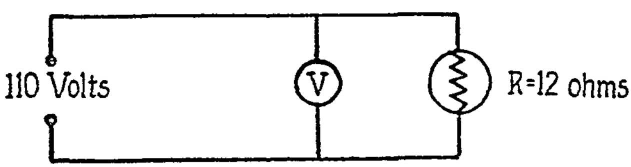


FIG. 43.—The circuit for example (*b*), section 5.4.

line. Find the power used by the lamp. From equation (6)

$$P = \frac{E^2}{R}$$

$$P = \frac{(110)^2}{12} = \frac{12{,}100}{12} = 100.8 \text{ watts}$$

6.4. Electric Fuses

The fact that the heating effect in a conductor depends on the square of the current makes the use of electric fuses

especially successful. A fuse is any piece of material which has a low resistance and which has a low melting point. It might be connected in a circuit as shown in Figure 44. As various appliances are connected to the circuit the current through the fuse increases. For small currents, the heat is also small. However, the heating effect increases as the square of the current, so that a value of current will finally be reached where heat developed in the fuse is sufficient to raise its temperature to the melting point. When the fuse does melt, it opens the circuit.

The purpose of fuses is to protect the entire electric circuit. If too much current is taken by electric appliances

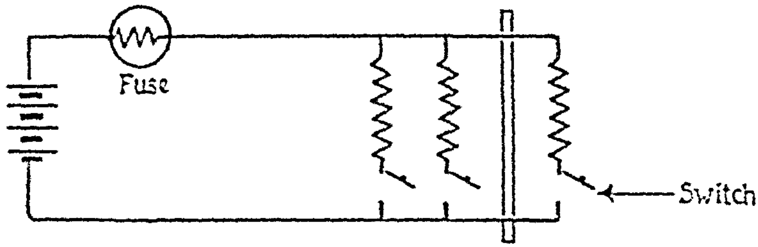


FIG. 44.—A diagram showing the location of a protective fuse in an electric circuit. If a metal bar is placed in the position of the rod (shown in black) the fuse will melt and thus open the circuit.

in comparison to the size of wires used to bring the power to these appliances, the wires themselves become heated since they also have some resistance. The fuses also protect the batteries or the generator against excessive loads such as might occur if some one should short circuit the leads running to the appliances.

The resistance of the average fuse is kept small so that there is not much power loss in it unless the current through it approaches the value at which it will melt. However, the material of which the fuses are made is cut in strips of various thicknesses and widths so that a variation of resistance does exist among fuses. This means that they can be designed for different currents. In buying fuses one must specify the current at which he wishes the fuse to melt.

Some Important Facts

1. Watts of power = Volts × Amperes. 746 watts is equivalent to one horsepower.

2. Electric energy, or work, in kilowatt-hours = Kilowatts of power × total hours.

3. In buying electricity, we pay for kilowatt-hours of electrical work, k.w.h × rate per k.w.h. giving the total cost.

4. Since one watt second of electrical work is equivalent to one joule, and since one joule = .24 calories, heat in calories = $.24I^2Rt$.

5. Since $E = IR$, and $P = EI$, power may be expressed as (1) $P = I^2R$, or (2) $P = E^2/R$.

6. A fuse is a conductor of low resistance and low melting point placed in series with the rest of the circuit to prevent dangerous overloading.

Generalization

The relationships of energy, work, time and power in electricity are essentially the same as in heat and mechanical energy, and each type of energy is readily convertible into either of the other types.

Problems

Group A

1. (*a*) What are commonly used units of electrical power?
   (*b*) What are commonly used units of electrical energy?

2. How much energy is a kilowatt-hour in terms of watts times seconds?
   3,600,000 watt seconds.

3. Does the meter in your home measure kilowatts or kilowatt-hours?

4. When an electric range is installed in a house it is usually necessary to put in a special line. Why?

5. What is the numerical relation between one horsepower and one watt? One horsepower and one kilowatt?
   1 h.p. = 746 watts. 1 watt = 0.00134 h.p. 1 h.p. = 0.746 kw.
   1 kw. = 1.34 h.p.

6. What facts are necessary to calculate the amount of power used in an electric iron such as is used for ironing clothes? The amount of energy?

7. What is the purpose of the fuse in an electric circuit? Make a diagram showing a number of electric appliances connected in parallel to the house lighting circuit. Show the proper location of a fuse.

8. A piece of copper wire and piece of iron wire each of the same length and cross section are connected, first in series, and second, in parallel to a battery of constant electromotive force. Tell which wire gets hotter in each case.

9. Electric power is sent over long distances at high voltage to reduce the amount of power lost in heating the line. Why is less power lost in the line than if the line operated at low voltage?

10. How much is the rate of heating decreased in a wire if the current in it is halved?

Group B

1. The filament of a certain radio tube requires a current of 1.25 amperes at 2.5 volts. What is the power rating of this filament?
3.13 watts.

2. Find the current required in an electric lamp rated at 75 watts if it is designed for use on a 115 volt line. 0.652 ampere.

3. A flat iron rated at 450 watts and a coffee percolator rated at 550 watts are frequently connected to the same power line. Make a diagram showing the connection of these appliances to the line and find the total current taken by them from a 115 volt source. 8.7 amperes.

4. What size fuse would you place in the line of problem 3 for protection? 10 amperes.

5. Find the minimum size of fuse for a line rated at 115 volts that supplies a chandelier with four 25 watt lamps and two 50 watt lamps, an indirect bridge lamp rated at 200 watts and a sun lamp rated at 450 watts. Make a diagram showing schematically how all of these devices are connected.
Current is 7.65 amperes. Probably a 10 amperes fuse would be best.

6. An electric toaster rated at 500 watts is used one-half hour per day 30 days per month. Find the number of kilowatt hours of energy used and find the cost where the price of electric energy is 6 cents per kilowatt hour. 7.5 kw. hrs. $0.45.

7. One mile of a certain street is lighted with 50 clusters of lamps using three 200 watt bulbs in each cluster. Find the cost of operating these lamps 12 hours per day 30 days per month at 0.75 cents per kilowatt hour. $81.00.

8. A certain storage battery can deliver 20 amperes at 6 volts for 8 hours. Find the energy in kilowatt hours and the value of it at 12 cents per kilowatt hour. 0.960 kw. hr. 11.5 cents.

9. At what rate in calories per second does the appliance of problem 8 produce heat? 28.6 calories per sec.

10. An electric heater takes 10 amperes from a 115 volt line. Find the monthly cost of operating the heater 5 hours per day 30 days per month at 8 cents per kilowatt hour. $13.80.

11. A coffee percolator is rated at 750 watts. Find the rate in calories per second at which it produces heat. If three-fourths of this heat is used to warm the water in the percolator, how long will it take to raise 1000 grams of water from 20°C. to the boiling point?
179 calories per sec. 595 sec. (approx. 10 min.).

Experimental Problems

1. Select several common household electrical devices. For each, measure or calculate; voltage, current, power, and cost per hour of use in your community. Arrange these results in a table.

2. Measure or calculate the watts of power input for several common electric motors. Translate these results into horsepower. How does this value compare with the rated horse-power output? Compute the approximate efficiency of each motor. Assume all electrical power input not recovered as useful output to be transformed into heat and calculate the rate at which heat is generated for each motor. Arrange all these results in a table.

ELECTROLYSIS

The molecules of many chemical combinations separate into two or more groups of atoms when the substance is dissolved in some liquid. Frequently this division takes place on an uneven electrical basis, some of the atoms taking extra electrons with them, and so leaving the others positively charged. Such charged atoms or groups of atoms are called ions.

If two electrodes are placed in a solution that contains ions, and if the electrodes are connected to a battery, electric forces will act on the ions. The negatively charged ions will move toward the positive electrode and the positively charged ions will move towards the negative electrode. Such a motion of ions forms an electric current in a solution.

If the atoms are those of metals, they tend to adhere to the electrode to which they go and so form a plating of this metal on the electrode. Useful applications of this process are found in the plating of gold, silver, nickel and other metals. It is also a method for separating some metals from their ores, provided the ore can be put into solution.

The amount of material deposited by electricity in this manner depends on the quantity of electricity transferred, the mass of the atoms, and on whether the ions carry one or more electric charges.

1.5. Motion of Electricity in Solutions

In earlier chapters we have explained the ability of *solids* (especially metals) to conduct electricity by suggesting that many of the electrons belonging to the atoms of such a piece of material leave their parent atoms and float about in the spaces between the atoms. The atoms themselves are more or less fixed in position in the solid. Of course, some of the atoms are positively charged, since they have lost the electrons that are floating about in the conductor. But since these atoms are not free to move they do not take any part in the motion of electricity in the conductor.

In the case of solutions, there is nothing to keep the atoms themselves from moving about. Hence, if two electrodes are

placed in a solution as shown in Figure 45 and if they are connected to a battery so that one has a positive potential with respect to the other, any positively charged atoms in the solution would be expected to move towards the negative electrode (cathode) and any negatively charged atoms would move towards the positive electrode (anode).

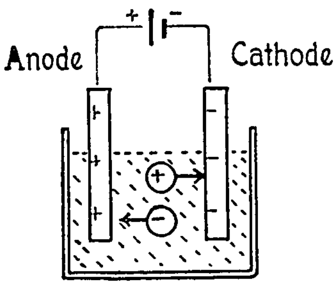


FIG. 45.—The motion of ions in a solution.

Atoms or groups of atoms which have lost or gained one or more electrons are called ions. A positive ion is an atom or molecule that has lost one or more electrons while a negative ion is one with

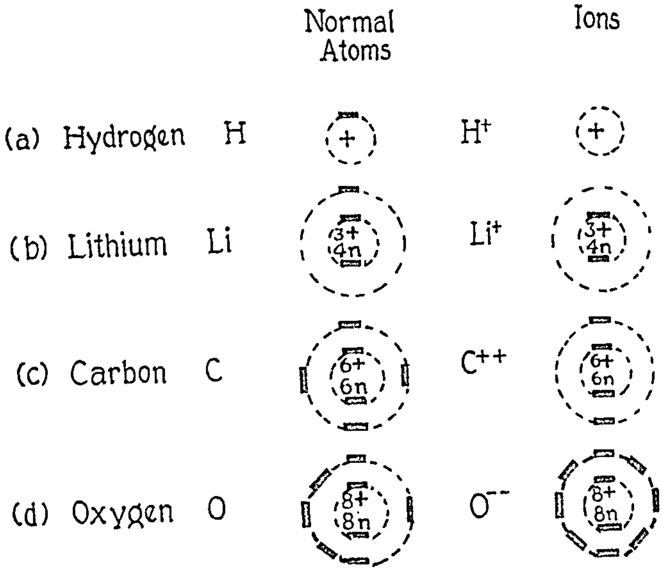


FIG. 46.—A diagram for comparing the electrons in normal atoms and ions of these atoms. (The number of protons and neutrons in the nuclei are indicated but the drawing is not intended to show the space occupied by each nucleus.) (*a*), (*b*) and (*c*) show positive ions; (*d*) negative. (*a*) and (*b*) are singly ionized; (*c*) and (*d*) are doubly ionized.

more than its normal number of electrons. The percentage of atoms that become ionized when in solution is fairly large for

many substances; so it is common to find that many solutions are good conductors of electricity. (See Figure 46.)

In the case of pure water, only a small percentage of hydrogen and oxygen atoms become ionized. The hydrogen ions have each lost an electron,—the oxygen ions have each picked up two electrons. So hydrogen moves towards the negative electrode and the oxygen towards the positive electrode. The hydrogen ions each pick up an electron from the negative electrode and the oxygen ions each deposit their electrons on the positive electrode. The atoms of the two gases then combine into diatomic molecules of hydrogen and

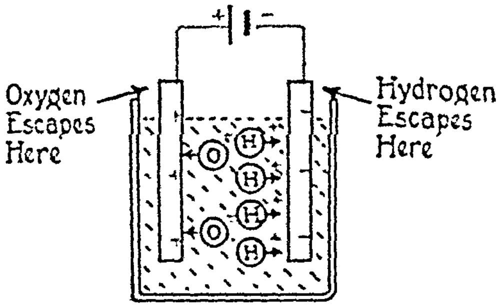


Fig. 47.—Oxygen and hydrogen can be obtained from water by electrolysis.

oxygen respectively. These molecules gather together in little bubbles of gas which at first cling to the electrodes and finally rise to the surface of the water. (See Figure 47.)

The number of ions in the water may be greatly increased by adding some substance such as sulfuric acid which is made up of hydrogen (H), sulfur (S), and oxygen (O). The amounts of these elements in a molecule of sulfuric acid are indicated in the symbols of the chemist as H_2SO_4, which means that two hydrogen atoms, one sulfur atom and four oxygen atoms make up one molecule of the acid.

When this substance is placed in water, the two hydrogen atoms break away from the remaining five atoms. Each hydrogen atom leaves one electron behind, so that each hydrogen atom becomes an ion with one positive charge and

the remaining group, SO_4, has two excess electrons. (See Figure 48.)

Solutions of many other substances also show excellent conductivity. For example if copper sulfate (sometimes called blue vitriol) is placed in water, the following ionization takes place. One molecule of copper sulfate is composed of one atom of copper (Cu), one atom of sulfur and four

H^+

$(SO_4)^=$

H^+

FIG. 48.—A molecule of sulfuric acid breaks up into ions when placed in solution.

$Cu^{++} \quad (SO_4)^=$

FIG. 49.—The ions from a molecule of copper sulfate.

atoms of oxygen. In symbols we write $CuSO_4$ for this compound. When placed in solution in water the copper atom leaves two of its electrons on the SO_4 group and moves away as a doubly ionized atom. (See Figure 49.)

Figure 50 shows a copper sulfate cell. The motion of positive and negative ions is indicated. All cells of this type

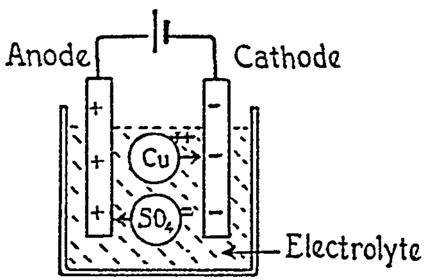


FIG. 50.—A copper sulfate cell.

are called *electrolytic cells,* the solution of ions is called the *electrolyte,* and the whole process is called *electrolysis.*

2.5. Electroplating

In the case of the copper sulfate cell, the copper atoms, in endeavoring to obtain two electrons each to make themselves neutral, become attached to the negative electrode so that the latter becomes plated with copper. The process is called

electroplating. It is used commercially for depositing thin layers of copper, nickel, silver, gold, etc. on other substances. To get a firm layer, a rather small electric current is used, so that the material is deposited slowly. This procedure is especially necessary with silver which will deposit in flakes and come off readily if it is put on too rapidly.

When electrolytic cells are used for electroplating it is customary to use an electrode connected to the positive terminal of the battery that is made out of the same kind of

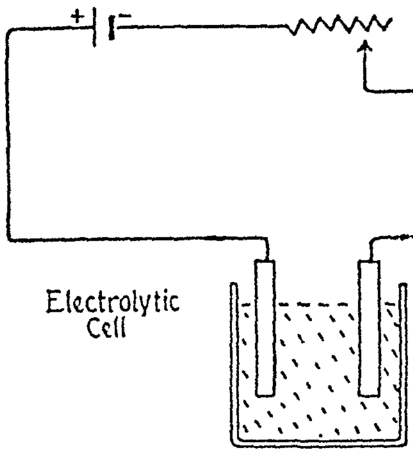


FIG. 51.—It is customary to use a variable resistor to control the current and an ammeter to measure it when an electroplating job is being done.

material as that which is to be deposited on the negative electrode from the solution. For example, if copper is to be plated from a solution of copper sulfate, a piece of copper is used for the electrode connected to the positive side of the battery. (See Figure 50.) As ions go out of the solution and are deposited on the cathode, new ions pass into the solution from the anode. So the supply of ions in the electrolyte is maintained.

This electrolytic method of separating one substance from a chemical combination is used in some industries for obtaining some substances from their ores. Refining aluminum from its ores is one example of this application of electrolysis.

3.5. Laws of Electrolysis

Suppose that a number of electrolytic cells are arranged in series as shown in Figure 52. The amount of electricity that goes through any cell must be the same as that which goes through any other. We will consider one cell to be a copper cell and another a silver cell. From the chemistry of these elements we know that the copper atoms will be doubly ionized (Cu^{++}) and the silver atoms will carry single charges (Ag^{+}).

At once we see that only half as many copper atoms will need to be transferred as silver atoms. Also, copper atoms

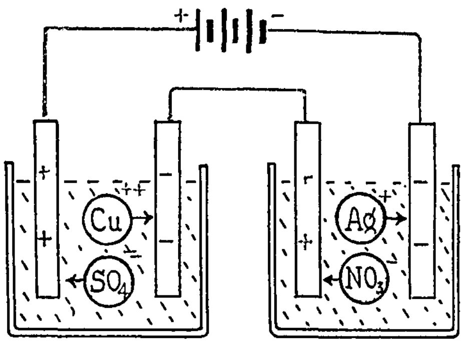


Fig. 52—Two electrolytic cells (one copper sulfate and one silver nitrate) in series.

weigh much less than silver atoms,—the ratio, as we learn from a table of atomic weights, is 63.57 to 107.88. Hence the weight of copper deposited is small compared to that of the silver both because there are fewer copper atoms and because the copper atoms are relatively light.

From this argument we can conclude that when equal quantities of electricity pass through cells, the amount of material deposited in each is:

(1) Proportional to the atomic weights of the substances.
(2) Inversely proportional to the valence (ionization) of each.
(3) In all cases the amount of any one material deposited is proportional to the amount of electricity passing through the cell.

The truth of these statements was discovered experimentally long before there was any satisfactory explanation for the nature of ionization and the conduction of electricity through a solution. They are called Faraday's Laws of Electrolysis.

4.5. Electrochemical Equivalent

The amount of any substance that will be deposited when one coulomb of electricity has been carried through the solution is called its electrochemical equivalent. Tables of these values for various elements have been compiled from experimental data and are useful in the electroplating industry to permit the operator to determine the amount of electricity that must be allowed to pass through a cell in order that he may get a desired amount of metal deposited.

ELECTROCHEMICAL EQUIVALENTS

Element	Valence	E.C.E. in Grams per coulomb
Aluminum	3	0.0000936
Chromium	3	0.0001796
Copper	1	0.0006588
Copper	2	0.0003294
Gold	3	0.0006812
Lead	2	0.001073
Nickel	2	0.0003040
Platinum	2	0.001010
Silver	1	0.001118
Zinc	2	0.0003387

Example. Current at the rate of 0.25 amperes passes through a silver cell for 30 minutes. How much silver will be deposited?

The amount of electricity that passes through the cell will be

$$\begin{aligned}\text{Coulombs} &= \text{Amperes} \times \text{Time}\\ &= 0.25 \times (30 \times 60) = 450\end{aligned}$$

From the above table we see that for every coulomb a mass of silver of 0.001118 grams will be deposited. The total

amount deposited by 450 coulombs will be

$$\begin{aligned} \text{Silver} &= \text{E.C.E.} \times \text{Coulombs} \\ &= 0.001118 \times 450 = 0.5031 \text{ grams} \end{aligned}$$

Example. Find the length of time needed to deposit 2 grams of copper from a solution of copper sulfate, with a current of 0.6 ampere.

In this case copper has a valence of two and so, from the above table, we see that its electrochemical equivalent is 0.0003294 grams per coulomb.

To obtain 2 grams of copper we must pass the following number of coulombs of electricity through the cell

$$\text{Coulombs} = \frac{\text{Grams}}{\text{E.C.E.}}$$

$$x = \frac{2}{0.0003294} = 6072 \text{ coulombs}$$

and

$$\text{Coulombs} = \text{Current} \times \text{Time}$$

From which

$$\text{Time} = \frac{\text{Coulombs}}{\text{Current}} = \frac{6072}{0.6} = 10{,}120 \text{ seconds} = 168.7 \text{ minutes}$$

5.5. The Mass of an Atom

Electrolysis also provides some information about the nature of atoms. The actual mass of a single atom can be found from such data. Experiment has shown, for example, that one coulomb of electricity in passing through a solution of silver will deposit 0.001118 grams of that metal.

The charge on one silver atom is a positive charge equal to the charge of one electron. We saw on page 392 that there are 6,285,000,000,000,000,000 electrons in one coulomb and since silver is singly ionized we must conclude that there are this number of silver atoms in the 0.001118 grams deposited by the one coulomb of electricity.

If we divide the number of grams by the number of atoms we obtain the mass of a single atom. In this example we find 17.8×10^{-23} grams for the weight of one silver atom.

From this number one can easily show that there are 5.62×10^{21} atoms in one gram of silver.

6.5. Direction of Electric Current

It is interesting to note that the motion of electricity in a solution always involves the motion of ionized atoms of matter, while motion of electricity in a solid requires only the motion of electrons. Also, since there are both positive and negative ions in a solution the two kinds of electricity move in opposite directions at the same time.

Before anything was known about electrons, it was assumed (without proof) that it is positive electricity that moves in all conductors. We now see that either or both positive and negative electricity may move. However, when the direction of an electric current is indicated, the direction in which positive charges would move is always given.

Some Important Facts

1. An ion is an electrically charged atom, or group of atoms. If an electric current passes through an ionized solution, positive ions go to the cathode and negative ions to the anode. Such a solution is called an electrolyte, and the process, electrolysis.
2. In an electrolytic cell, if the electrolyte contains ions of a metal, then that metal can be "electroplated" on to the cathode.
3. In electroplating, the amount of metal deposited varies: (1) directly as the atomic weight; (2) inversely as the valence; (3) directly as the amount of electricity passed through the cell.
4. The weight of any substance deposited on the cathode by one coulomb of electricity is called its electrochemical equivalent.
5. In solid conductors, electricity travels in only one direction at any one time since only electrons are free to move. In ionized fluids, electricity travels in both directions at the same time since positive ions and negative ions are free to move.

Generalization

When normally neutral atoms, or atomic groups, gain or lose electrons, they become electrically charged, that is, ionized: and, when in a fluid condition, they can serve as carriers of electricity.

Problems

Group A

1. How is an atom changed into an ion?
2. How is the passage of electricity through a solution explained?

3. With the aid of a labelled diagram explain the process of electrolysis using the copper sulfate cell as an example.

4. Why do you use a copper anode in a copper sulfate cell that is used for electroplating?

5. Would you expect the value of the concentration of copper sulfate in a cell to affect its use in electrolysis? How?

6. Explain the nature of a positively ionized atom and a negatively ionized atom.

7. What is meant by a singly ionized atom and a doubly ionized atom?

8. State examples where an ion consists of a group of atoms instead of a single atom.

9. In what way is the chemical "valence" of an atom or a group of atoms related to its possible state of ionization?

Group B

1. A copper and a nickel electrolytic cell are connected in series and are operated until 0.75 gram of copper is deposited on the cathode. How much nickel will be deposited? (The atomic weight of copper is 63.57 and that of nickel is 58.69.) The valence of each element is two. 0.692 gm.

2. A silver and a nickel electrolytic cell are connected in series and are operated until 0.50 gram of silver is deposited on the cathode. How much nickel will be deposited? (The atomic weight of silver is 107.88 and its valence is one.) 0.136 gm.

3. A silver and a copper electrolytic cell are connected in series and are operated until 0.85 gram of silver is deposited on the cathode. How much copper will be deposited? 0.250 gm.

4. The amount of gold deposited by one coulomb of electricity is 0.0006812 gram. The valence of gold is three. How much does one atom of gold weigh? (Suggestion: First find the number of gold atoms required to carry one coulomb of electricity.)

0.000,000,000,000,000,000,000,325 gm. (3.25×10^{-22} gm.).

5. For how long a time would you require a current of 0.25 ampere in a gold cell to produce a deposit of 1.0 gram of gold? (Remember that: amperes $\times$ time = coulombs.) 5,872 sec. (1 hr. 37 min. 52 sec.).

6. If one coulomb of electricity deposits 0.0003294 gram of copper, how much copper would be deposited by a current of 0.5 ampere in one hour? 0.593 gm.

Experimental Problems

1. Set up an electroplating cell and plate several objects. Formulate any general principles about electroplating.

2. Using an electric light bulb in series with two conveniently spaced electrodes, test the conductivity, and hence the degree of ionization, of solutions: $NaCl$; HCl; $CuSO_4$; $NaHCO_3$; Sugar ($C_{12}H_{22}O_{11}$); alcohol (C_2H_5OH); etc.

BATTERIES

When some objects are dipped into solutions a chemical action takes place with the solution in such a manner as to carry electrons from the object into the solution or vice-versa. In either case these objects (called electrodes) become charged electrically and have an electrical potential with respect to the solution. If two electrodes of different materials are used, a potential between them will exist and if they are connected by a wire, electrons will flow from one to the other.

Such an arrangement is called a voltaic cell or a battery. The electrodes are usually metals and the electrolyte is usually an acid. Sometimes, however, non-metals and non-acids are used.

If electricity is forced to reverse its direction of flow in a cell, the chemical action between the electrodes and the solution will reverse for some combinations. When this is true the battery may be restored to its initial condition. It is called a storage (or secondary) battery, while one in which the chemical action will not readily reverse is called a primary battery.

Cells may be connected in series to increase the total voltage. They may also be connected in parallel to increase the total current that can be supplied.

1.6. Simple Cells—Charging the Electrodes

In the introductory chapter on electricity, page 364, we learned that many substances, when immersed in certain liquids, especially acids, become electrically charged, and hence have an electrical potential with respect to the liquid. We also saw that if two dissimilar substances are immersed, electricity will flow from one to the other through an external conductor, and the device is known as an electric cell or battery. (See pp. 364 and 398.)

A simple battery may be made by using zinc and copper for electrodes with dilute sulfuric acid as the electrolyte. Zinc atoms each leave two electrons behind on the electrode and go into the electrolyte to form zinc sulfate. In this manner the zinc electrode becomes negatively charged and the solution

positively charged. A potential difference is built up until the chemical action is stopped.

Meantime the copper electrode loses electrons to positively charged hydrogen atoms in the solution. The copper electrode becomes positively charged in this manner until no more hydrogen ions can approach it.

If the copper and zinc are connected by a wire, electrons drift into the wire from the zinc and they drift out of the other end of the wire into the copper electrode. The action in the cell then begins again in an attempt to get sufficient charge on the electrodes to restore them to the same electrical potentials with respect to the solution that they had before the drifting of electrons from one to the other took place.

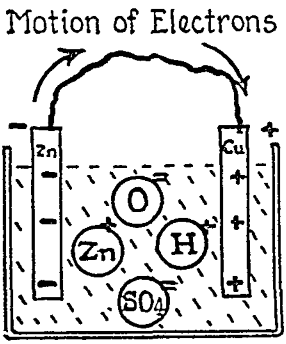


FIG. 53.—A battery of one cell. In this cell one electrode is copper and the other, zinc. Dilute sulfuric acid is used for the electrolyte.

The difference in the potential between the two electrodes of a cell is called the electromotive force of the cell as was pointed out on page 399. When it is measured in volts, it gives the work per coulomb that can be done by the chemical action in the battery working against the electric forces between the solution and the electrodes.

2.6. Polarization in a Simple Battery

When the hydrogen ions become neutralized by gaining electrons from the copper electrode, they unite to form ordinary diatomic molecules of hydrogen. These molecules cluster together and adhere to the copper plate until the bubbles become large enough to float up through the solution and escape. While they stay on the plate, they insulate that part of it from the solution and so reduce the active area of the plate. The chemical action of the battery is then not able to supply electricity to the electrodes at the same rate as before and the current in the connecting wire is reduced. The effect is known as polarization.

It is a simple matter to perform this experiment in the laboratory. The effect of wiping the gas bubbles from the copper plate after the battery has been in operation for a short time is easy to observe.

Batteries as simple as the one described above are seldom used, but the complete chemical action even in this one is more involved than the above simple explanation shows.

3.6. The Dry Cell

A more commonly used battery has electrodes of carbon and zinc and the electrolyte is a solution of ammonium chloride. Usually the container of the cell is made of zinc and serves as one electrode. The electrolyte is held in a paste, so that there is no free liquid to spill. The top of the cell can be covered with wax. These cells are called *dry cells*. Some additional chemical is always added to these cells to unite with the hydrogen gas so that it cannot cover the positive electrode. The effects of polarization are reduced in this manner and the cell gives more satisfactory performance. The normal electromotive force of this cell is approximately 1.5 volts.

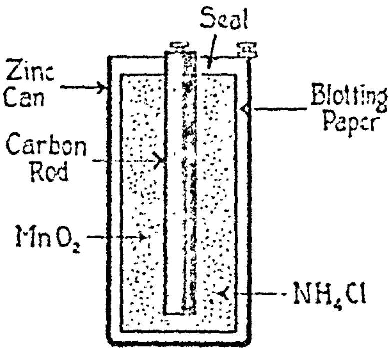


FIG. 54.—Cross-sectional view of a dry cell.

4.6. Storage Batteries

If a battery of large electromotive force is connected in parallel to one with smaller value, the motion of electrons into the electrodes of the latter will be reversed as compared to the normal action. The potentials of the electrodes with respect to the solution will become too large, and electricity will be forced through the cell in reverse direction. (See Figure 55.)

In some kinds of cells this action will reverse the chemical process and thus restore the electrodes and the solution to

their original state. In other cases the reverse action will not proceed fully.

In the batteries first described in this chapter, the chemical action does not reverse efficiently and the batteries are called *primary* batteries. Batteries in which the chemical processes are readily reversible are called *secondary* or *storage* batteries.

When the chemicals in a primary battery are all converted into other forms as a result of keeping the electrodes charged when the battery is in use, the battery is thrown away. When

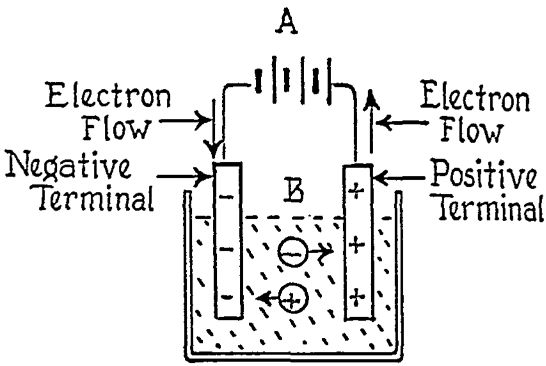


FIG. 55.—Charging a storage battery. The battery *A* has a larger electromotive force than battery *B*, so that ions are forced to move in a reversed direction in the electrolyte of *B* as compared to the motion when *B* is supplying current. In this manner the chemical action in the Battery *B* is reversed.

the same condition exists in a storage battery, the battery is connected to a source of electric power and the chemical condition is restored. The process is called *charging*.

5.6. Lead Storage Battery

The two best known types of storage batteries are called "lead" batteries and "Edison" batteries.

The lead battery has one electrode made of pure lead in spongy condition. The other electrode is lead peroxide. The first is gray in color and the second is a deep red. The electrolyte is dilute sulfuric acid.

The lead peroxide loses electrons to the electrolyte and becomes the positive electrode, while the pure lead gains electrons and is the negative electrode. When electrons move

from one electrode to the other through a connecting wire, the chemical action is such that each electrode gradually turns into lead sulfate. When the current is reversed to charge the battery, electrons are forced through the battery in the opposite direction to the original flow, the chemical action is reversed, and the initial condition of the electrodes and the electrolyte is restored. (See Figure 56.)

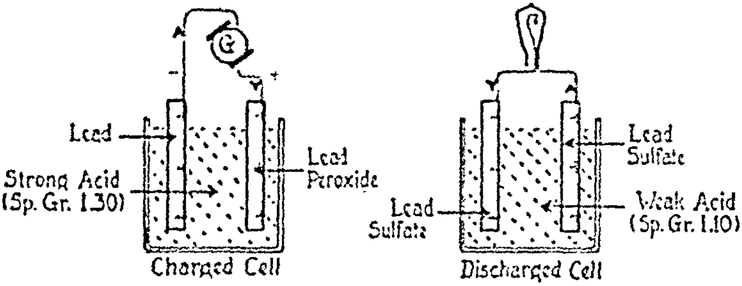


FIG. 56.—A lead storage battery being charged is shown at the left. At the right the battery is supplying current through a lamp load. In these diagrams the arrows on the connecting wires indicate the direction of conventional current. The electrons drift in the opposite directions.

The reversible chemical reaction for the positive and negative plates may be given in symbols:

$$\underset{\text{negative}}{Pb} + 2H_2SO_4 + \underset{\text{positive}}{PbO_2} \rightleftharpoons \underset{\substack{\text{discharged}\\ \text{negative}}}{PbSO_4} + 2H_2O + \underset{\substack{\text{discharged}\\ \text{positive}}}{PbSO_4}$$

When the cell is exhausted the electrolyte is weak in acid, for the SO_4 molecules from the sulfuric acid have united with the lead atoms in the electrodes to form the lead sulfate. When the battery is fully charged the acid content of the electrolyte is normal. Since sulfuric acid has a high density in comparison to water, the condition of the battery can be found by reading the density of the electrolyte with a hydrometer.

The electromotive force of this type of cell is approximately 2.0 volts. The actual value depends on the strength of the acid solution. It is common practice to use a solution with a specific gravity of 1.30. With the battery fully

charged, the electromotive force is then about 2.05 volts. The specific gravity falls to about 1.10 on nearly complete discharge and the voltage drops to 1.9 or less.

6.6. The Care of Lead Storage Batteries

The materials of which the electrodes of a lead storage battery are made are spongy so that the chemical action between the electrolyte and the material may take place rapidly throughout the electrode. The spongy mass is packed into a grid-like plate made of lead alloyed with a little anti-

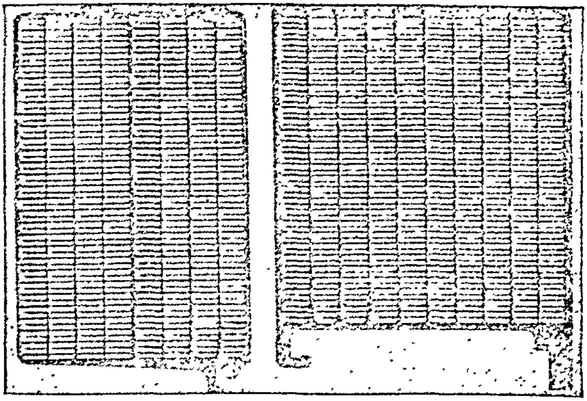


FIG. 57.—Metal grids which hold the soft lead compounds for making spongy plates for a lead storage battery. (*Courtesy Gould Storage Battery Co.*)

mony to give it mechanical strength. Several positive and negative plates are placed alternately in each cell, the positive ones being connected together, and similarly, the negative ones. This arrangement simply increases the size of each electrode in the cell. (See Figure 58.)

As the battery is used and charged repeatedly the active material tends to crumble and to fall out of the grids. It collects in the bottom of the cell and when a sufficient amount has fallen so that it touches the positive and negative electrodes, it may serve as a connector between them and so discharge the battery.

Charging the battery at too high a rate and discharging it too rapidly both tend to increase the falling out of the active material and so to shorten the life of the battery.

The average battery is built to be discharged in about eight hours. Batteries are rated in terms of the number of ampere hours they can deliver. For example, a one hundred ampere hour battery can deliver 12.5 amperes for eight hours. The normal charging rate is about the same as the discharging rate.

However, many storage battery engineers believe that better results are obtained if the charging rate is considerably

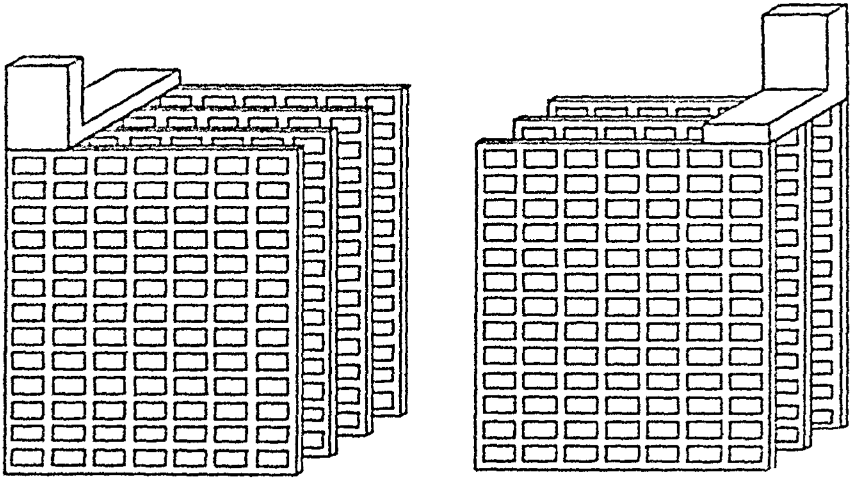


FIG. 58.—To increase the ability of a lead storage battery cell to provide current, it is customary to fasten several plates together for both the positive and negative electrodes. The two sets of plates interleave one another and are kept from touching by porous insulators.

higher than the eight hour rate at the beginning of charging when the battery is nearly exhausted. The charging current is then gradually reduced as the charge builds up.

The storage battery in an automobile is subjected to much abuse, for the starting motor requires from 100 to 200 amperes. If the car starts in a few seconds, the battery is not harmed by this load. However, when an unskilled person stands on the starter for several minutes on a cold morning, the battery takes considerable punishment.

On long distance driving, the storage battery is called on but little to start the motor, and the car generator easily keeps

it charged. During city driving the storage battery must operate the starter frequently in comparison to the amount of charging that the generator can do. Meantime we use lights, sleet wands, heater fans and radio sets. The drain on the battery is high and we try to keep it charged by setting the car's generator so that it will charge at higher than the normal rate.

Meantime the car battery takes a severe jolting as we ride over bumps on the road. The surprising thing is not that a car battery wears out on the average in about two years but that it lasts so long under this abuse. Many similar batteries in use around laboratories will last from 5 to 20 years depending on the care that they receive.

When batteries are used or when they are charged, some electrolysis of the water in the solution takes place. Hydrogen and oxygen gases escape. There is also evaporation of water. So it is necessary to add water to the cells of a lead storage battery from time to time. Acid is not lost unless the electrolyte actually spatters out of the cell. The hydrogen and oxygen gases given off are in correct proportions to form a highly explosive mixture. Lights and open fires should be kept away from batteries and care should be taken to avoid sparking at the connectors on a battery.

If a lead storage battery is used until it is completely discharged, the chemical action on the electrodes may form a double sulfate instead of the ordinary type. This is a chemical action which is not reversible. The cell is now said to be "sulfated" and is worthless since it cannot be recharged. The moral of this story is "Charge your batteries before they are completely exhausted."

7.6. Edison Storage Batteries

The electrolyte of the Edison storage battery is caustic potash (potassium hydroxide, KOH) which is an alkali. The positive electrode is a compound of nickel (nickel hydrate, $Ni(OH)_2$), and the negative electrode is an oxide of iron. The electromotive force of this cell is about 1.3 volts.

Some engineers feel that this cell will stand more rough usage than the lead battery; but its principal virtue is that it has larger ampere hour capacity in proportion to its weight than the lead battery.

The principal disadvantage of the Edison battery is that the electromotive force falls off considerably as the battery discharges, so that it is not very useful if a steady value of voltage is required.

8.6. Internal Resistance of Batteries

Even though we think of an electric cell as a device for pumping electricity, it is not surprising for us to find that the

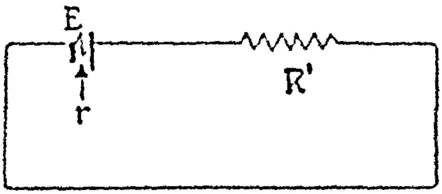


FIG. 59.—Internal resistance, r, in a battery is added to the external circuit resistance.

cell itself offers some resistance to the flow of electricity. If we connect a simple cell to a resistor as shown in Figure 59 the resistance of the battery is easily seen to be in series with the circuit resistor. So the total resistance R of the circuit would be

$$R = r + R'$$

And if we should apply Ohm's law to find the amount of current in the circuit we would write

$$\text{Current} = \frac{\text{electromotive force}}{\text{total resistance}}$$

$$I = \frac{E}{R} = \frac{E}{r + R'}$$

Some cells, particularly those that have a tendency to polarize, have internal resistance that increases when large currents are taken from the cell. Cells also show increasing internal resistance as they become discharged. Internal

resistance values may be found all the way from as little as 0.001 ohm or less in a freshly charged lead storage battery to 100 ohms or more in an old worn out dry cell.

9.6. Combinations of Batteries

When two or more batteries are connected in series, as shown in the diagram of Figure 60, the electromotive forces

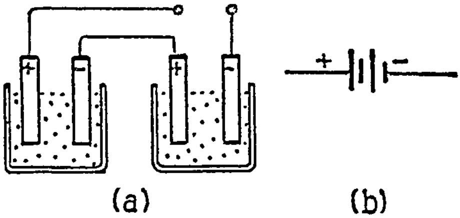


FIG. 60.—(*a*) Battery cells connected in series. (*b*) Symbol for series connected batteries.

add. It is common practice to mount three lead storage cells in a single case and to connect them in series so that approximately 6 volts are available. Opposite electrodes on successive batteries are connected when they are put in series. This arrangement increases the electromotive force but does not increase the ability of any cell or the combination to deliver current. The internal resistance of each cell is in series with all of the others and so the combined internal resistance is the sum of that of all the cells in the series combination.

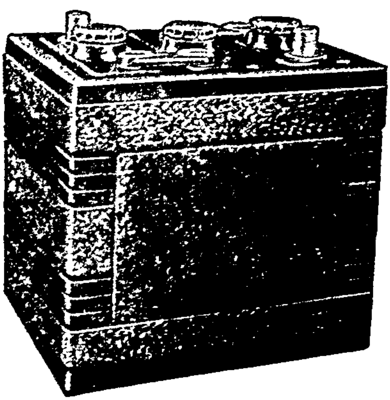


FIG. 61.—An automobile type lead storage battery usually consists of three cells. (*Courtesy Gould Storage Battery Company.*)

If two or more cells have the same electromotive force they may be connected in parallel as shown in Figure 62. Positive terminals are connected together and negative terminals also are connected together. The electromotive force of the combination is the same as that of a single cell, but the current taken from the combination is the sum of the individual currents from the cells.

The internal resistance of each cell is in parallel with the others and so the combined internal resistance is less for the parallel combination than for a single cell.

If the electromotive force of one of the batteries in Figure 62 is greater than that of the other, this battery will force electricity through the other in reverse direction as if to charge

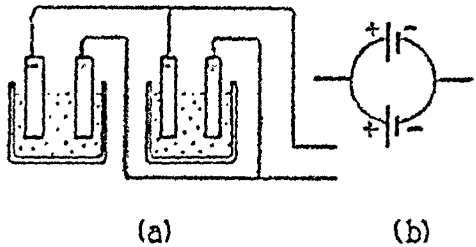


FIG. 62.—(*a*) Battery cells connected in parallel. (*b*) Symbol for parallel connected batteries.

it. Such a condition is usually undesirable when the batteries are to be used for supplying current to some external device.

10.6. Batteries and Their Uses

One of the most common uses to which batteries are put is automobile service. The battery is called on to operate the starter, to light the lamps, to sound the horn, and to supply electrical energy for the spark plugs. In addition, many car owners add additional electrical devices, such as a radio set, a heater using an electric blowing fan, an electric fan for defrosting the windshield and sometimes electrically operated windshield wipers. It is almost universal practice to use a six volt lead storage battery for this service in passenger Three cells are connected in series and mounted in a case as mentioned in the above section. Figure 61 typical battery of this type.

Batteries are often used in small indepen systems. Here the cells are sometimes mou and arranged as shown in Figure 63. S are used in some telephone circuits

plants where either very steady currents are required (as is the case with the telephone) or where some standby source of power is needed.

FIG. 63.—Large storage batteries in glass jars for stationary service. (*Courtesy Gould Storage Battery Company.*)

Many radio broadcasting stations obtain some of their power from storage batteries. In some cases large voltage but small current is needed. For such purposes small cells, usually in glass, are assembled in cases.

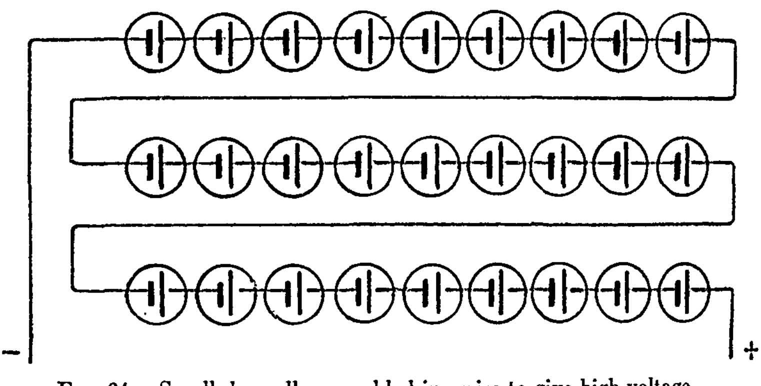


FIG. 64.—Small dry cells assembled in series to give high voltage.

Very large storage batteries may be used as sources of power on submarines especially when the submarine is submerged.

Many needs for electricity from batteries are small in terms of power requirements. Here dry cells may serve the purpose. In communities that do not have electric power lines, one or two dry cells are used for such simple services as ringing the door bell. Still smaller units are used for lighting flashlight bulbs.

For very small currents at moderately high voltages, such as may be needed for running a radio set in a place where no electrical power supply is at hand, small dry cell units are connected in series, packed into a pasteboard box, and sealed with wax. A common arrangement is 30 cells which, in series, gives approximately 45 volts. Several of these blocks may be connected in series to give the desired high voltage.

Some Important Facts

1. When two unlike electrodes are immersed in an electrolyte which reacts with at least one of them, there is built up a potential difference between the electrodes which is termed the electromotive force of the cell.

2. The collection of hydrogen bubbles on the anode "polarizes" a simple cell.

3. In the common dry cell, the zinc can is the cathode; the carbon rod, the anode; and ammonium chloride, the electrolyte. Manganese dioxide is added to oxidize the hydrogen around the anode, thus depolarizing the cell. Its e.m.f. is 1.5 volts.

4. In the secondary cell, or storage battery, the chemical action of discharging can be reversed by forcing electricity through it in reverse direction.

5. In the common lead storage cell, the anode is lead peroxide; the cathode, spongy lead; and the electrolyte, dilute sulfuric acid. Its e.m.f. is about 2 volts.

6. A lead storage battery should be charged and discharged at a moderate rate, and never completely discharged. The specific gravity should be kept at about 1.30 and distilled water added so as to keep the plates covered. Stationary storage batteries with good care should last many years. An automobile battery does well to last 2 to 3 years, due to the abuse it receives, such as vibration and the wide range of charging and discharging rates.

7. In the Edison storage cell the anode is nickel hydrate; the cathode, iron oxide; the electrolyte, potassium hydroxide; and the e.m.f. about 1.3 volts.

8. A battery not only pumps electricity through a circuit, but it adds some resistance to the circuit. It is called "internal" resistance.

9. When cells are connected in series, their e.m.f.'s are added, but so, too, are their internal resistances. When like cells are connected in parallel, there is no increase in e.m.f. but the internal resistance of one cell is divided by the number of cells.

10. Although much more expensive than generator current, battery current is used;—(1) where generator current is unavailable; (2) in emergencies when generator current fails; (3) for special voltage requirements where current is small; (4) for convenient intermittent use.

Generalization

Any primary or secondary cell consists of two dissimilar electrodes immersed in an electrolyte, and while delivering current converts chemical energy into electrical energy. A cell "stores" chemical energy rather than electricity.

Problems

Group A

1. What are the essentials of a simple or primary cell? Show a diagram and give an explanation.

2. How do you distinguish between a primary cell and a storage cell?

3. Make a list of the things to be done in order to keep a lead storage battery in good condition.

4. What is meant by "polarization" in a simple cell?

5. Try to find several reasons why taking excessive currents from a lead storage cell may damage the electrodes.

6. Describe the chemical action that takes place where a piece of zinc is put into dilute sulfuric acid, and show how the zinc becomes negatively charged with electricity.

7. What do you think stops the chemical action in the case described in problem 6?

8. How is the charge on the electrodes of a battery built up when electrons are allowed to drift from one electrode to the other through an external circuit?

9. Describe the changes in the strength of acid in the electrolyte of a lead storage cell as current is taken from the cell.

10. What serious objection is there to using a lead storage cell until it is completely exhausted?

11. How many dry cells (1.5 volts each) will be required to obtain an electromotive force of 45 volts? Indicate how they would be connected.

30.

b. If the external resistance in *a* is increased, which battery delivers more current? If this resistance is decreased, which?

c. Repeat the above procedure, using batteries of 3, 4, 5, and 6 cells. Can you formulate any general conclusions? If you can, is such a conclusion sufficiently general to apply to any number of practically identical cells?

d. Show how a general conclusion can be reached mathematically without recourse to experiment.

4. In view of what you have learned in connection with this chapter, devise a method for determining the positive and negative electrodes of a battery charging device without using the magnetic effect of current electricity.

MAGNETISM

When electric charges are in motion, they exert forces on other moving electric charges in addition to the simple forces of attraction or repulsion which we have already studied. For example, if two wires run close to one another for some distance, it is easy to see that they are drawn towards one another when there is current in both wires in the same direction. If one of the currents is reversed, the wires are pushed apart.

This effect of moving electricity is called *Magnetism*.

Many centuries before the above effect due to the motion of electricity was known, attractive and repulsive forces between some pieces of iron ore had been discovered. This effect had also been named *magnetism*, and we are familiar with such magnets now made out of steel. They are commonly called "permanent" magnets and are often named according to their shape; for example, bar magnets and horseshoe magnets.

Experiments show that forces of attraction or repulsion may easily be found between a conductor carrying electric current and a permanent magnet. These experiments suggest that the magnetism of an electric current and that of a permanent magnet may be of a similar nature. The effect in the permanent magnet can be explained in terms of moving electricity by assuming that the atoms of the magnet contain electrons and protons and that these electrons and protons are moving within each atom.

Materials (such as iron and some of its alloys) which are capable of being magnetized are often used in combination with electric currents to build various devices. Some well known examples are the electric doorbell, the telegraph sounder, the electric relay, and large electro-magnets for handling pig iron.

1.7. Forces between Electric Currents

We will now study a number of experiments to learn effect two currents of electricity may have on one anc

Two wires may be stretched loosely as shown i and connected to a battery so that the current same direction in each. If the switch is c it is easy to see that the wires are att, when electricity is flowing in them.

The connections to the wires should now be changed as shown in Figure 66 so that the current will be in opposite directions in the two wires. With this arrangement the wires will be pushed apart when the current is on, and this effect of a repellant force may be observed as easily as the attractive effect first described.

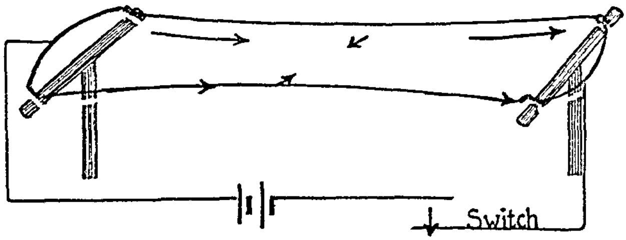


FIG. 65.—Parallel conductors carrying currents in the same direction are attracted toward one another.

A more exaggerated motion of electrical conductors may be produced by winding a loose coil and hanging it on a glass rod as shown in Figure 67. When the switch to this circuit is closed, the coil will pull together, and then, when the switch is opened, the elastic forces in the wire will make the coil

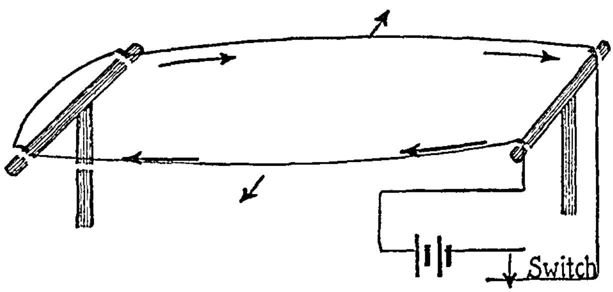


FIG. 66.—Parallel conductors carrying currents in opposite directions are repelled from one another.

spring apart. Opening and closing the switch will make the coil behave something like an accordion.

If the long wires first shown in Figures 65 and 66 are placed cross-wise to one another as indicated in Figure 68, no effects of forces pulling them together or pushing them apart can be

seen. However, there will be forces tending to turn the wires about as if to make them parallel to one another and to make the currents be in the same direction in the two wires. If the wires are allowed to turn so that they are no longer

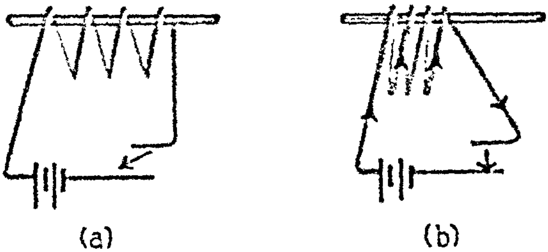


FIG. 67.—Turns of a coil are attracted to one another when electricity flows in them. (*a*) The coil with no current in it. (*b*) The coil with current in it.

perpendicular to one another, the attractive forces will be found again.

These experiments lead one to the conclusion that some kind of a force is associated with moving electricity in addition to the ordinary forces of attraction and repulsion of stationary charges about which we studied in the first chapters on electricity. This electrical effect due to the *motion* of the electricity is called *magnetism*. The space around an electric current is said to contain a *magnetic field* just as the space around any charge of electricity is said to contain an electric field.

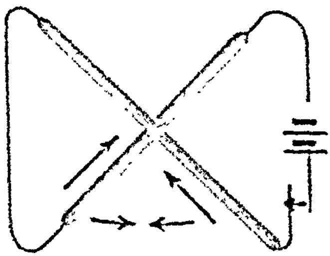


FIG. 68.—When conductors carrying current are perpendicular to one another, no forces of attraction or repulsion seem to act between them. However forces act in such a direction as to make them rotate from the perpendicular direction to a parallel direction as indicated by the two lower arrows.

An important property of a magnetic field is that any other source of magnetism (for example, another conductor with a current in it) will experience a force on itself when brought into the region in which the magnetic field exists.

2.7. Forces between Permanent Magnets

In about 600 B.C. the Greeks discovered small pieces of an ore now called "magnetite" that had the property of attracting other bits of the ore and also bits of iron. This property could be passed on from the magnetite to the bits of iron under its influence so that they in turn would attract still other pieces of similar material. This action is called magnetization by induction. Later it was learned that iron in the form of steel which can be tempered would retain this property after being removed from the magnetite. These magnetized pieces are commonly called permanent magnets.

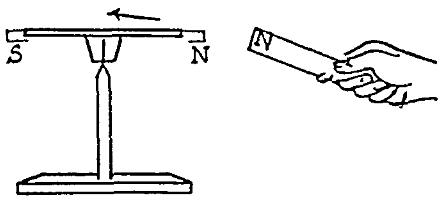


FIG. 69.—A small magnet mounted on a pivot may serve as a simple compass. When one end of a second magnet is brought near one end of the compass magnet, a force of repulsion will be found for one end of the second magnet and of attraction for the other end.

If a permanent magnet in the form of a small bar is suspended by a thread tied about its center, or if it is mounted on a pivot as indicated in Figure 69, it will point approximately in a north-south direction. When a magnet is used in this manner it is called a compass. Its value to indicate geographical directions was early recognized, and it came into general use especially in navigation.

If a second bar magnet is brought near the pivoted one in Figure 69, one can easily see that one end of the second magnet shows an attraction with one end of the pivoted magnet, and a repulsion for the opposite end of the pivoted magnet. These effects are reversed when the other end of the second magnet is used.

The phenomena described above were about all that was known of magnetism for many centuries. Not much progress

was made either in explaining magnetism or in making practical applications of the effect until after the discovery that magnetism was really of an electrical nature.

3.7. Forces between Currents and Magnets

The discovery that forces exist between a current of electricity and a permanent magnet was made by a Danish physicist, Oersted, in the year 1819.

A simple magnetic compass was lying on his lecture table while he performed experiments with electric currents. He chanced to notice that the compass changed direction whenever he closed the switch to an electric circuit which ran close to the compass. A little experimenting showed that the compass needle tended to set itself crosswise to the electric wire carrying current, and that the needle completely reversed itself when the current in the wire was reversed.

We can now compare three phenomena.

1. Electric currents exert forces on one another.
2. Permanent magnets exert forces on one another.
3. An electric current and a permanent magnet exert forces on one another.

From these experimental relations we may conclude that it is at least probable that the causes of all these forces are similar. Many modifications of these simple experiments and many more complicated experiments involving electricity and magnetism have confirmed this point of view; so that all magnetism is now considered to be electrical in nature and to involve the motion of electricity.

4.7. Electrical Nature of Magnets and Magnetic Substances

At first the idea that a piece of solid material may be able to produce effects similar to the magnetism of a current in a conductor may seem strange. Then if we remember that the atoms of which iron is made are themselves composed of electrons and protons which may be moving about, we see that the real nature of magnetism in a bar magnet and in the space surrounding a current in a conductor may quite likely be just the same. (See Figure 70.)

On this belief we conclude that every atom of iron behaves like a miniature coil of wire with current in it. When the iron is unmagnetized, the atoms are arranged in helter skelter fashion; but if they can be lined up, we would expect to get a magnetic field around the iron.

We have already learned that atoms of all elements contain protons in their nuclei and electrons in their external orbits. It seems probable that the electrons travel around the nucleus in these orbits; and it is also probable that each electron spins about its own axis. The nucleus containing the protons may also be spinning about its axis. All of these motions of electricity contribute to the magnetic characteristics of the atom.

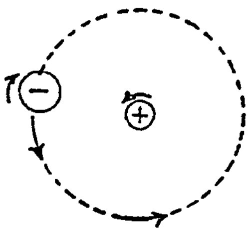


FIG. 70.—It is likely that the electrons of an atom spin about their own axes as well as move about the nucleus just as the earth spins about its own axis at the same time that it moves around the sun. The nucleus may also be spinning. All of these motions of electricity give the atom its magnetic characteristics.

It might seem that atoms of all elements could be expected to have magnetic properties if the above theory is correct, whereas only a small number of elements are known to be strongly magnetic. This state of affairs is easily accounted for if we assume that in the atoms of most elements, the motions of the individual electrons and the nucleus are in such directions as to tend to neutralize one another.

There are a number of elements in which the magnetic neutralization seems to be quite complete; for example, bismuth, also all the rare gases (helium, neon, argon, krypton, xenon, radon). A great many other substances show feeble magnetism (called paramagnetism); for example, oxygen.

Only a small number of elements show high magnetic characteristics. The best known of these is iron, but cobalt, nickel and manganese are also classed as magnetic materials. Some alloys of iron are considered superior to pure iron for magnetic purposes.

Ordinarily the atoms of a magnetic substance are arranged in hit or miss fashion. When the substance is brought into a magnetic field, the atoms tend to line up so as to produce a combined effect. (See Figure 71.) If this process takes place easily, the substance is said to have high *permeability*. Soft iron and some of its alloys have very high permeability; but they tend to lose their magnetic arrangement of atoms as soon as the magnetizing field is removed. Hence they are said to have low *retentivity*. Tempered steel in general is less permeable than soft iron, but once its atoms are properly arranged, they tend to stay in position. Permanent magnets are made from tempered steel or from alloys that have the property of high retentivity.

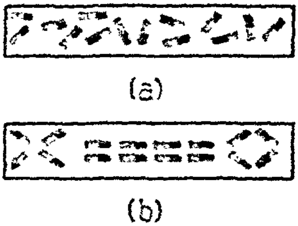


FIG. 71.—Bars of iron. (*a*) Unmagnetized. (*b*) Magnetized.

5.7. Magnetic Fields Due to Electric Currents

In the first section in this chapter we studied forces of one electric current on another. The space surrounding any electric current we described as having a property called magnetic field.

In a later section we learned that Oersted discovered that a compass in the near vicinity of an electric current would point crosswise to the current.

As one possible way in which to describe a magnetic field, we may use the behavior of a very tiny magnet mounted as a compass just as in the case of Oersted's experiment. The direction of the magnetic field will be considered as the direction in which the north seeking end of the compass points.

In Figure 72 the direction of conventional current through the conductor is indicated by arrows. If a compass is placed near the wire, it will point in such positions as are shown on the paper.

If a compass is made free to point in any direction instead of just in a horizontal plane, and if it is moved around on the

circumference of a circle drawn about the wire as center, the compass needle will tend to point along the circumference at all points. So the magnetic field surrounding current in a single conductor is often illustrated by a set of circles

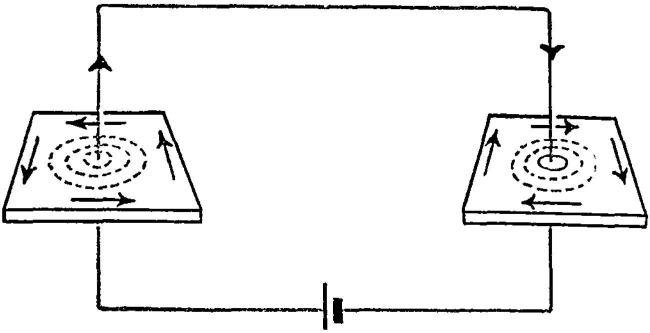


FIG. 72.—A small magnet placed in the positions of the arrows on the plates tends to set itself crosswise to the direction of a nearby current.

with direction indicated as that in which the compass would point as described above.

The small black circles in Figure 73 indicate conductors running perpendicularly into the paper. In (*a*) conventional current is flowing into the paper away from the reader. In (*b*) the current is flowing towards the reader. The dotted circles indicate magnetic field in the space around each conductor.

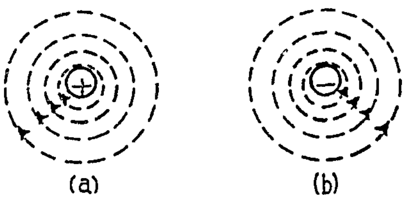


FIG. 73.—(*a*) The magnetic field around a current flowing from the observer. (*b*) The magnetic field around a current flowing toward the observer.

Of course the magnetic field around a current extends much farther away from the current than is shown by the small number of magnetic field circles in these drawings. The strength of field is often indicated by the closeness of the field lines. In Figure 73 the field is indicated in this manner to be much stronger near the currents than at more remote points.

A simple way in which to remember the direction of a magnetic field of a current in comparison to the direction of current is to grasp the conductor with the right hand in such

a manner that the thumb indicates the direction of current. The fingers then indicate the direction of magnetic field. The student may readily apply this rule to the diagrams of Figures 72 and 73.

Obviously, if two conductors as those in (*a*) and (*b*) of Figure 73 are in the same vicinity, their

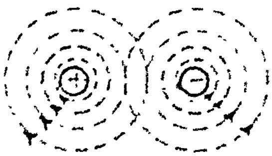


FIG. 74.—The overlapping magnetic fields of the two currents of Figure 73 when they are near one another.

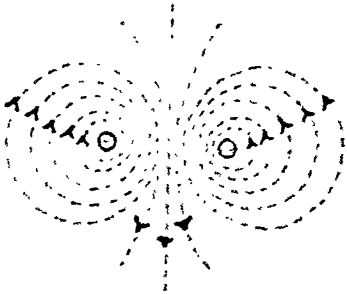


FIG. 75.—The resultant magnetic field for the case of Figure 74.

magnetic fields will overlap, and the magnetic field surrounding them will be the resultant of the two fields.

Figure 74 shows this effect when the wires of Figure 73 are placed moderately close together. Obviously, these fields aid one another in the space between the wires and tend to annul one another beyond the wires. The resultant field is shown in Figure 75.

This latter case corresponds to that shown in Figure 66 where we learned that the wires are pushed apart. With the aid of Figure 75 we can imagine that the intense magnetic field between the two wires exerts a repulsive force on the wires.

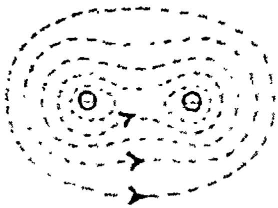


FIG. 76.—If, in Figure 74, both currents had been in the same direction, the resultant of the two over

the wires it is increased as a result of the overlapping of the two fields. See Figure 76. Again we can say that forces act tending to move the wires away from the intense fields and towards the weak field regions.

These are useful observations, but must not be taken as complete explanations of the forces between currents. The exact nature of magnetic forces, as well as the exact nature of simple electric forces, is still understood only imperfectly.

6.7. Magnetic Fields around Permanent Magnets

The magnetic field in the space about a permanent magnet may be explored by using a small compass in the manner suggested for studying the magnetic field due to an electric

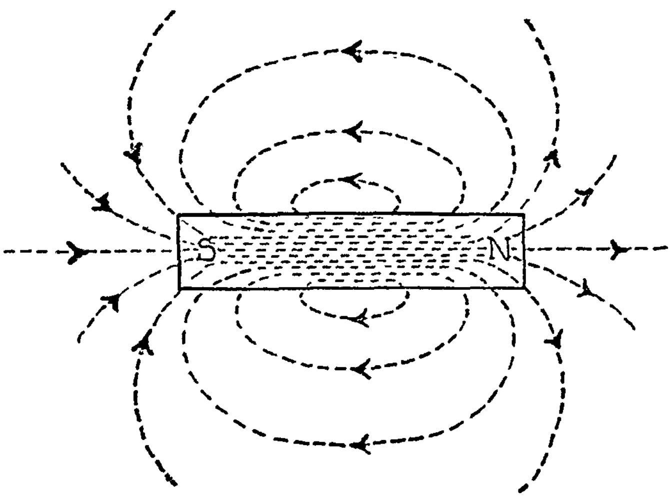


FIG. 77.—Lines and arrows to show the directions in which a compass will point in the vicinity of a large bar magnet.

current. A bar magnet can be placed on a large sheet of paper and a small compass placed at any point near it. Pencil marks may be made on the paper to indicate the position of the ends of the compass. The compass is then moved so that one end will be near the dot just made for its opposite end. A third dot is now added for the new position of this end of the compass. This process can be carried out until the whole region near the magnet has been explored. Lines con-

necting the consecutive pencil dots will represent the field and should give a graph similar to that shown in Figure 77. For

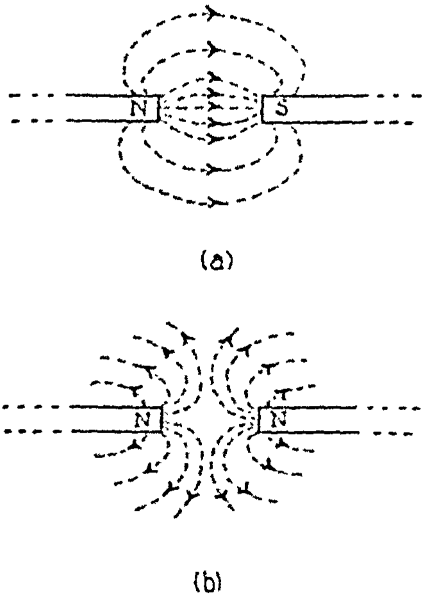


FIG. 78.—Magnetic fields for adjacent poles. (*a*) Unlike poles. (*b*) Similar poles.

points at a considerable distance from the magnet the symmetry of the field plotted may show some variations, because the small compass actually explores the combined effects of the field due to the magnet and that due to the earth.

Another way to determine the field about a magnet involves the use of iron filings. These filings may be placed on a sheet of paper held just over the magnet. When the paper is tapped gently the filings arrange themselves along the field lines. Blue print paper may be used in place of ordinary paper. With the aid of sunlight a permanent record of the position of the filings may easily be obtained.

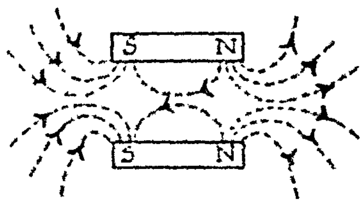


FIG. 79.—These magnets repel one another.

Either of the above experiments may be repeated with two or more magnets to determine their combined fields. In Figure 78 we find two bar magnets that are long in comparison to the distance separating their nearer ends. In (*a*) we find the opposite kind of poles near one another and in (*b*) similar poles are placed near one another. In case (*a*) we learn by experience that attractive forces exist, and similarly in case (*b*) we find the forces to be repulsive. The shapes of the magnetic fields as found either with a compass or with the aid of iron filings are indicated in Figure 78.

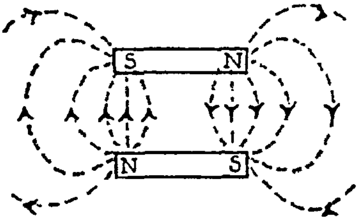


FIG. 80.—These magnets attract one another.

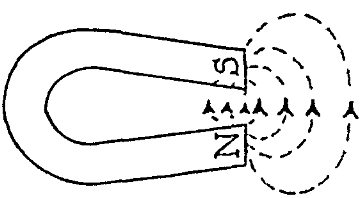


FIG. 81.—A permanent magnet is frequently made in the shape of a horseshoe.

Other combinations of magnets are shown in Figures 79 and 80.

Figure 81 shows a permanent magnet bent into the form of a horseshoe. Magnets in this form are used extensively in the electrical industries and may also be obtained as toys. Because the opposite poles of the magnet are close together intense magnetic fields may be obtained by this arrangement, as indicated in the figure.

7.7. Magnetic Poles

The shapes of the magnetic fields just described are what one would expect if these fields were due to charges of something or other placed near the ends of the magnets where the field lines converge. In an earlier section in this chapter we learned that the magnetic effect in materials was due to the motions of electrons and protons in the atoms; but long before anything was known about the nature of atoms, early scientists

jumped to the conclusion that there really was a charge of something near each end of a magnet.

These charges were called magnetic "poles"—the term poles being used because there were always two of them, one at each end of the magnet.

We have already seen that if such a bar is suspended at its center, it will point in a north-south direction. From this effect the assumed charges of magnetism were named "north seeking" pole and "south seeking" pole respectively. These names are often shortened to "north" pole and "south" pole.

If two bar magnets are placed as shown in Figure 78*b* or in Figure 79 they are found to repel each other, while in the

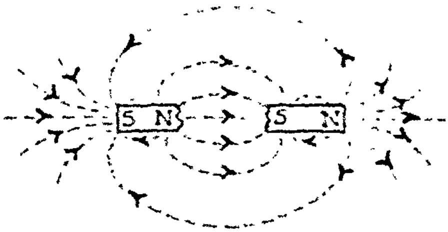


FIG. 82.—When a bar magnet is broken, new "poles" appear on each side of the break.

positions shown in Figure 78*a* or in Figure 80 they attract one another. These experiments increase our belief that the two ends of a magnet have unlike poles, that like poles repel one another and that unlike poles attract one another.

If a bar magnet such as we have been discussing is broken in the middle as shown in Figure 82 new "poles" appear as indicated. This experiment adds to our belief that a "pole" of a magnet is not really a charge of anything, but is an effect due to the arrangement of the magnetic atoms of the material.

Sometimes magnetic effects due to currents in electrical circuits are called electromagnetism and effects due to magnetic materials are called simply magnetism.

8.7. Terrestrial Magnetism

One easy way to account for the fact that a suspended bar magnet will point in a north-south direction is to assume that

the earth itself is a huge magnet with one magnetic pole in the general location of the geographic north pole and the other near the geographic south pole. Actually we do not know to what the magnetic field of the earth is due.

Since experiment shows that unlike poles attract one another, we must conclude that the earth's magnetic pole in the vicinity of the geographic north pole must be a south pole. Similarly a north magnetic pole must be located near the earth's geographic south pole.

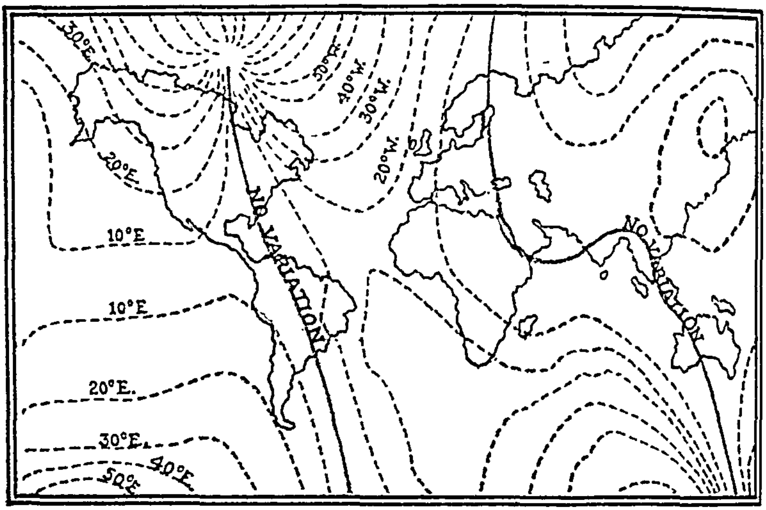


FIG. 83.—Isogonic map of the earth showing the deviation of the magnetic north from the true north.

These magnetic poles do not appear to be located exactly at the geographic poles, so that for most points on the earth, a magnet will point either east or west of true north. The deviation from true north is called magnetic "declination." Not only does it vary from place to place, but it also varies from time to time at a given place. Hence a map showing magnetic declination must be revised every few years. On such a map, all places having the same declination are connected by a line, called an isogonic line. The line connecting all places of zero declination is termed an agonic line. These

declination maps are frequently called isogonic maps or charts. (See Figure 83.)

An ordinary compass is mounted so as to swing only in a horizontal plane. Actually, at nearly all geographical locations, the earth's magnetic field makes an angle with the horizontal as indicated, for example, in Figure 84. Therefore, in the northern hemisphere, if a small magnet is suspended at its true center of mass, its north end will point downward, while in the southern hemisphere its south end will point downward. This effect is known as "dip" or "inclination."

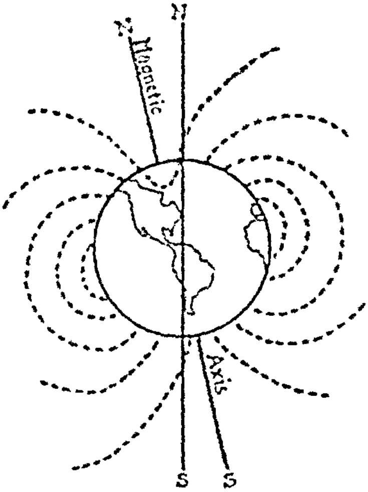


FIG. 84.—The earth's magnetic field in the northern hemisphere and also the southern hemisphere dips downward as compared with the horizontal. The dotted lines show the field directions.

The conception of the earth itself behaving as a huge magnet was first advanced by William Gilbert in the latter half of the 16th century.

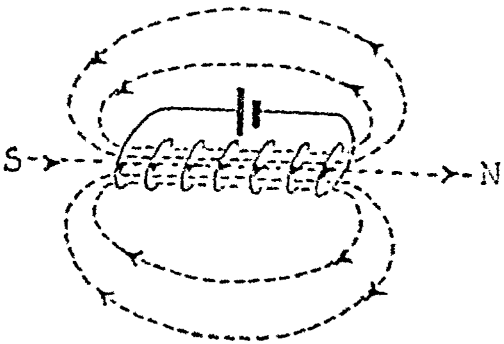


FIG. 85.—The magnetic field due to current in a long coil. The arrows on the turns of the coil indicate the direction of conventional current. The dotted lines indicate the field as it could be determined with the aid of a small compass.

9.7. Electro Magnets

Figure 85 shows a coil of wire in which the conventional direction of current is indicated by arrows on the turns of the

coil. If we grasp any turn of this coil with the right hand in such a manner that the thumb indicates the direction of current then the fingers indicate the direction of the magnetic

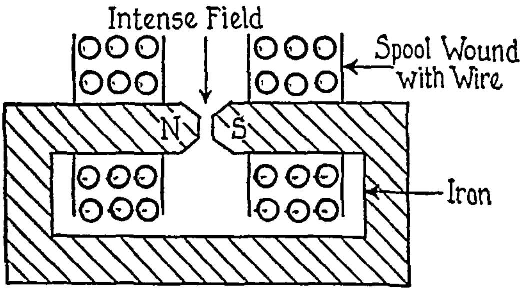


FIG. 86.—A large electromagnet such as is used in scientific laboratories. Coils of wire are wound on iron.

field. This field direction for the coil is also shown in the figure.

If a piece of iron or other magnetic material is placed inside this coil its atoms can be oriented by the magnetic field of the coil, and so the substance becomes a magnet. Ordinarily soft iron or other materials having high permeability and low retentivity are used in such systems. The effect is often called electromagnetism to distinguish it from the behavior of permanent magnets.

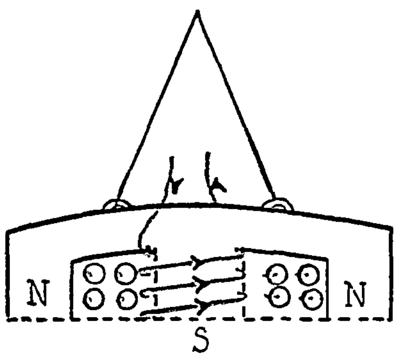


FIG. 87.—A form of electro-magnet used to handle pig iron and iron products.

A bar of iron inside the coil of Figure 85 would be magnetized with a north pole at the end of the coil marked N in the figure and a south pole at its opposite end.

Electromagnets are made in various shapes and sizes depending on the uses to which they are to be put. Figure 86 shows a form commonly used in laboratories where an intense field in a small space is needed. Sometimes these magnets

are made quite large, the iron of the magnet and the copper wire often weighing half a ton or more.

Figure 87 shows a modified form of such a magnet which may be used for picking up iron. Pig iron is loaded and unloaded by means of such a magnet attached to a crane. When there is current in the coils of this magnet, large forces of attraction hold the pieces of iron. When the current is stopped by opening a switch, the iron falls.

10.7. The Electric Doorbell

Small electro magnets may easily be made by winding coils on iron. These magnets have many uses, one of which is in

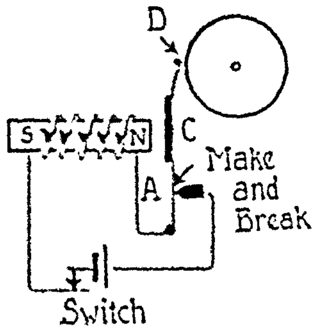


Fig. 88.—An electric doorbell.

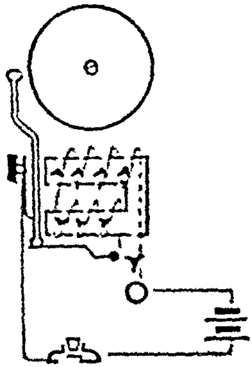


Fig. 89.—A horseshoe type electric magnet used with a doorbell.

the operation of an ordinary doorbell. Figure 88 shows such an electromagnet. A strip of spring brass, *A*, is fastened at one end, and has a piece of iron, *C*, attached to it just opposite the ends of the electromagnet. When there is current in the coil the iron is attracted to the iron core of the magnet. This motion causes the make and break switch to open, so that the current stops and the springiness of the strip, *A*, pulls the iron, *C*, back. At the same time the clapper, *D*, hits a bell. The contact switch also closes so that the current is started again. The process repeats at a rate depending on the strength of the pull on the iron, *C*, the stiffness of the spring

strip, *A*, and the mass of this strip and the parts attached to it. The moving system is called the armature.

Electromagnets for door bells are often shaped somewhat in the form of a horseshoe magnet. This arrangement is shown in Figure 89. Since both a north pole and a south pole of the electromagnet are brought near the iron on the movable arm, greater forces are exerted on this arm for a given size of electromagnet than in the case of the single bar magnet type shown in Figure 88.

11.7. Telegraph Instruments

Instruments for sending messages by telegraph are made along lines similar to those described above for the electric doorbell. Figure 90 shows an electromagnet mounted with the coils vertical, although they could also be mounted horizontally as shown in Figure 89. The device is called a telegraph "sounder." The essential differences as compared to the doorbell system are, first, that the bell is omitted and clicks are made as the armature strikes metal stops, and second, there is no make and break switch on the armature.

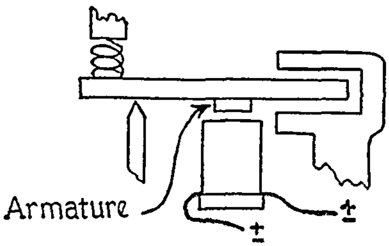


FIG. 90.—A telegraph sounder.

A key and sounder may be connected with a second key and sounder as shown in Figure 91, where the ground has been substituted for one wire. When the distance between key and sounder is great, the use of the ground for one side of the electric circuit may represent an appreciable saving in cost of installation.

When large distances are involved, the current that can be sent through a key and sounder system with batteries of reasonable size is small. The clicks produced by the sounder are then too feeble.

To remedy this condition still another instrument similar in design to the electric doorbell is used. It is called a

Some Important Facts

1. Two parallel conductors attract each other if their currents are in the same direction, but repel each other if their currents are opposite in direction.

2. Magnetism was discovered as a property of lode stone in about 600 B.C.

3. Oersted, in 1819, discovered the magnetic field about an electric current to be of the same nature as the field about a permanent magnet.

4. In some atoms, such as those of iron, cobalt and nickel, the motion of the planetary electrons is considered as constituting an electric current and hence generating a magnetic field. When such magnetic atoms are similarly aligned, the substance is magnetized.

5. The end of a bar magnet tending to point geographically north is called the north pole of the magnet; the end tending to point geographically south is called the south pole of the magnet.

6. The direction of any magnetic field is considered that indicated by the north end of a small magnet free to orient itself in the field.

7. The magnetic field of an electric current in a long straight wire may be represented by concentric circles in the space about the wire using the wire center as a center for the circles.

8. The magnetic field of a permanent magnet is of such a shape as to suggest the presence of charges of magnetism near the ends of the magnet. These apparent charges of magnetism have been called magnetic poles.

9. A magnetic field is found all over the earth and its presence suggests that the earth may be a huge magnet.

10. Iron or other easily magnetized materials are often used in connection with coils carrying electric currents for making various devices including magnetic cranes, doorbells, telegraph sounders, relays, and many others.

Generalization

Moving electricity, either as a current in a circuit, or as the motion of individual electrons in atoms, is the cause of magnetism.

Problems

Group A

1. If the electrons in two long parallel wires move in the same direction in each wire, what motion of the wires is observed?

2. If the electrons in one of two long parallel wires move in one direction and those in the other wire move in the opposite direction, what motion of the wires is observed?

3. What was one of the first practical uses to which a bar magnet was put?

7. If the magnetism of the earth is due to something like a large magnet, what kind of magnetic pole must there be near the geographic north pole? Why?

8. Imagine yourself to be looking along a conductor in the direction in which current is flowing. The conductor points north and south. A magnetic compass is placed beneath the wire. Which way will the north seeking end of the compass move? If the compass is now placed above the wire, which way will the north seeking end move?

9. For what reasons is a magnetic crane especially useful?

10. Make an actual wiring diagram for a doorbell installation showing the location of the doorbell, the push button, and the battery, as they would be arranged in a house.

11. Make a drawing similar to that of question 10 but with a push button at each of two doors to operate the same doorbell.

12. Make a drawing for a doorbell system for two doors where a different bell is sounded for each door but where the same battery is used for each bell.

13. Make a list of the similarities and differences in the construction of doorbells, telegraph sounders, and relays.

Experimental Problems

1. Given two bar magnets, a horse-shoe magnet, sufficient blue-print paper, and iron filings, plot the following magnetic fields:

   a. A single bar magnet.
   b. Like adjacent poles.
   c. A single horse-shoe magnet.

2. Repeat Experiment 1 using a compass and a plain sheet of paper instead of iron filings and blue-print paper.

3. Using iron filings, magnetic compasses, sufficient insulated wire, soft iron rod and dry cells, determine concerning the magnetic field around an electric current:

   a The relation of current direction to the direction of lines of magnetic force.
   b The factor determining the strength of a magnetic field.

4. Using a demountable electric bell, source of current and suitable connections, trace the circuit, so as to explain the intermittent action, and alter the circuit so as to make a single stroke or gong type of bell.

5. By means of compasses and dipping needle, determine as accurately as possible the compass declination and dip. Compare experimental results with those given on isogonic charts.

6. Attempt to magnetize a steel rod by pounding it when it is held approximately tangent to the earth's magnetic lines of force. Also, demagnetize and reverse the polarity of the steel rod.

7. Given two telegraph sounders, two relays, two keys, four dry cells and sufficient insulated wire, arrange apparatus as a working two station, two way telegraph system.

Chapter 8

ELECTRIC MOTORS AND METERS (FOR DIRECT CURRENT)

The forces between electric currents or between an electric current and the field of a magnet can be used to do mechanical work. A device arranged to rotate under the influence of these electromagnetic forces is called an electric motor.

It is also possible to make a device in which the electromagnetic forces turn a pointer against the resistance of a spring. Such a piece of apparatus can be used for measuring electric currents or voltages and is called an electric meter.

1.8. Electric Motors

We have seen that there are mutual forces (1) between pieces of magnetized iron, (2) between conductors carrying electric currents and (3) between magnetized iron and conductors carrying electric current. These forces tend to move the conductor or the iron if either is free to move, and such motions may easily be seen as described in the preceding chapter.

It would seem that with a small amount of inventive genius, one might arrange a system of conductors and magnets so as to produce continuous motion. A simple arrangement can be constructed as follows.

In Figure 93 we have an iron magnet which may be a permanent magnet or it may be made of soft iron and magnetized by means of current in the coil, A. A strong magnetic field, H, will exist between the poles of the magnet, NS. A coil, B, is mounted on a horizontal axis so that the sides c and d are at right angles to the magnetic field. When electricity flows through this coil, B, forces will act on c and d tending to move one up and the other down. The coil will turn until one of these wires is at the top of the path and the other at the bottom.

If the current connections to the coil *B* are reversed, the side which was forced upward at first will now have a force acting downward on it and the other side will have an upward force on it.

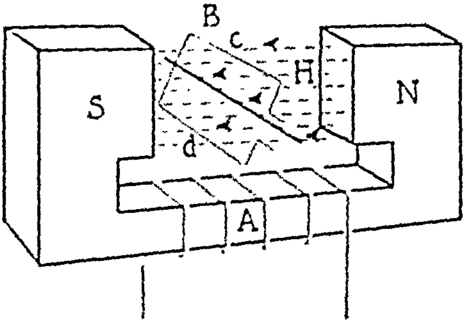


FIG. 93.—A simple direct current motor.

So it follows that the coil, *B*, will revolve provided the current is reversed every time the turns *c* and *d* reach the up and down limits of the motion. Reversing the current by hand would be a great nuisance even if one could do it, so by exercising a little more inventive genius one can make the system do its own current reversing.

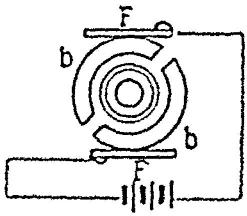


FIG. 94.—A commutator with sections, *b*, *b*, and with contact brushes, *F*, *F*.

The end of the shaft on which the coil, *B*, is mounted is shown in Figure 94. The sections, *bb*, are made of brass or copper and are insulated from the shaft. The ends of the wire in the coil *B* are fastened to these sections. Spring strips of brass, *FF*, touch these sections as shown. As the coil revolves, the contact between strip and section changes from one section to the other in such a manner as to reverse the current. The assembly of sections, *bb*, is called a commutator and the contacting strips are called the "brushes."

A simple motor of this type can be built in any physics laboratory and hence some first hand knowledge of the operation of a direct current motor may be obtained.

The coil A of Figure 93 is called the field coil; the coil B is called the armature. There are various ways in which these coils may be connected to a source of electric power. If each coil is connected directly to the source of power the motor is said to be "shunt" connected. If one end of coil A is connected to one end of coil B and the combination connected to the power source, the arrangement is called a "series" wound motor.

In some motors there are two field coils. One of these may be connected in series with the armature coil as in a series wound motor and the other field coil may be connected independently to the power source. This arrangement is known as a "compound" wound motor. Compound winding is fairly common since it permits of easier adjustment of the operating characteristics of the motor than the simpler series or shunt connections.

In large motors a number of coils are used in place of the single coil, B, and a corresponding increase is made in the number of sections in the commutator. Brushes are often made of graphite instead of brass and are held against the commutator by separate springs. The connections to the coils in a large motor are more involved than in the case of the simple motor described here and students who are interested in this subject should look in elementary electrical engineering texts or in an encyclopedia.

2.8. Electric Meters

The same arrangement shown for the moving coil, B, of a simple motor as illustrated in Figure 93 may be changed slightly for use as a current measuring meter.

Suppose that a flatly coiled spring similar in appearance to the escapement spring in a watch is fastened to the coil so as to oppose its motion when electricity is flowing in it. Such a device is pictured in Figure 95. The amount that the coil, B, turns will depend on the current in the coil, for it is the force on the current in the magnetic field that provides the turning torque.

Of course no commutator is used with this arrangement, and the coil is not expected to turn through a very large angle. A permanent magnet is used and the pole ends may be shaped as shown in Figure 96. A piece of soft steel is sometimes fastened so as to be located in the middle space of the coil. Then the magnetic field in which the coil moves is

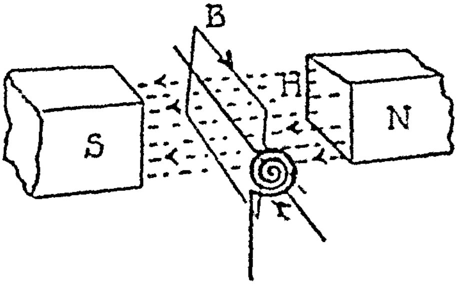


FIG. 95.—A coil that can turn in a magnetic field can be used as an electric meter. A coiled spring (shown in black) opposes the turning of the coil.

strengthened. Such a piece of iron is indicated at *A* in Figure 96.

An electric meter employing a moving coil in a magnetic field due to a permanent magnet is called a D'Arsonval type galvanometer. It is quite sensitive; that is, small currents will move it quite readily unless too strong a spring is used.

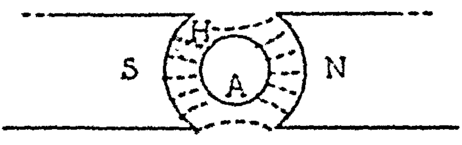


FIG. 96.—Specially shaped pole pieces and an additional section of iron, *A*, are often used to give a magnetic field of the type shown here for use in electric meters.

For very small currents, the coil is fastened to a light metal ribbon and suspended vertically. See Figure 97. A mirror is placed on the coil holder so that a spot of light may be reflected on a scale, or a printed scale may be read through a small telescope after reflection from this mirror, or a pointer may be attached as shown in Figure 98. Such a galvanometer, although very sensitive, is not very rugged.

and is usually mounted permanently on something solid, like a wall.

Most portable galvanometers have their coils mounted on pivots working in jeweled bearings. They employ the escapement type of spring described above. A metal pointer, usually made of aluminum, is fastened to the coil and as the

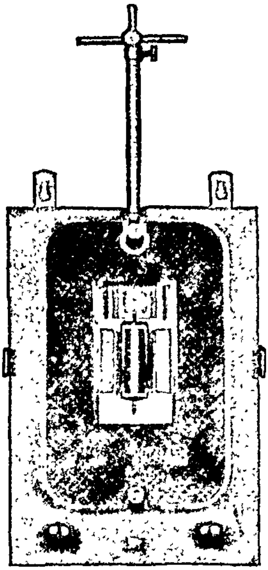


FIG. 97.—Galvanometer for wall mounting. The coil is suspended by a long metal ribbon running up through the tube shown. A small round mirror mounted above the top of the coil indicates the motions of the latter. (*Courtesy Leeds and Northrup.*)

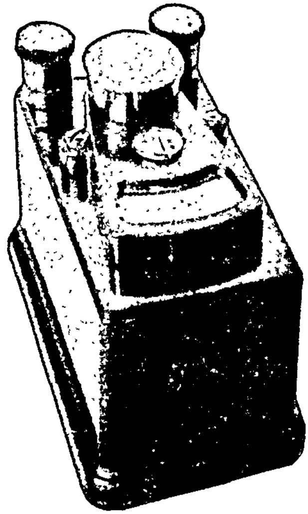


FIG. 98.—Galvanometer for table use. The coil is suspended from a short fiber in the rear. A pointer attached to the moving coil indicates the motion of the coil. (*Courtesy Leeds and Northrup.*)

coil turns, the end of the pointer moves over a printed scale. See Figure 99.

3.8. Voltage and Current Sensitivity of Galvanometers

This D'Arsonval type of galvanometer movement is used quite generally in all of the better grade of ammeters and voltmeters for direct current measurements.

The galvanometer itself may be thought of as both an ammeter and a voltmeter. It measures small currents, but if the resistance of the coil is known the voltage that exists across the coil for any known current can be found from Ohm's Law:

$$\text{Volts} = \text{Ohms} \times \text{Amperes}$$
$$E = RI$$

(See page 41, equation 1.)

For example, if a current of 0.001 ampere gives a deflection of one scale division on a galvanometer whose coil resistance is 100 ohms, the voltage across the meter must be

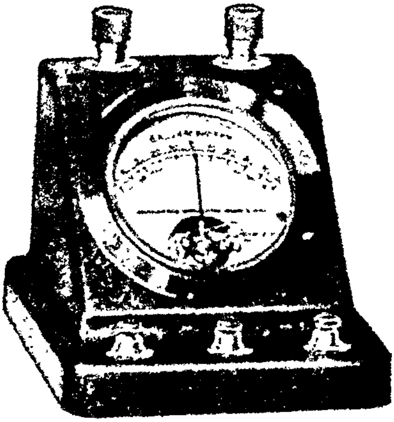


Fig. 99.—A table type galvanometer in which the coil is mounted between pivots. (*Courtesy Weston Electrical Instrument Corp.*)

$$E = RI = 100 \times 0.001 = 0.1 \text{ volt}$$

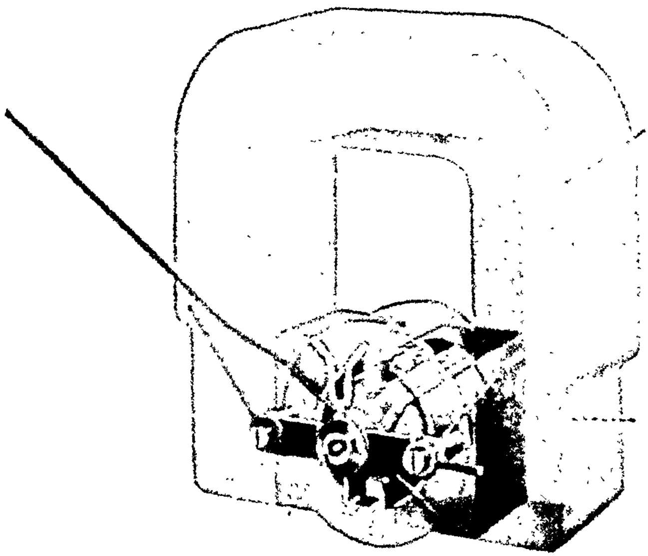


Fig. 100.—Interior construction of a pivot mounted moving coil type electric meter. (*Courtesy Weston Electrical Instrument Corp.*)

So this galvanometer may be thought of as an ammeter with a sensitivity of 0.001 ampere per scale division and also as a voltmeter with a sensitivity of 0.1 volt per scale division.

4.8. Voltmeters

If it is desired to have a voltmeter reading 1 volt per scale division the following arrangement with the galvanometer described above may be used. In Figure 101 a resistance, R, is shown in series with the galvanometer. When 1 volt exists across the points AA the meter will read one scale division provided the resistance, R, in series with the galvanometer resistance, is sufficiently large.

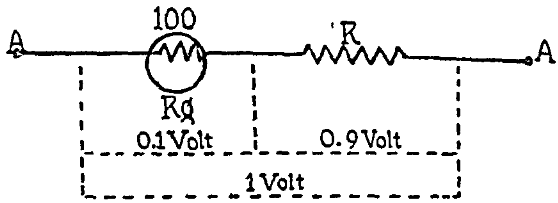


FIG. 101.—A galvanometer may be converted into a voltmeter by adding a series resistance.

There will then be 0.1 volt across the galvanometer and 0.9 volt across the resistance, R. Since the current through the galvanometer for a deflection of one scale division is known to be 0.001 ampere there must be a current of 0.001 ampere through R and by Ohm's law

$$R = \frac{E}{I} = \frac{0.9}{0.001} = 900 \text{ ohms}$$

Another way to look at this problem is to see that if the current through the galvanometer and through the series resistance are the same, as they must be, the series resistance will have the same ratio to the galvanometer resistance as the corresponding potential differences. In this case we may write, in symbols,

$$\frac{R}{R_g} = \frac{E}{E_g}$$

or

$$\frac{R}{100} = \frac{0.9}{0.1}$$

From which

$$R = 900 \text{ ohms}$$

5.8. Ammeters

Galvanometers usually give full scale deflections for very small currents, whereas it is often desired to measure relatively large currents. If the current range of a galvanometer is to be increased, a shunt resistance may be placed across it so that a large part of the current to be measured goes around

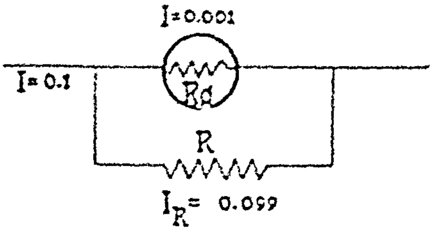


FIG. 102.—A galvanometer may be converted into an ammeter by adding a shunt resistance.

the galvanometer instead of through it. This arrangement is shown in Figure 102.

Suppose that the galvanometer of the above example is to be used and suppose that we want one scale deflection to represent 0.1 ampere. This current must be divided so that only 0.001 ampere goes through the galvanometer and the shunt must carry the difference, which is 0.099 ampere.

When two conductors are in parallel, (in this case the galvanometer and the shunt resistor) the voltage across each of the two is identical, and so the currents through them are inversely as the resistances (as may be proved by using Ohm's Law). So we write

$$\frac{R}{R_g} = \frac{I_g}{I_s}$$

or

$$\frac{R}{100} = \frac{0.001}{0.099}$$

From which

$$R = 1.01 \text{ ohms}$$

Sometimes a galvanometer is built with a number of series and shunt resistances in the same box so arranged that by means of different binding posts or several switches the same moving element may serve for a voltmeter of a number of ranges and also an ammeter of various ranges.

Some Important Facts

1. A machine for converting electrical into mechanical energy is an electric motor.

2. The operation of an electric motor depends on forces experienced by electric currents in conductors lying in magnetic fields.

3. If current carrying conductors and magnetic fields are arranged so that the amount of motion of the conductors is proportional to the current in them, the arrangement is called an electric meter.

4. When a high resistance is placed in series with a current sensitive meter, the device may be calibrated as a voltmeter.

5. When a shunt conductor is placed around a current sensitive meter, the device may be calibrated as an ammeter.

Generalization

When a conductor carries current across a magnetic field, forces exist which can be made to produce motion between the conductor and the field. The arrangement can be used for converting electrical into mechanical energy continuously as in the case of the electric motor, or it can be used for the measurement of electric current or voltage.

Problems

Group A

1. How are the magnetic forces described in the previous chapter adapted to produce circular motion as in the cases of motors and meters?

2. Why is a commutating switch needed on a direct current motor?

3. Show by means of a labelled diagram the action of a galvanometer.

4. Redraw the diagram in Problem 3 so as to show the action of:

a. An ammeter.
b. A voltmeter.

5. Suggest two or more ways of arranging a spring to restrict the motion of the coil in the case of a simple electric meter.

6. Show that a galvanometer may be thought of either as a current or a voltage measuring device.

Group B

1. A galvanometer with a coil resistance of 50 ohms gives a deflection of one scale division for a current of 0.01 amperes. How many volts are there across this meter when the deflection is one scale division? 0.5 volt.

2. Show how to make a voltmeter out of the galvanometer of Problem 1. It is to read 1 volt per scale division. 50 ohms.

3. Show how to make an ammeter out of the galvanometer of Problem 1. It is to read 1 ampere per scale division. 0.505 ohms.

4. A galvanometer with 100 scale divisions gives full scale deflection for 0.001 ampere. How much current does each scale division represent? 0.00001 ampere.

5. The coil of the galvanometer in Problem 4 has 250 ohms resistance. What voltage across the galvanometer will give full scale deflection? 0.25 volt.

6. Make a voltmeter out of the galvanometer of Problems 4 and 5 which will give 500 volts full scale deflection. 499,750 ohms.

7. Distinguish with the aid of labelled diagrams; shunt, series and compound wound direct current motors.

Experimental Problems

1. Using any available materials, such as iron, cardboard, insulated wire and dry cells, construct, operate and explain a simple direct current motor:

   a. With a shunt field.
   b. With a series field.

Compare these home made motors with similar commercial ones and account for the chief structural differences.

INDUCED ELECTRIC CURRENTS

Experiments show that when a current is started or stopped or simply moved in the neighborhood of a closed conductor, electric currents are induced in this conductor and it experiences forces that tend to move it.

A description of experiments that may be done with simple apparatus is given in the first part of this chapter. The latter part of the chapter describes methods for applying the results of these experiments to the building of electric generators and electric motors.

1.9. Constant Currents and Changing Currents

In the last two chapters the magnetic effects described were those due to steady electric currents or to the atoms in magnetic materials such as iron. The thing that could be detected in each case was a force, or the motion of a conductor or magnet due to this force.

In this chapter we are to study magnetic effects of currents or magnets when the magnetic fields vary either because the currents get larger or smaller or because the magnets or conductors are moved about. These chapters may be compared by saying that Chapters 7 and 8 covered the magnetic effects of steady currents and this chapter describes the magnetic effects of currents varying in amount or position.

2.9. Induced Currents and Forces on Conductors

Figure 103 shows a coil, A, suspended on a long thread. Coil B is placed parallel to A and close to it. When the switch K is closed, sending current through B, coil A will show a slight tendency to move away from B. When the key is opened, so that the current in B dies down and stops, the coil A moves slightly towards B. These effects are small and are hard to observe unless much care is taken.

The attraction and repulsion of A for B could be explained according to Chapter 7 if there were an electric current

in A in the opposite direction to that in B when the switch is first closed and if the current in A is in the same direction as that in B as the switch is being opened.

We can look for such a current by opening the coil A and connecting it to a sensitive galvanometer as shown in Figure 104. The galvanometer readily shows that such currents actually exist in coil A while the current is starting and stopping in B although these currents die out very quickly once the current in B is either established or completely stopped. Although the forces between the coils in the experiment suggested in Figure 103 are hard to observe, it is a simple matter to detect the currents with a galvanometer as shown in Figure 104.

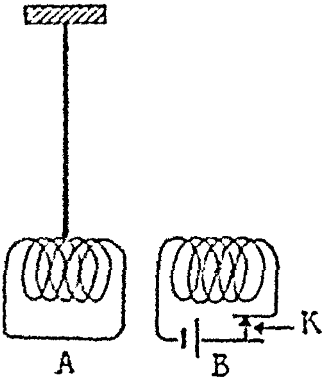


Fig. 103.—Starting and stopping a current in coil B makes coil A move from or towards coil B.

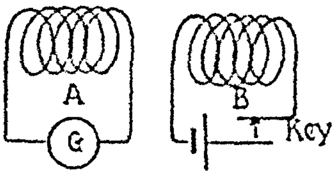


Fig. 104.—Starting and stopping a current in coil B produces currents in coil A.

The experiment may now be repeated in a slightly different form by leaving the switch to the coil B closed and by moving this coil towards and away from A. Again induced currents are easily detected in A, the direction being opposite to that in B as B approaches and being in the same direction as in B when B moves away.

Still further observations may be made by substituting a bar magnet for the coil B. The directions of induced current in A will now depend not only on whether the magnet is moved to or from A but also on which end of the magnet is nearest A. (See Figure 105.)

These currents are called "induced" because they are due to things happening nearby but without having any actual

contact with the conductor. If currents are induced in a conductor such as the coil A, it must also be true that electromotive forces are induced in it since an electromotive force is needed to cause the flow of electricity in a conductor.

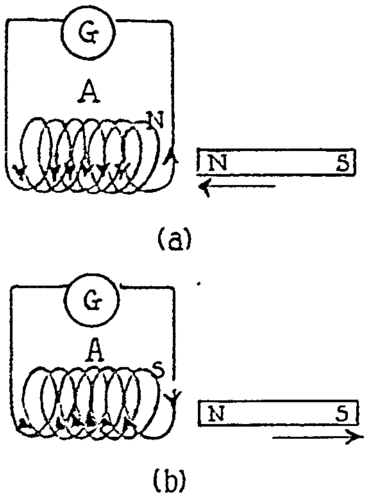


FIG. 105.—Electric currents may be induced in a coil by moving a bar magnet near it. (*a*) Bar magnet being inserted. (*b*) Bar magnet being withdrawn.

3.9. The Nature of Induced Currents

Induced currents like all other electric currents have magnetic fields. The direction of an induced current is always such that its magnetic field tends to oppose the change in the magnetic field that is inducing the current. For example, in the first case (see Figure 103) where the key is closed to coil B, the current in A is in the opposite direction to that in B and hence produces a magnetic field through A opposite to the field that B is trying to establish.

When the induced current in A dies out, the field of the current in B penetrates through the coil A. When the switch is opened in an attempt to stop the current in B, the induced current in A is in the same direction as that in B so that it tends to establish a magnetic field through itself in the same direction as the field which is being removed.

In each case, the induced current is in such a direction as to try to keep the magnetic conditions in its own neighborhood in an unchanged condition. This description of the phenomenon of induced currents is sometimes called Lenz's Law.

If a coil could be made without any resistance, a current induced in it would not die out, but would continue forever. For example, in the first experiment described above, the current in A would keep up until the switch was opened in B.

lamps or a toaster or some other electrical device we would expect to get electrical energy out of the coil.

The boy might not think much of the job of holding a heavy magnet and moving it to and fro; and moreover he probably would not maintain a steady to and fro motion.

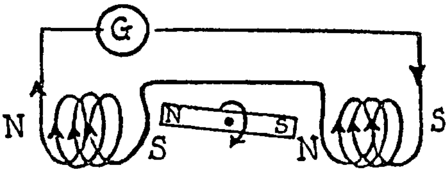


FIG. 106.—An electric generator with a revolving magnet.

So we might improve the arrangement by mounting the magnet on an axle with a crank attached and by winding two coils and arranging them as shown in Figure 106. If the windings in one of the coils are reversed as compared to those in the other, the electromotive force of the two coils will be in the same direction in the electrical circuit even though the

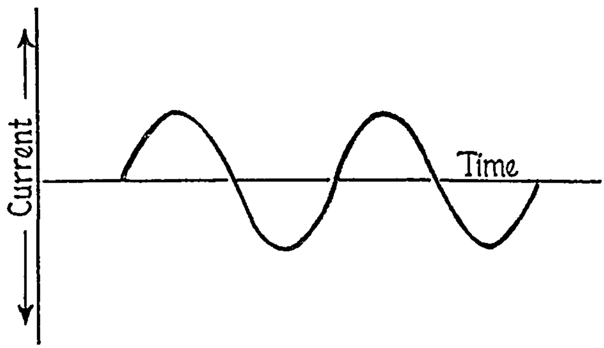


FIG. 107.—A graph to show the variation of current magnitude and direction in the generator of Figure 106.

north end of the magnet approaches one coil while the south end approaches the other. The to and fro direction of the electric current that this generator can supply may be indicated as in the graph of Figure 107, provided the magnet is turned uniformly.

Of course, since it will not prove very satisfactory to have a boy provide the energy for turning the magnet, we may

mount a pulley on the axle of the magnet and attach it to a water wheel or a gasoline engine or some other source of steady power.

In any case we see that the scheme is one for putting in mechanical energy and for obtaining electrical energy.

As a further change in the arrangement of our generator we might try making the magnet stationary and letting the coils revolve. Figure 108 shows this arrangement. The principal difficulty is now to make contact with the ends of the revolving coil. This is done by mounting two conducting

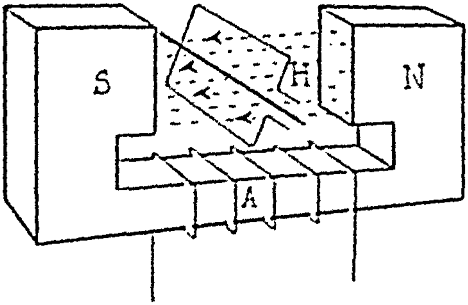


FIG. 108.—An electric generator with a stationary magnetic field and revolving coils.

rings on the revolving shaft and insulating them from it. One end of the coil is fastened to one ring and the other end of the coil to the second ring. Brushes similar to those described on page 471 for a direct current motor make contact. The rings are called "slip rings."

In Figure 108 the magnet may be a permanent one of proper shape or it may be made of soft steel which can be magnetized by a direct current through a coil wound about it as shown. In very tiny generators permanent magnets are sometimes used, while in all large generators the magnets are of the electromagnet type. For such magnets direct current must be supplied from some other source.

6.9. Frequency of Alternation of Current

In the generators described above one complete to and fro motion of induced current occurs for one revolution of the

rotating part. In large generators that revolve at high speeds this same arrangement is sometimes used.

Generators driven by water power usually run slowly and there are a number of electromagnets and a number of coils so arranged that several complete cycles of alternating electromotive force are developed for one revolution of the rotor.

Generators that have the magnet or electromagnet revolve are described as having a rotating field. If this magnet stands still as shown in Figure 108, the field is said to be stationary.

The windings in which the currents are induced are called the armature no matter whether they are stationary as shown in Figure 106 or whether they revolve as pictured in Figure 108.

Small generators such as those with which most of us are familiar usually have stationary fields and revolving armatures. Large generators such as those used in great power plants usually use the other type with the revolving field and the stationary armature.

7.9. Direct Current Generators

The output current or voltage of a simple alternating current generator of any of the types described above may be

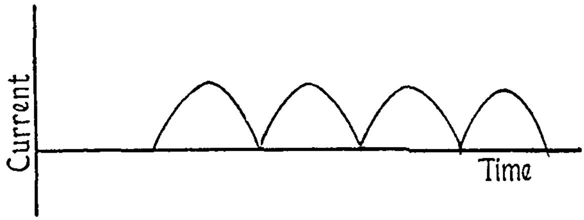


FIG. 109.—The commutated output of a simple electric generator gives pulses of current in one direction in contrast to the alternating current produced by the generator without commutation.

represented by the graph of Figure 107. The curve above the middle line represents either currents or voltage in one direction and the curve below the line represents current or voltage in the opposite direction.

If a switch could be arranged to reverse the output of such a generator just as the induced voltage reverses, the voltage

curve would appear as shown in Figure 109. Instead of using a hand operated switch, one can build a switch like the commutator described for a simple direct current motor on page 471. Since the voltage and current supplied by such a system would always be in the same direction, the device would be called a direct current generator.

Of course the voltage would be far from steady, as is obvious from Figure 109. However, it is possible to build a generator with several coils and with a commutator having many sections so that the output voltage and current are quite steady.

Any student interested in the construction of either alternating or direct current generators should consult additional text books on this subject.

8.9. The Work Done by Generators

The student should notice that all of these generators are devices whereby energy of a mechanical nature can be converted into energy of an electrical nature. He should also notice that a generator does not "make" electricity. It is simply a device in which, at the expense of mechanical energy, electrons already in the conductors of the device can be forced to drift out through an external circuit and back into the other end of the conductor in the generator. Of course it is not important whether or not it is the same electrons that make the complete trip so long as the same number enter one end of the coil in the generator that leave the other end.

In the case of alternating current, the electrons barely get started drifting in one direction until they are stopped and set drifting in the opposite direction. When one buys a lamp bulb and screws it into a socket the electrons already in the lamp filament are jerked to and fro as a result of the electromotive forces supplied by the distant generator. No new electrons need be supplied to the lamp by the power line or taken away by it.

Obviously one buys his electricity from the hardware store in this case and not from the power company. We pay the

power company for shaking the electrons which we already own. This shaking takes *work,* and it is *work* for which the power company must be paid.

9.9. Induction Motors

In the case of the suspended coil, *A*, of Figure 103, we found not only that currents were induced, but that the coil moved to and fro as the switch in *B* was opened and closed.

Instead of supplying current from a battery through a switch to the coil *B* we might connect this coil to a source of

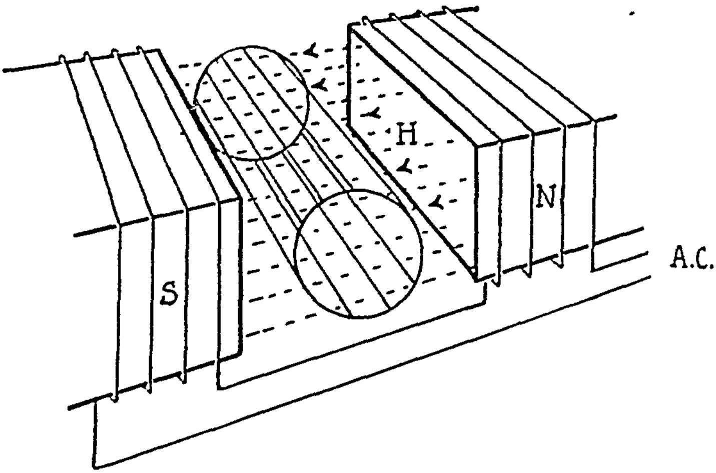


FIG. 110.—A "squirrel cage" induction motor.

alternating current. The induced currents in the coil *A* reacting with the currents in *B* will produce a to and fro motion of this coil *A*.

If a little inventive genius is again available, one should be able to modify this arrangement of coils so that some kind of steady rotating motion could be produced. Such a device we would call an induction motor.

Figure 110 shows an electromagnet to replace the coil *B* of Figure 103. Alternating current in the coil of this magnet will produce a rapid reversal of the poles of the magnet and of the direction of the magnetic field between them.

The rotating part of the motor consists of a network of large copper rods mounted in iron to increase the magnetic flux from the electromagnet across the rods. (Only the copper rods are shown in the figure.) These rods are connected at the ends and the general appearance of the arrangement (without the iron core) resembles a cage. In fact, this type of rotor is often called "squirrel cage."

It is the network of copper conductors forming the "squirrel cage" that acts as the coil *A* in Figure 108. Through the medium of their magnetic fields the currents in the squirrel cage and in the field coils react so as to produce repellant forces.

This motor operates successfully once it is started, but in the simple form described here it will not start itself. Many devices are used to make these motors self-starting, and the interested student is again referred to more advanced texts on electrical subjects.

10.9. Telephone Transmitters

The principles of induced electric currents may also be applied to the problem of converting sound vibrations into

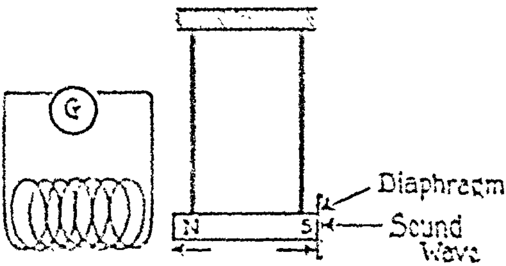


FIG. 111.—A simple telephone transmitter.

corresponding alternating electric currents. In this manner, a telephone microphone (or transmitter as it is often called) may be developed.

For a simple case, let us redraw Figure 105 to show the magnet supported on strings and a light flat disk fastened across it. Sound waves striking the disk could give the

magnet a slight to and fro motion, and induced currents would then be developed in the coil.

Actual practice in the construction of a telephone transmitter operating on the principle of induced currents has followed somewhat different lines. Figure 112 shows a small horseshoe magnet on which is wound a coil of many turns of fine wire. A diaphragm of thin iron is placed near the poles

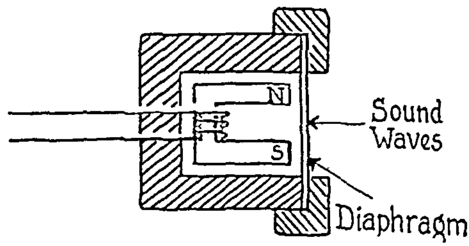


FIG. 112.—A two pole electromagnetic telephone transmitter.

of the magnet. It is supported about its rim, but the center can vibrate to and fro when sound waves strike it. The magnetic path from the N to S pole of the magnet changes as the iron diaphragm vibrates. The flux throughout the iron magnet changes accordingly and alternating currents are induced in the windings.

Figure 113 illustrates an electromagnetic telephone transmitter operating on the same general plan as that of Figure 112 but using a bar magnet instead of a horseshoe type magnet.

11.9. Telephone Receivers and Systems

The telephone transmitters of Figures 112 and 113 will also operate as receivers. When alternating current is supplied

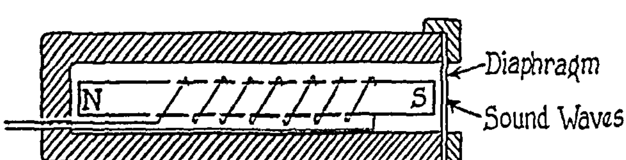


FIG. 113.—A single pole electromagnetic telephone transmitter.

to the coil of either instrument, the current will alternately weaken and strengthen the flux of the permanent magnet.

The magnetic attraction for the iron diaphragm will fluctuate accordingly and the diaphragm will move back and forth.

Figure 114 shows a simple telephone system consisting of two identical instruments. If a person talks into either instrument, his speech should be reproduced by the other.

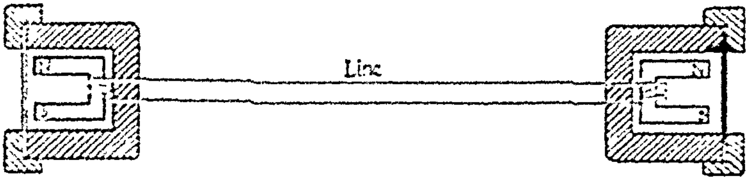


FIG. 114.—A simple telephone system.

Microphones and telephone receivers can also be built to operate on principles other than electromagnetic. Some of these constructions will be discussed in later chapters.

Some Important Facts

1. Whenever you change the magnetic conditions in the neighborhood of a closed circuit, electric voltages and currents are induced in the circuit.

2. The magnitude of the induced electromotive force depends on the rate at which the total number of magnetic lines threading through the closed circuit are changed and on the number of turns of wire in the closed circuit.

3. An electric generator uses the principles of induced voltage and current to supply electric power. It is operated by mechanical energy and converts this energy into electrical energy.

4. The amount of the electromotive force set up by a generator depends on the number of turns of wire on the armature, the speed at which the armature (or field) rotates, and the strength of the magnetic field.

7. In an induction motor, the field of the induced armature current exerts forces of mutual repulsion on the stationary field.

8. The principles of electromagnetic induction may be used for changing sound waves into electric currents.

Generalization

Whenever there is relative motion between an electric circuit and a magnetic field, an e.m.f. is generated in the circuit, and the circuit experiences forces.

Problems

Group A

1. Describe as many methods as you can for producing an induced current in a coil of wire.

2. Re-draw Figure 103 with only one turn in each coil. Then indicate the direction of current in each coil and show by an arrow along the axis of each coil the direction of the magnetic field through each coil for the cases of closing and then opening the switch to the battery.

3. Why does the current induced in coil A in Figure 103 soon die out?

4. Compare the electrical effect in a coil when a bar magnet is moved slowly near it with the effect when the magnet is moved rapidly.

5. What is the essential difference between a motor and a generator?

6. (*a*) What are the essential parts of an alternating current generator? Give the function of each part.

(*b*) Describe how it is possible to change mechanical energy into electrical energy with such a device.

7. What is the essential difference between a direct current generator and an alternating current generator?

Group B

1. Re-draw the electric generator of Figure 106 showing a single turn of wire in each coil. Indicate clearly the way in which the coils must be connected so that the current supplied to the circuit may be in the same direction from each coil.

2. If the coils of question 1 are connected incorrectly what will happen to the electro-motive force in the circuit?

3. Why is the alternating current motor described on page 488 called an induction motor?

4. Why does it require work to move the electrons in a lamp filament to and fro?

5. In a direct current system, the electrons drift in one direction, so that the same number of electrons that come into a house over one wire leave the house over the return wire. Why should the power company receive any compensation from the householder?

6. In the case of an alternating current system electrons are neither supplied nor taken away by the power company. For what does the householder pay in this case?

Experimental Problems

1. Using several feet of insulated wire in series with a galvanometer, move the wire in a fairly strong magnetic field so as to determine the direction relationship of

   a. Lines of magnetic force
   b. Mechanical motion.
   c. Induced current.

Formulate your conclusions into a single concise sentence.

2. Using essentially the same apparatus as in No. 1 above, discover three factors affecting the intensity of the induced current.

Chapter 10

INDUCTANCE AND ALTERNATING CURRENT

Since alternating current and voltage keep changing in value and reversing in direction it is necessary to define a method for obtaining average values for measuring volts and amperes. This is done by assuming that an alternating current is equivalent to a direct current if it produces the same heating effect in a resistance.

The facts of induced electromotive forces described in the preceding chapter can be applied to the construction of a device called a transformer. The transformer is especially useful with alternating currents for changing the voltage from *one value to another.*

Transformers can be used with direct current only when some arrangement is supplied for opening and closing a switch to the direct current source. Such arrangements are used in connection with automobile radio sets for taking power from a six volt storage battery and producing alternating current of several hundred volts. Another such arrangement is used for taking power from the storage battery of the car and producing small currents at large voltages for operating the spark plugs in the cylinders of the engine.

Electromotive forces are also induced in the original coil where a current is changing. This induced electromotive force opposes the change of current. This property of a coil, which is called self-inductance, hinders the flow of alternating current and so it may be used as a control device in an alternating current circuit.

1.10. Alternating Currents and Voltages

In the case of alternating electromotive forces, the voltage builds up to a maximum value, then decreases until it becomes zero and then builds up with opposite polarity. Similarly the current increases from zero to a maximum value, decreases to zero and then builds up in the opposite direction to a maximum and then again decreases to zero.

We have already seen that these changing values of current or voltage can be represented by a graph as in Figure 107 of Chapter 9, page 484. This graph is repeated in Figure 115.

*Specifying the voltage or current in an alternating elec*trical system requires some special definition since the values vary from zero to a maximum in both positive and negative

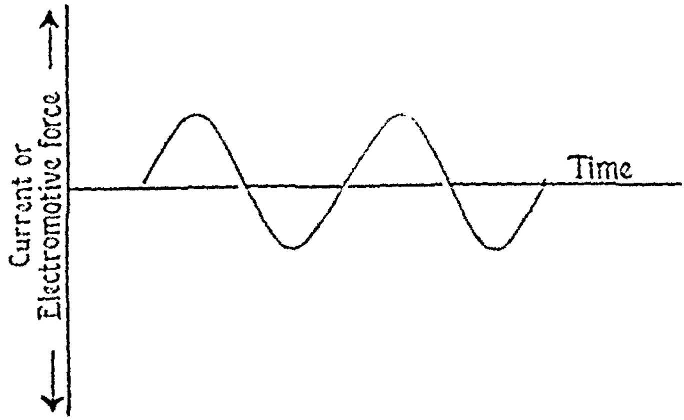


FIG. 115.—Alternating current or voltage plotted against time.

directions. A simple average over a complete cycle would give zero for a value.

The method used to define values of alternating current or voltage is based on the heating effect of the current in a

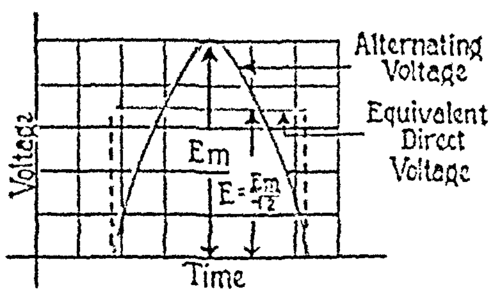


FIG. 116.—Graph to show the relation between an alternating voltage (or current) and the equivalent direct voltage (or current).

resistance as compared to the same effect of a
An alternating current is said to be the equi
current if it produces the same heating effe
as the direct current would produce.

An alternating electromotive force is said to be equivalent to a direct electromotive force if it produces a current in a resistance such that the same amount of heat is produced as would occur if the direct electromotive force were applied to the resistance.

From these definitions it turns out that the value of the alternating electromotive force (or current) called the *effective value* is equal to the maximum value divided by the square root of 2. In symbols we would write

$$E = \frac{E_m}{\sqrt{2}} = 0.707E_m$$

Or

$$I = \frac{I_m}{\sqrt{2}} = 0.707I_m$$

From this statement we can also say that the maximum value of alternating current or voltage is equal to the effective value multiplied by the square root of 2, that is,

$$E_m = E\sqrt{2} = 1.41E$$

Conversion of peak values to effective values for alternating currents and voltages by dividing the peak value by the square root of 2 is based on the assumption that a graphical representation of these alternating forms has a shape known as sinusoidal, as illustrated in Figure 115. For other shapes, the dividing factor will differ from 2. All of the discussions of alternating current and voltage in this text are concerned only with currents and voltages of the simple form discussed above.

Example. If a house is supplied with alternating current power at 110 volts, the peak voltage is

$$E_m = 1.41 \times 110 = 155.1 \text{ volts}$$

Watts are computed in the alternating current case by multiplying the effective values of volts and amperes; that is

$$\text{Watts} = \text{Volts} \times \text{Amperes}$$

Or, in symbols

$$P = E \times I$$

Just as in the case of direct current and voltage (see page 415) we can write:

$$\text{Watts} = (\text{amperes})^2 \times \text{ohms}$$

Or

$$P = I^2R$$

Or

$$P = \frac{E^2}{R}$$

and in each case we would use the *effective* values of the alternating current and voltage. Volts and amperes for alternating currents are always measured in "effective" values unless otherwise stated. In some texts effective values of voltage and current are called "*root mean square*" values (R.M.S.).

2.10. Transformers

In the early part of the last chapter we saw that when an electromotive force was applied to a coil of wire so that electricity began to flow in the coil, an electromotive force would be induced in a neighboring coil. This electromotive force would produce a current of electricity in the second coil in just the opposite direction to that in the first coil.

Further experimenting with such a set of coils shows that the induced electromotive force and current are larger when the coils are close together than when they are far apart. When the coils are close together most of the magnetic flux due to the current in the first coil threads through the turns of the second coil. If it were possible to make all of this flux thread through the second coil, and if the two coils had the same number of turns each, the electromotive force induced in the second coil would be equal to that applied to the first coil.

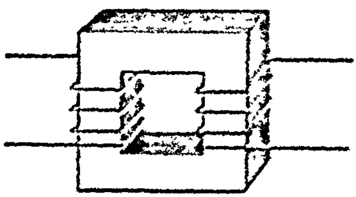


FIG. 117.—A simple core type transformer.

This ideal condition can be nearly obtained by winding both coils on a magnetic material such as iron. Figure 117 shows a closed figure made of iron and called a "core." On this core may be wound two or more sets of coils. They may be wound on different parts of the core as shown in this diagram, or one coil may be wound on top of another. The magnetic material makes such a good path for magnetic flux that nearly all of the flux due to a current in any coil will also pass through the other coils.

This arrangement of coils is called a transformer.

3.10. Voltage, Current, and Power in Transformers

If alternating electromotive forces are applied to one coil of a transformer so that an alternating current is produced in that coil, alternating electromotive forces will be induced in the second coil as described above. Also, under the ideal conditions given above, the alternating electromotive force induced in the second coil will be nearly equal to that applied to the first coil provided the number of turns in the two coils is the same.

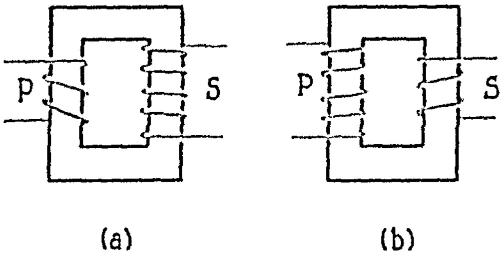
We may also say that the electromotive force *per turn* has the same value in the second coil as in the first coil. So if the two coils have a different number of turns, the total electromotive forces across the coils will have the same ratio as the number of turns in the coils. In symbols we may write:

$$\frac{E_1}{E_2} = \frac{n_1}{n_2} \tag{1}$$

where the E's are the electromotive forces and the n's the number of turns in the two coils.

The chief use of a transformer is to change the voltage either up or down as compared to the supply voltage. However, neglecting small losses in the transformer, the *power* delivered by the secondary winding is just equal to that supplied to the primary winding. So if the voltage of the secondary is higher than that of the primary, the current supplied by the secondary to a load must be proportionately

er than the current in the primary, and vice versa. The ng state for an ideal transformer is that the product of



118.—Transformers are used to change the voltage from that in the source. tep up transformer. (*b*) A step down transformer. (Simplified drawings l in this figure and those that follow. See Figure 117 for a three dimensional the core.)

ge and current in the secondary may just equal that in rimary. In symbols we may write:

$$E_1I_1 = E_2I_2$$

$$\frac{E_1}{E_2} = \frac{I_2}{I_1}$$

Transformers with More than Two Windings

gure 119 shows a transformer with three windings. If ' is used as the primary we have left one low voltage dary and one high voltage dary. For example, suppose P has 440 turns, S_1, 20 turns, $_2$, 2200 turns.

the primary is connected to a olt alternating current line the ge for the two secondaries may ind by comparing each separate- th the primary by means of ion (1) above.

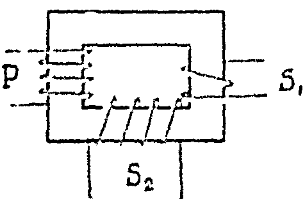


FIG. 119.—A simple core type transformer with three windings.

nother way to attack the problem is to notice that if 110 is applied to a 440 turn coil there must be an electro- e force of ¼ volt per turn. Since this same voltage is

induced in each of the secondary turns a coil of 20 turns will have

$$20 \times \frac{1}{4} = 5 \text{ volts}$$

and a coil of 2,200 turns will have

$$2{,}200 \times \frac{1}{4} = 550 \text{ volts}$$

5.10. Transformer Designs

Transformers wound as described above are called core type. A modification of this simple core is shown in Figure 120 where the coils are wound on the middle leg and the magnetic flux may follow either of the other two legs to make a closed magnetic path.

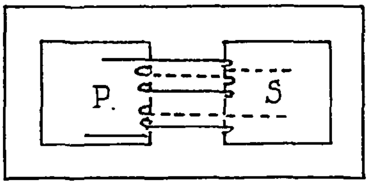


FIG. 120.—A transformer using a core with a center leg on which the windings are placed and with two return legs for the magnetic flux.

This design may be extended so that these outside legs are built as a cylindrical wall completely enclosing the windings. Such a construction is called "shell" type. It is used chiefly on very large transformers.

6.10. Autotransformers

Occasionally transformers are made with a single winding but with connections to this winding so that in some ways it behaves as two separate coils. Such a transformer, called an autotransformer, is shown in Figure 121.

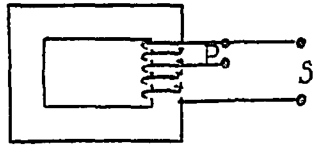


FIG. 121.—An autotransformer.

The voltage between any two pairs of leads will be proportional to the number of turns included; so that, with the connections shown, more volts will be found across the secondary than are put into the primary. Of course the connections marked *S* could be used for the primary and those marked *P* could be used as the secondary. In this case fewer volts would be found on the output connections than are supplied to the input.

7.10. The Use of Transformers

Transformers are useful chiefly because of the ease with which the voltage of a supply line can be changed. This same fact is one of the reasons why the use of alternating current is so desirable as compared to direct current. There is no simple way for changing the voltage of a direct current power supply. Transformers, of course, are useless on direct current since voltages are induced in the secondary winding only when the current in the primary is changing.

Practically all electrical power is sent from the generators where it is produced to the place where it is to be used at moderately high voltages. In this manner the current required is small, since, as we saw in Chapter 4, page 411, power is equal to volts multiplied by amperes. ($P = EI$.)

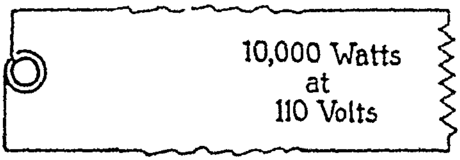


FIG. 122.—Energy losses in a long line are large if power is supplied over the line at high current and low voltage.

The advantage of keeping the current small is that some energy is always lost in the connecting wires and this energy is equal to the resistance of the line *multiplied* by the current squared. (Power lost in heating the line = I^2R. See page 415.)

Example. Suppose that a two wire power line is 10 miles long and that each wire has a resistance of 0.25 ohms per mile. This will give a resistance of 5 ohms for the entire line. Suppose that we try to take from this line 10,000 watts at 110 volts. (See Figure 122.)

First we find the current from the relation

$$\text{Power} = \text{Volts} \times \text{Current}$$
$$P = EI$$
$$10{,}000 = 110 \times I$$

From which

$$I = \frac{10,000}{110} = 90.9 \text{ amperes}$$

The heating loss P_1 in the line will be

$$\text{Power}_1 = \text{Current squared} \times \text{Resistance}$$
$$P_1 = I^2R = (90.9)^2 \times 5 = 41,314 \text{ watts}$$

But the line delivers only 10,000 watts and this result shows that more than four times as much energy will be lost in the line as is delivered by it.

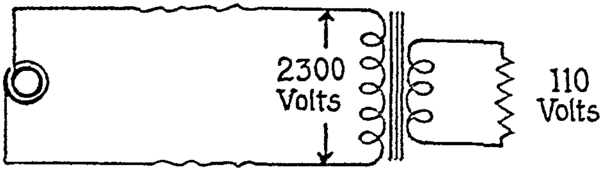


Fig. 123.—Energy losses in a long line can be reduced by supplying power at high voltage and low current. A transformer can be used at the end of the line to supply the voltage desired.

If we were willing to take this 10,000 watts at 2,300 volts instead of 110 volts, the current would be

$$P = EI$$
$$10,000 = 2,300I$$
$$I = \frac{10,000}{2,300} = 4.35 \text{ amperes}$$

The heating loss in the line will now be

$$P_1 = I^2R = (4.35)^2 \times 5 = 95 \text{ watts}$$

Since it would be an easy matter to put 2300 volts into the primary of a step down transformer which would deliver 110 volts at the place where the power is to be used, it would represent a great saving to have the power sent over the line at the high voltage instead of the low one. (See Figure 123.)

Where large amounts of electrical power are to be delivered over long lines, voltages of 23,000 are often used and in some cases voltages as high as 100,000 or more are in operation.

8.10. Transformers Used with Direct Current

While transformers are ordinarily thought of only in connection with alternating currents, it is possible to use them with direct current power supplies if some scheme is provided for opening and closing a switch to the direct current so that the current may be started and stopped repeatedly. Such an arrangement is used on some automobile radio sets where the power supply for the radio must be obtained from the car's six volt battery.

Figure 124 shows a battery connected to the primary of a transformer through an arrangement that looks like the

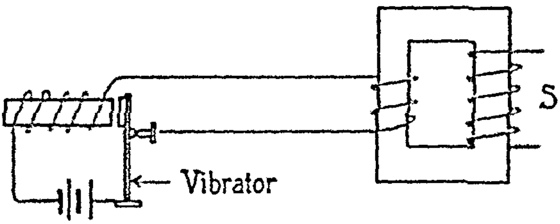


FIG. 124.—A vibrator may be used for interrupting direct current through the primary of a transformer.

electric doorbell described on page 463. Fundamentally it is quite similar except, of course, that the bell is omitted.

The contact points are much better made than those of the doorbell mechanism and the rate at which the strip will vibrate to and fro is adjusted to some desired value by the massiveness of the strip and the strength of the spring.

The primary of the transformer, of course, simply receives pulses of current always in the same direction. As this current starts and stops, alternating voltages are induced in the secondary. By using a large number of turns in the secondary, the alternating electromotive force may easily be as much as several hundred volts.

This alternating current may be used to obtain direct current at several hundred volts as is done in radio sets that operate directly from the electric lighting power supply in our homes. This feature will be described in a later chapter

9.10. Transformers in Telephone Systems

Electromagnetic telephone transmitters were described in the previous chapter, but nearly all modern telephone systems use what is known as a carbon button transmitter in combination with batteries and a transformer. Such an arrangement connected to an ordinary 2 pole magnetic receiver is shown in Figure 125.

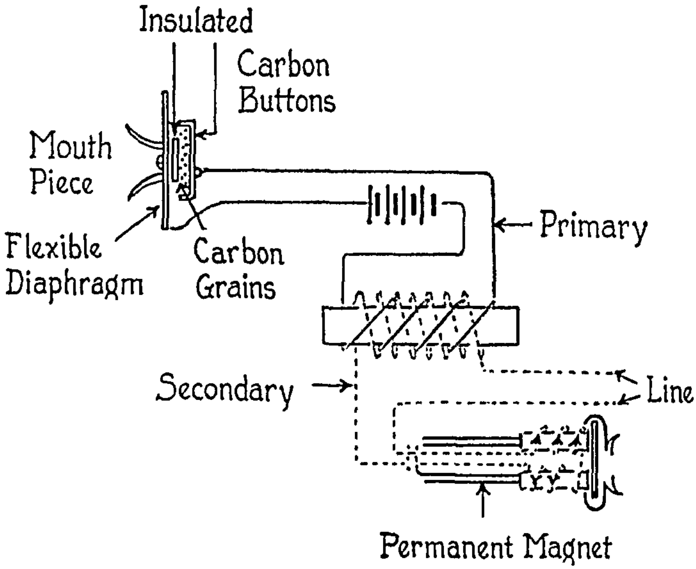


FIG. 125.—A carbon button microphone in a telephone system.

The carbon granules that make up the "button" offer considerable resistance to the flow of electricity and the amount of this resistance depends on how tightly the granules are squeezed together. Sound waves striking the diaphragm cause a varying pressure on the carbon button and so the current which the battery supplies is not steady but changes in synchronism with the sound waves.

This pulsing direct current in one winding of the transformer induces corresponding alternating currents in the other winding and it is these alternating currents that travel over the telephone line to the receiver at the other end.

10.10. Induction Coils

A slightly different arrangement of the type of transformer described above has been used for many years for obtaining pulses of high voltage current from a low voltage direct current source.

The iron core used in the vibrator mechanism is made larger than the one described in the above section and a secondary coil is wound over the primary. The secondary coil has a great many turns of small wire as indicated in Figure 126. The voltage induced in the secondary depends on the abruptness with which current can be started or stopped in the primary.

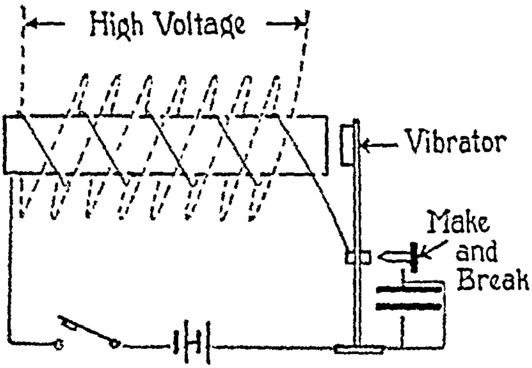


FIG. 126.—A coil and vibrator system for obtaining pulsating high voltage from a low voltage direct current source that can be interrupted by a vibrator.

For reasons that we shall talk about later in this chapter, the current will not start abruptly in this circuit when the contact is closed but it will build up gradually. On the other hand, it can be stopped quite quickly as the contact opens. The abruptness of stopping can be increased by the use of a condenser as shown in the drawing. (The nature of condensers will be described in the next chapter.)

We can see that the voltage induced in the secondary when the current in the primary is interrupted will be large in comparison to that induced when the contact is made. This difference can be made so great that the secondary is often said to produce "unidirectional" electromotive forces.

The induction coil is not very efficient, for not nearly as much energy can be gotten from the secondary as is put into the primary. On the other hand, it is a very convenient and simple device for obtaining small pulses of current at high voltage from a low voltage direct current source.

The induction coil may be used on automobiles for obtaining sufficient voltage to jump the spark gaps that are used for firing the explosive mixture of gas vapor and air in the cylinders. In most cars the arrangement is modified so that the make and break is operated by a mechanically driven device instead of a simple vibrator. The contact is opened at just the instant that a particular cylinder is to fire. The same contactor works for all cylinders and a rotating lever closes the high voltage current to the spark plugs one at a time in the order in which they are to operate. This arrangement is called a "timer."

11.10. Self Induction

Since changing a current in a coil induces a voltage in a neighboring coil, one might expect some similar effect to take

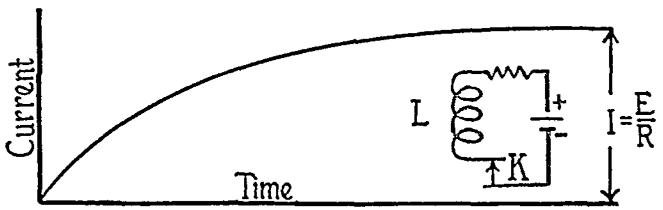


FIG. 127.—Growth of current in an inductive circuit.

place in the various turns of the original coil in which the current is being changed. Experiment shows that this is true.

An induced electromotive force is built up in this primary coil itself and this electromotive force is in the opposite direction to that which is being applied to the coil. This fact accounts for the slowness with which a current builds up to full value in a coil of many turns, especially when wound on an iron core.

Figure 127 is a graph showing current plotted against time for the case of connecting a direct electromotive force to a

coil. If the coil is large and is wound on iron, several seconds pass after the switch is closed before the current reaches the full value as given by Ohm's Law, $I = E/R$.

This induced voltage and current phenomenon within the primary coil is called *self-inductance* whereas the effect from one coil to another is simply called inductance, or mutual inductance.

Mutual inductance was discovered by Michael Faraday in 1831 while self inductance was discovered at about the same time by an American physicist, Joseph Henry. The amount of self inductance that any circuit has is measured in a unit called the henry, named after the discoverer of the effect. The same unit is also used to measure mutual inductance.

The total amount of magnetic flux associated with any electric circuit is proportional to the current in the circuit.

$$\text{Total Flux} = \text{Proportionality factor} \times \text{Current}$$
$$N = LI$$

The proportionality factor, L, varies from one circuit to another and is descriptive of a circuit's ability to produce magnetic flux. In other words, L is the self inductance of the circuit. It is measured in terms of a unit called the *henry* when I, the current, is measured in amperes and N, the flux, is measured in the practical unit of magnetic flux (called the weber).

The self inductance, L, of any circuit may be defined as the ratio of the total magnetic flux associated with the circuit to the current in the circuit. That is

$$L = \frac{N}{I}$$

Some idea of the magnitude of a henry may be obtained by noting that 70 turns of wire wound in a single layer on a tube of 2 inch diameter will have an inductance of approximately .00017 henry: 5000 turns of small size wire wound c

a small iron core might easily have an inductance of 30 henries or more.

12.10. Effect of Self-inductance on Alternating Current

Since self inductance retards the growth of current in a circuit, we would expect it to offer considerable hindrance to the motion of alternating currents. This effect would depend on the frequency of alternation. In advanced texts on this subject, it is shown that the hindrance of an inductance to the flow of alternating current is,

$$X_L = 2\pi f L$$

where X_L is called the "reactance" of the inductance and is measured in ohms, f is the frequency of alternation of the

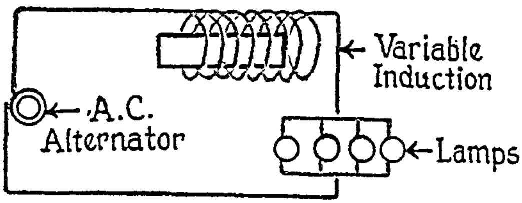


FIG. 128.—A variable inductance may be used to regulate alternating current.

current in cycles per second and L is the self inductance in henries.

In the circuit of Figure 128 is shown a coil (called a "choke" coil) in series in a line supplying some lamps from an alternating current source. This particular coil is arranged so that an iron rod may be moved in or out of the coil, thus changing the self inductance of the coil. The student should build up such an arrangement in the laboratory and observe the dimming effect of the lamps when the iron rod is pushed into the coil.

When a choke coil is in an alternating current circuit, the motion of electricity is hindered both by the reactance of the inductance and by the resistance of the windings of the coil. However, it can be shown that the combined effect is not the simple sum of the reactance and the resistance but is the square root of the sum of the squares of these two quantities.

The relation may be shown graphically by using a right angled triangle as in Figure 129 where the horizontal leg represents the *resistance*, the vertical leg, the *reactance*, and the diagonal, the combined effect of the two. This combined effect

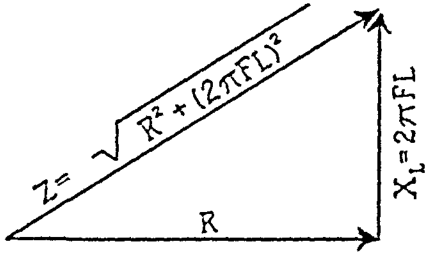


FIG. 129.—Graph to show the relations among resistance, inductive reactance, and impedance.

is called *impedance* to distinguish it from simple *resistance* or *reactance*.

Example. Find the current from a 110 volt 60 cycle line supplied through a choke coil of 0.5 henry and through a lamp of 100 ohms resistance. (See Figure 130.) (We will neglect

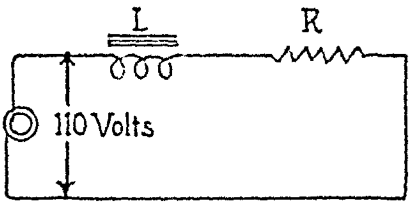


FIG. 130.—An alternating current circuit containing inductance, *L*, and resistance, *R*.

the resistance of the connecting wires and the wire in the choke coil in this example.)

In Ohm's law we substitute for the resistance R the letter Z to indicate impedance so that we write

$$\text{Current} = \frac{\text{Voltage}}{\text{Impedance}}$$

$$I = \frac{E}{Z}$$

and from the above discussion,

$$Z = \sqrt{R^2 + (2\pi fL)^2} = \sqrt{(100)^2 + (2\pi 60 \times 0.5)^2}$$
$$= \sqrt{10{,}000 + 35{,}600} = \sqrt{45{,}600} = 213 \text{ ohms}$$

And so the current

$$I = \frac{E}{Z} = \frac{110}{213} = 0.516 \text{ ampere.}$$

If there had been no choke coil in the circuit the current, I', would have been

$$I' = \frac{E}{R} = \frac{110}{100} = 1.1 \text{ amperes.}$$

Choke coils that can be varied are often used in the lighting circuits of theatres where it is desired to turn the lights up or down gradually. The initial cost of such equipment is greater than it would be to put in large variable resistors but energy is always lost in resistance, while the choke coils regulate the flow of electricity without absorbing much energy. They simply prevent the energy from being supplied by the line to the load.

Some Important Facts

1. If an alternating current and a direct current each has the same heating effect in a pure resistance, they are considered equivalent in voltage, amperage, and hence, wattage.
2. In a transformer, an alternating current in the primary is able to induce in the secondary another alternating current of nearly equal power.
3. In any transformer coil, the voltage varies directly, and the amperage inversely, as the number of turns.
4. A transformer may have several secondary coils, with a corresponding variety of voltages.
5. In case the same coil in a transformer serves as both primary and secondary, the device is called an auto-transformer.
6. The chief use of transformers is to change the voltage of a source of a.c. power.
7. When varying direct current exists in the primary of a transformer, the secondary will deliver alternating voltage and current.

8. If an automatic interrupter is inserted in the primary circuit of a transformer, a direct current may be used in the primary. Such a device is often called an induction coil.

9. The self inductance of a circuit may be defined as a factor relating the total magnetic flux in a circuit to the current producing that flux in the circuit.

10. The self inductance in a coil hinders, or "chokes," the original inducing current.

11. The vector sum of resistance and inductive reactance is the total impedance which a circuit containing resistance and inductance offers to the flow of alternating current.

Generalization

Inductance and self inductance are important factors in circuits in which there is alternating current or direct but pulsating current.

Questions and Problems

Group A

Unless otherwise specified alternating voltages and amperes are always given in effective (R.M.S.) values.

1. If the effective value of the electromotive force on a power supply is 120 volts, what is the peak value? 169.7 volts.

2. The peak, or maximum, value of electromotive force from a certain power supply is 80 volts. Find the effective value. 56.6 volts.

3. An electromotive force of 115 volts supplies 4.5 amperes (all R.M.S. values) through an electric heater. Find the power supplied to this circuit. 517.5 watts.

4. A radio set can deliver a maximum peak voltage of 18 volts to a loud speaker that presents 15 ohms resistance. Find the power delivered to the speaker. (Suggestion: First reduce the peak volts to effective volts.) 10.8 watts.

5. A transformer is wound with 550 turns in the primary and 3,000 turns in the secondary. What effective voltage will be found across the secondary when 110 volts is applied to the primary? 600 volts.

6. The winding of an autotransformer contains 330 turns. The entire coil is used as a primary and one-third of the turns is used as a secondary. Find the voltage developed on the secondary when 220 volts is on the primary. 73.3 volts.

7. Why cannot a transformer be used on simple direct current to change the voltage?

8. Make a wiring diagram showing an alternating current generator connected to a step-up transformer. The secondary of this transformer is to be connected to a long line the far end of which is connected to a step-down transformer.

9. Explain why arrangements such as that of problem 8 are used for the transmission of electric power over long distances.

10. What is meant by self induction as compared to mutual induction?

Group B

1. Why is a magnetic material such as iron used in the construction of transformers?

2. If the coils on a transformer are made very large in comparison to the iron core, less voltage per turn is induced in the secondary than is applied to the primary. Why should this be true?

3. A transformer is wound with 330 turns in the primary and it is to be used on a 110 volt supply. Find the number of turns in two secondaries, the first to deliver 10 volts, and the second to deliver 500 volts.

30. 1500.

4. The small secondary of problem 3 is connected to 5 ohms and the other secondary is open. Find the current in the primary. (Suggestion: First find the current in the secondary.) 0.182 ampere.

5. The large secondary of problem 3 is connected to 5,000 ohms and the other secondary is open. Find the current in the primary.

0.454 ampere.

6. Find the current in the primary of the transformer of problem 3 if both secondaries are operating into the loads given in problems 4 and 5 at the same time. 0.636 ampere.

7. Explain the action of each part of the vibrator device used on auto radio sets for obtaining high voltage alternating current from low voltage direct current.

8. Over a 20 mile line whose total resistance is 25 ohms, 10,000 watts of power must be delivered.

(*a*) If the power delivered is at 10,000 volts, find the current.

1.0 ampere.

(*b*) Find the rate at which power is wasted in the line. 25 watts.

9. Repeat problem 8 for the case of delivering the power at 50,000 volts. 0.2 ampere, 1 watt.

10. Point out all the differences in construction and in principle between the system used for converting low voltage direct current to high voltage alternating current as for an auto radio and the induction coil.

11. Explain the action of the make and break and the timer mechanism in an automobile.

12. Find the reactance of 5 henries in a 60 cycle circuit. 1885 ohms.

13. Find the reactance of 0.25 henry in a 500 cycle circuit.

785.4 ohms.

14. Find the impedance of a line that contains 2 henries and 200 ohms in series. The power supply is 60 cycles. 780 ohms.

15. Make a diagram showing a 110 volt alternating current source connected to a resistance of 50 ohms and an inductance of 1 henry in series. Find the reactance of the inductance, the impedance of the combination, and the current through the combination if the frequency is 60 cycles.

377 ohms. 380 ohms. 0.289 ampere.

Experimental Problems

1. Measure the D.C. resistance of several coils. Then measure the A.C. impedance of the same coils by means of an A.C. ammeter and an A.C. voltmeter and by using the equation given on page 509. Account for any differences.

2. Using your data of Exp. Prob. 1, compute for each coil:

(*a*) The inductive reactance in ohms.
(*b*) The inductance in henries.

3. If possible, include in the above selection of coils a few of known inductance, as a check on your experimental results and calculations.

Chapter 11

CONDENSERS AND ALTERNATING CURRENT

Electricity can be placed on any piece of conducting material (for example, any piece of metal) and it will stay on the object until it can leak away or else be neutralized by attracting electricity of the opposite kind.

After some electricity has been placed on an object, it will exert a repelling force on any other electricity of the same kind that we may try to bring up to the object. It will be easier to get a lot of electricity on a large object than on a small one; but in any case, the amount that we get on will depend on how hard we are willing to work to push more electricity on to the object.

The capacity of a conducting object to hold electricity is defined as the electrical charge on it divided by the electrical potential that the object has as a result of this charge.

Any group of conductors arranged to increase their ability to hold electricity is called a condenser.

A condenser, after becoming charged, behaves as an open circuit in a direct current circuit. However, since it can easily be charged in either direction, it can be placed in an alternating current circuit without stopping the alternating flow of electricity.

A condenser in combination with an inductance has the property of letting alternating current of one particular frequency pass more easily than currents of other frequencies. By changing the size of the condenser or inductance, or both, the circuit can be "tuned" to any desired frequency.

Condensers may be connected in series or in parallel, and the total capacity of such combinations can easily be found.

Some condensers are made of metal plates that interleave one another and that can be varied in position so as to change the capacity. Others are made with metal foil separated by mica or paraffin paper. Various other constructions are also possible, and some of them are described in this chapter.

Condensers may be used as simple storage capacities for electricity and some experiments to show this effect are described at the end of the chapter.

1.11. Electrical Capacity

If electricity is placed on any conducting object such as the metal sphere shown in Figure 131 it will exert a repelling force on any additional electricity of the same kind that is brought up. It will require work to place additional elec-

tricity on the sphere and the work per unit charge is called the potential of the sphere.

Of course the amount of electricity that can be placed on the sphere may be almost any quantity provided only that one is willing to do the necessary work to place it there. The actual capacity of an object for holding electricity is a rather flexible and indefinite quantity and so electrical capacity is defined in terms of the potential that is built up in comparison to the amount of electricity placed on the object.

FIG. 131. A metal sphere charged with negative electricity.

The exact definition of the electrical capacity, C, of an object is the ratio of the amount of electricity, Q, on the object to the electrical potential, V, of the object with respect to its surroundings. In symbols this definition becomes

$$C = \frac{Q}{V}$$

If the quantity of electricity is measured in coulombs and the potential in volts, the unit of capacity is called the *farad.*

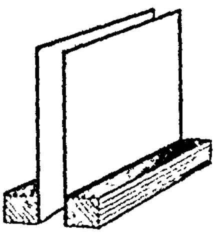


FIG. 132.—Two flat metal plates placed close to one another form a simple condenser.

A capacity of one farad is a very large amount in terms of the capacity of most of the electrical objects with which we are familiar and so the one one-millionth part of the farad, called the *micro-farad* is a more commonly used unit. In radio circuits where very small capacities are often of great importance, units such as the milli-micro-farad and the micro-micro-farad are used. The prefix, milli, means the one-thousandth and the prefix, micro, means the one-millionth part of the unit to which they are attached.

2.11. Condensers

Whenever a set of conductors is arranged in such a manner as to improve their ability to hold electricity, the arrangement is called a *condenser.*

A simple sort of condenser may be made by placing two sheets of metal a small distance apart. The condenser may be charged by carrying electrons from one plate and depositing them on the other. In this manner one plate becomes negatively charged and the other positively charged. The amount of electricity on one plate is equal to that on the other, although it is opposite in kind.

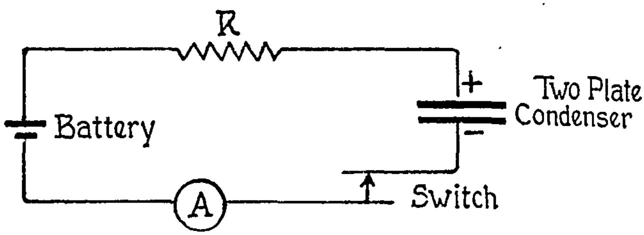


FIG. 133.—A circuit containing capacity and resistance.

When the equation for capacity

$$C = \frac{Q}{V}$$

is used for a condenser, the quantity, Q, is the amount of electricity on either the positive plate or the negative plate of the condenser; and the voltage, V, is measured across the condenser, that is from one plate to the other.

The transfer of electrons from one plate of a condenser to the other may conveniently be made by means of a battery connected as shown in Figure 133.

When the battery is first connected, electrons are transferred rapidly, but as the charge builds up on the condenser plates, the current in the circuit decreases. (This effect is shown in Figure 134.) Transfer of electrons by the battery stops entirely when the potential difference across the plates of the condenser reaches the same value as the electromotive force of the battery.

If the battery could be reversed at about the time the condenser becomes charged to this value, the electrons would

drift back through the battery until no charge of electricity was left on the condenser plates. The process would then be repeated with the condenser plates charging up as before but with opposite polarity.

The battery could now be reversed again and the process described above would repeat itself. An ammeter placed

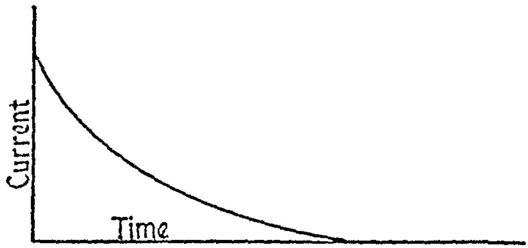


FIG. 134.—The current in a circuit similar to that of Figure 133 is large when the switch is first closed to a direct current source and then decreases with time.

in the circuit as shown would indicate a to and fro current even though a condenser has no connection through it.

3.11. Condensers and Alternating Current

From this description we can see that if an alternating source of current is used instead of a battery, to and fro currents can take place in a circuit which contains a condenser. So, although a condenser does cause an open circuit for direct current (except while the condenser is being charged) it does not stop alternating current. There will be, however, some hindrance to the flow of the alternating current in comparison to the same circuit with the condenser omitted.

The larger the condenser, the less will be the hindrance to the alternating current; and the faster the alternations of electromotive force, the less will be the hindrance. Advanced texts show that the hindrance to the alternating flow of electricity (which is called reactance as in the case of the choke coil) is given by

$$X_c = \frac{1}{2\pi fC}$$

If the frequency, f, is expressed in cycles per second and the capacity, C, in farads, the reactance, X_c, will be given in ohms.

4.11. Capacity and Resistance in A.C. Circuits

The reactance due to a condenser, like that due to a choke coil, cannot be added directly to the resistance of the circuit. The combined effect is equal to the square root of the sum of the squares as in the choke coil case. We may represent the quantities by a right triangle as before, where the resistance is the horizontal leg, the reactance the vertical leg and the

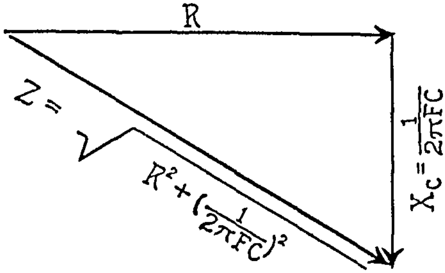


FIG. 135.—A graph to show the vector relations among resistance, capacitative reactance, and impedance.

impedance is represented by the diagonal. See Figure 135. In symbols we may write for the impedance, Z,

$$Z = \sqrt{R^2 + \left(\frac{1}{2\pi f C}\right)^2}$$

The graph shows the vertical leg representing the reactance of the condenser drawn down instead of up, for the effect of a condenser and that of an inductance are not the same.

5.11. Capacity, Inductance and Resistance in A.C. Circuit

A review of the actions of capacity and inductance (see Figure 127 of Chapter 10 and Figure 134 of this chapter) shows that a choke coil has more hindering effect on the flow of electricity at high frequencies of alternation of the electromotive force than at low frequencies and a capacity has less effect at the high frequencies. Also a large choke coil

hinders the flow of electricity more than a small one while a large capacity stops the flow less than a small one. In these respects inductance and capacity have opposite effects in an alternating current circuit.

If a circuit contains both inductance and capacity in series (as is shown in Figure 136) the reactance due to both sources together is found by subtracting that due to the con-

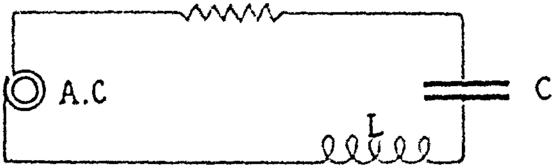


FIG. 136.—An alternating current source supplying a circuit containing resistance, capacity and inductance in series.

denser from that resulting from the inductance. In symbols the reactance is given by

$$X = 2\pi fL - \frac{1}{2\pi fC}$$

The impedance of the entire circuit may be found from the diagonal of a right triangle where the horizontal leg, as before,

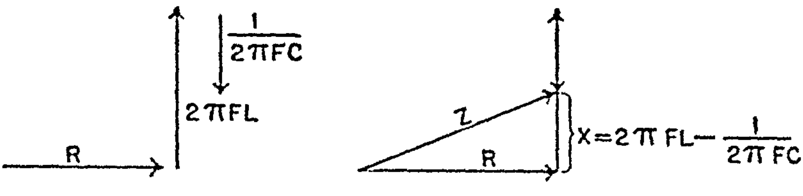


FIG. 137.—Vector relations among resistance, capacitative and inductive reactance and impedance in the circuit of Figure 136.

represents the resistance and where the vertical leg represents the net reactance. (See Figure 137.)

The impedance, Z, may be expressed in symbols as

$$Z = \sqrt{R^2 + \left(2\pi fL - \frac{1}{2\pi fC}\right)^2}$$

The current in such a circuit will be

$$\text{Current} = \frac{\text{Voltage}}{\text{Impedance}}$$

$$I = \frac{E}{Z}$$

6.11. Resonant Circuits

Of course it is possible to find some value of frequency so that the reactance due to the condenser in a given circuit is just equal to that due to the inductance. In this case the net value is zero and the impedance of the circuit is simply equal to the resistance. In the expression for current above, we would expect to get the largest current when the impedance is least. So we see that the greatest current will occur when the relations between frequency, capacity and inductance are such that the net reactance is zero.

This condition is called resonance, and so we may speak of electrically resonant circuits. Such circuits are not often used in electrical power engineering, but they are of the greatest importance in radio engineering. One tunes a radio set by turning one or more adjustable condensers to such a value of capacity that the net reactances of the circuits vanish for the frequency of the particular radio signal that he wishes to listen to.

Later we shall see that such circuits are also used in connection with radio tubes to create alternating currents at the resonant frequencies of the circuits.

The resonant frequency of a circuit can be found by setting the expression for reactance equal to zero and solving for f.

$$X = 2\pi fL - \frac{1}{2\pi fC} = 0$$

From which

$$f = \frac{1}{2\pi\sqrt{LC}}$$

7.11. Combinations of Condensers

Sometimes several condensers are used in the same circuit. If they are placed in parallel (see Figure 138) the combined value, C, is simply the sum of the individual values. That is

$$C = C_1 + C_2 + C_3 + \cdots$$

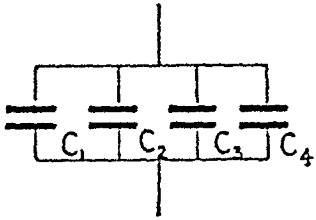


FIG. 138.—Condensers in parallel.

When they are in series, (see Figure 139) the combined value is less than that of the smallest one present. In other texts it is shown that the capacity, C, of a combination of condensers in series may be found from the expression

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \cdots$$

Series or parallel combinations are used mostly at times when the right size of the condenser for a particular job is

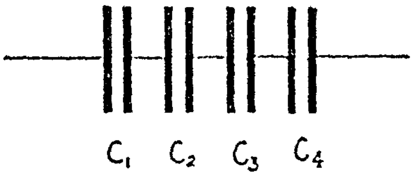


FIG. 139.—Condensers in series.

not available. For example, if you needed a capacity of 0.5 microfarad and had none in stock smaller than 2 microfarad, you could get the desired value by placing four of the larger ones in series, for

$$\frac{1}{C} = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{4}{2} = 2$$

From which

$$C = \frac{1}{2} = 0.5 \text{ microfarad}$$

8.11. Types of Condensers—Dielectrics

Types of condensers using air spacing between the plates are used chiefly in places where it is desired to move the plates easily and so change the value of the capacity.

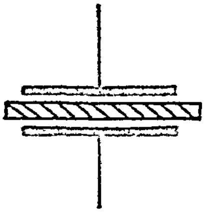


FIG. 140.—A piece of glass or hard rubber placed between two metallic plates increases their electrical capacity.

However, the capacity of a condenser can be increased by using various insulating materials between the plates. A piece of hard rubber placed between two metallic plates will appreciably increase their capacity for holding electricity. Such a condenser is shown schematically in Figure 140. A condenser of the same type but of different shape is shown in Figure 141. It has been named a "Leyden" Jar. The one illustrated in Figure 142 is designed for laboratory demonstrations and may readily be taken apart. However, any student can easily construct a Leyden jar by putting interior and exterior coatings of metal foil on any glass jar.

Mica and paraffined paper are also often used between the plates of condensers. The plates can be put close together without danger of touching and the capacity is increased both by this closeness and by the electrical nature of the atoms of the insulating material.

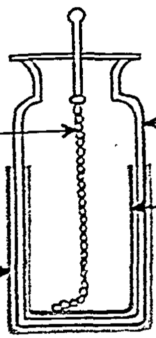


FIG. 141.—A type of condenser known as a Leyden jar.

All such materials are rated in terms of the proportional amount that they increase the capacity of the condenser in comparison to air (or more properly, a vacuum). The rating is called the "dielectric constant" of the material. Dielectric constants of from 2 to 4 are fairly common.

Condensers using paper between the plates usually have some flexible material for plates, such, for example, as tin

foil. These condensers may be rolled into compact form and placed in paper cartons, moulded mica forms, or metal cans.

Condensers of the movable plate type may have their capacities increased by immersion in oil. Ordinary mineral oil is often used, but castor oil is better in the sense that it has a larger dielectric constant.

FIG. 142. A modified Leyden jar type condenser so constructed that it can easily be disassembled.

Condensers may also be made by making use of the fact that although aluminum is a good conductor of electricity, the compound, aluminum oxide, is a very good insulator. If a sheet of aluminum is immersed in a water solution of borax and boric acid, and if the aluminum is connected to the positive side of the battery while the solution (through its metal container) is connected to the negative side, this insulating oxide will form on the aluminum plate. The oxide is very thin and hence the plate, at a positive potential, is separated by only a very small distance from the electrolyte which acts as the negative plate. The capacity of such an arrangement is very large.

Various substances other than the ones given above may be used for the electrolyte and some metals other than aluminum act in a similar manner. Aluminum is quite generally used, however, in the condensers of this type now on the market. The electrolyte may be supplied in paste form instead of liquid.

In general these electrolytic condensers are used only on circuits where the direction of the electromotive force does not change, although there are now some improved forms available which can be used on alternating current.

These condensers find their chief commercial use at present in the part of radio sets in which alternating current power is converted into direct power for use in the radio set proper.

9.11. Condensers Store Electricity

In this chapter the use of condensers in alternating cu... circuits has been emphasized because of their importan

such service. The tuning of a circuit to a particular frequency as in the case of radio receivers and the ability of a condenser to stop the flow of direct current and permit the flow of alternating current are two of the more important uses in such circuits.

Condensers, however, may serve as simple storage capacities for electricity. This effect is easily seen by taking a simple condenser made of two sheets of thin metal separated by a glass plate as shown in Figure 143. If the two plates are connected to the terminals of a static machine, the condenser may be charged to a high voltage.

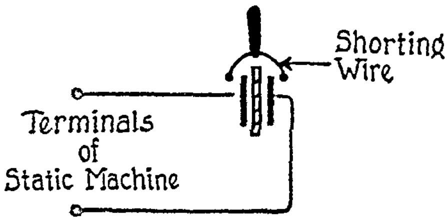


FIG. 143.—Condenser plates may store electricity from a static machine. When the plates are shorted with a wire, a spark is produced.

Before charging the condenser a wire should be fastened at one end to one of the plates. If, after the condenser is charged, the other end of the wire is moved by means of an insulating rod until it touches the other plate, a violent spark will be seen and heard.

The experiment may now be repeated several times, waiting longer and longer intervals after the condenser has been charged before the wire is touched to the second plate. If the condenser is a good one, that is, one that has its plates well insulated from one another and from any other conducting material, a good spark can be obtained several minutes after the condenser is charged.

This experiment may be repeated with much lower voltages if a condenser of larger capacity is used. In Figure 144 is shown an electrolytic condenser with a capacity of about 30 microfarads. It can be charged with the 90 volt battery of dry cells shown in the diagram. Although this voltage is not large, a considerable amount of electricity will

be required to charge so large a condenser even to this value. A wire or a screw driver (with a wooden handle to protect the experimenter) may be used to short circuit the terminals of the condenser after a switch in the battery line has been

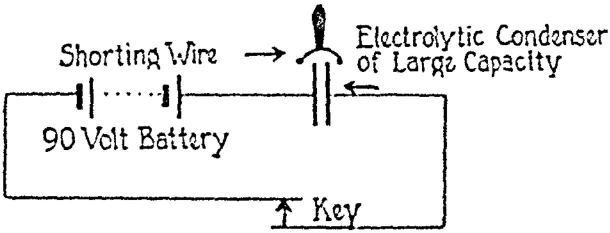


FIG. 144.—When a condenser of large capacity is charged to a moderate voltage, sufficient energy may be stored to produce a hot spark several seconds after the key to the charging battery has been opened.

opened. A hot spark should be easily obtained with this equipment.

Some Important Facts

1. The ability of any electrically conducting object to hold electricity is called electrical capacity.

2. Electrical capacity is defined as the ratio of electrical charge on an object to the electrical potential of that object as a result of the charge. When the charge is measured in coulombs and the potential in volts, the capacity is given in farads.

3. When two or more conductors are arranged in such a manner as to increase their individual electrical capacities the assembly is called a condenser.

4. When a condenser is charged, one plate or set of plates is charged negatively and the other plate or set of plates is charged positively to an equal amount.

5. When the capacity of a condenser is computed, the amount of electricity is taken as that on either one, but not both sets of plates; and the voltage is the potential difference between the plates.

6. A condenser passes alternating current (with some hindrance to its flow), but stops direct current.

7. The total impedance to the flow of alternating current in a circuit containing resistance and capacity in series is the vector sum of the resistance and the capacitative reactance.

8. Inductance and capacity, although having a hindering effect on the flow of alternating current, react in opposite manners so that the presence

of one tends to offset the presence of the other in a circuit. When the two effects are equal the circuit acts as a simple resistive circuit, and the circuit is said to be tuned to the particular frequency for which the effect exists.

9. The combined capacity of condensers in parallel is the sum of their separate capacities; and in series, the reciprocal of the combined capacity is the sum of the reciprocals of the individual capacities.

Generalization

Assemblies of conductors designed to hold electrical charge are called *condensers and may be used as storage devices* on direct current sources and control devices in alternating current circuits.

Questions and Problems

Group A

1. What is meant by the electrical "capacity" of any metal object?

2. Why does it require work to place more and more electricity on a conducting object?

3. How much electricity can be placed on any conducting object?

4. Define a condenser and suggest a simple construction for a condenser.

5. Describe the effect that a condenser will have if it is placed in series in a direct current circuit.

6. Make a wiring diagram of a 3 and a 2 microfarads condenser connected in parallel and find the capacity of the combination.
5 microfarads.

7. Make a wiring diagram and find the capacity of a 3 and a 2 microfarads condenser connected in series with one another. 1.2 microfarads.

8. Show by a labelled diagram the distribution of electrons on a Leyden jar when the inner conductor is negative with respect to the outer conductor.

9. Repeat No. 8 above, but with the charges reversed.

Group B

1. Why cannot the electrical capacity of an object be stated simply in terms of the amount of electricity that can be placed on it?

2. Explain why a condenser in a circuit does not completely stop the to and fro motion of alternating current.

3. A condenser of 10 microfarads is charged to 100 volts. How much electricity is on each plate of the condenser? 0.001 coulombs.

4. Suppose that you have a supply of condensers in the following sizes: 1, 2, 5 and 10 microfarads.

(*a*) Make up a combination in parallel that will give 19 microfarads.

(*b*) Make up a series combination that will give 0.5 microfarad.

(*c*) Make up any kind of combination that will give 2.5 microfarads.

(*d*) Make up any kind of combination that will give 0.667 microfarad.

Show wiring diagrams and calculate the capacity to prove that your combinations are correct.

5. Find the capacitative reactance of a 50 microfarads condenser in a 60 cycle circuit. (Remember that capacity must be expressed in farads—not microfarads—for this kind of problem.) 53 ohms.

6. Find the impedance of a series circuit containing 100 ohms resistance and a 50 microfarads condenser. The frequency is 60 cycles per second. 113 ohms.

7. An inductance of 0.25 henries is in series with a 40 microfarads condenser in a 60 cycle line. Find the reactance of the combination. 27.94 ohms.

8. What is meant by electrical resonance?

9. Find the reactance of a 2 microfarads condenser in a 500 cycles per second circuit. 159.2 ohms.

10. Find the size of inductance to put in series with the condenser of problem 9 to make the circuit resonant to 500 cycles per second. (Suggestion: Remember that the inductive reactance, $2\pi fL$, must be equal to the capacitative reactance.) 0.0507 henry.

11. Find the resonant frequency of a circuit that has an inductance of 5 henries in series with 3 microfarads of capacity. 41.1 cycles per sec.

12. Find the resonant frequency of a circuit that has an inductance of 0.0002 henries in series with 0.0002 microfarad. 795,800 cycles per sec.

Experimental Problems

1. Charge a Leyden Jar, or other simple condenser

a. By means of a static machine.

b. By a 90-volt battery (two 45-volt radio "*B*" batteries in series).

In each case note the length and thickness of the spark, when the condenser is discharged with a conductor insulated from the hand.

2. Connect the terminals of a 45 volt battery to two posts of a commutator switch. Connect the other two posts of the switch in series with a galvanometer and a variable condenser. Note any changes in galvanometer readings when:

a. Current direction in the condenser circuit is reversed.

b. The capacity of the condenser is varied.

3. Collect several types of old condensers, both fixed and variable. Dissect them sufficiently to identify all necessary parts. Explain in terms of capacity, compactness, convenience, cost, etc., their differences in mechanical structure.

WIRELESS

When alternating electric currents occur in circuits that have an open section, as in the case of a wireless antenna and a ground, some electric energy tears loose from the circuit and travels out through space. This phenomenon we call wireless or radio.

Sending energy through space in this manner is not very efficient and so will probably never be used in place of wires for power transmission. However, speech, or music, or telegraph signals can easily be impressed on this radiated energy and so wireless becomes a convenient method for sending messages.

The essential parts of radio transmitters and radio receivers are listed in this chapter.

The latter part of the chapter describes the passage of wireless waves through the earth's atmosphere as they travel from a transmitter.

1.12. Radiating Electrical Energy

All of the electrical circuits that we have studied up to this point are known as closed circuits. In the present

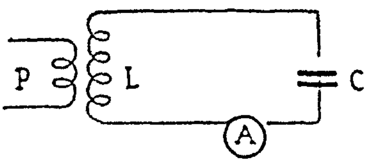


FIG. 145.—A simple circuit in which alternating current may flow.

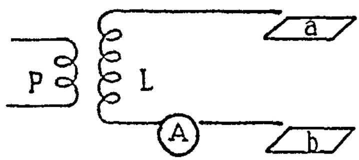


FIG. 146.—A circuit similar to that of Figure 145 but with the condenser plates moved far apart.

chapter we become acquainted with a slight change in some of the circuits studied in the last chapter. Technically they are called "open" circuits and the particular new effect is that energy can be flipped off to radiate away through space.

In Figure 145 is shown a circuit containing an inductance, *L*, and a condenser, *C*. The inductance is the secondary winding of a transformer and currents are easily induced in this circuit by alternating currents supplied to the primary

winding. We have seen circuits of this type in the last chapter and we know that alternating current will flow in this circuit and that an alternating current ammeter placed as shown would read the current.

Suppose now that the plates of the condenser *C* are moved far apart so that, in quite an ordinary sense of the word, the circuit really is open. (See Figure 146.) Every time the current alternates and the voltage on the plates *a* and *b* reverses, pulses of energy from the region between the plates will be flipped off from the system and will start through space with the speed of light.

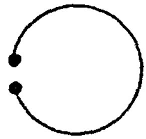


Fig. 147. The first detector of wireless waves. Enough energy had to be caught in the loop to cause a visible spark to jump between the ends of the wire.

Although these pulses of energy travel with the speed of light they do not illuminate objects as light does, and unless the amount of energy in the pulses is very large it is something of a problem to discover whether or not any energy is being radiated. The first device used to detect such radiated energy was a single loop of wire with the ends close together. (See Figure 147.)

If such a loop of wire with ends almost touching is in the neighborhood of a radiating circuit, it is possible to see sparks jump the gap between the ends of the loop. This experiment proves that energy radiated from the circuit of Figure 146 causes electric currents in nearby conductors. The frequency of alternation of current in the loop will be the same as that in the original circuit. The radiating circuit we will call the transmitter, and the loop will be called the receiver, or detector.

Since we have nothing but electric fields, and electric currents (which have magnetic fields) in the transmitter, we can say that the radiated energy is electromagnetic.

2.12. Discovery of Wireless

Clerk Maxwell, a famous English physicist, predicted in the early 1860's that ordinary visible light was of an electrical and magnetic nature, and as a result of his writings various

scientists tried for years to make electrical contrivances which would send electrical energy through space just as light travels.

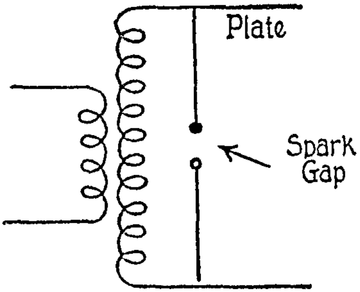


FIG. 148.—Hertz's transmitter. The circuit is similar to that of Figure 146 but with the addition of a spark gap. In the circuit of Figure 146 high frequency alternating current is fed directly into the primary coil, *P*, and current of the same frequency is induced in *L*. In the circuit of Figure 148 low frequency alternating current or pulsating direct current may be supplied to the primary coil. The action of the spark gap in the secondary circuit in combination with the circuit's inductance and capacity will generate high frequency current.

One of these groups of people centered about the German physicist, Helmholtz. In his laboratory there was a bright young man known as Heinrich Hertz and it was young Hertz who first set up a simple apparatus very much like the one described in the section above and who succeeded in sending energy by wireless across a laboratory. This was in 1888. (See Figure 148.)

Another group of scientists worked with the Italian physicist Righi. Righi, like Helmholtz, had a bright young pupil. His name was Marconi. Marconi thought of using a large out-of-door antenna for one of Hertz's electric plates and the ground for the other one. (See Figure 149.)

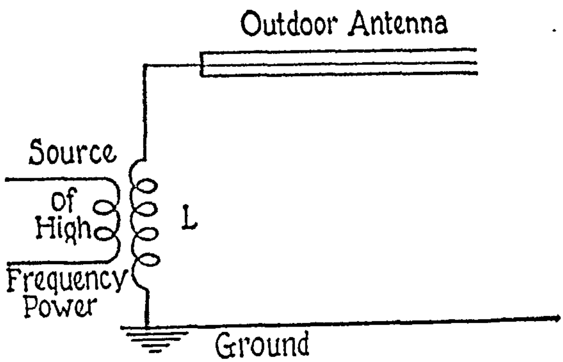


FIG. 149.—An outdoor antenna was substituted by Marconi for one plate of Hertz's apparatus, and the ground served as the other plate.

These improvements in the transmitter greatly increased the distance over which the energy could travel and still be strong enough to be detected. The *transmitter* was now greatly improved in comparison to the loop *detector*.

If wireless waves cause electric currents in a simple loop conductor one might at first suppose that a good detector could be made by simply using an antenna and a ground with a sensitive ammeter connected as shown in Figure 150. The difficulty is that currents in the antenna-ground system due to wireless waves are very small (unless the transmitter is near by), and it is impossible to get a good enough ammeter to make the plan practical.

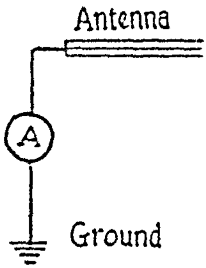


Fig. 150.—An A.C. ammeter connected between an antenna and the ground forms a simple detector for wireless waves.

3.12. Uses for Wireless

The amount of power that can be sent *through space* by *wireless in any one direction* and picked up by a receiving antenna is small in comparison to the amount radiated in various directions by the transmitter. The sending of electrical power by wireless is so inefficient that it seems unlikely that it will ever be used in place of ordinary wire lines in the power industry.

On the other hand, if the transmitted wireless signal is made the conveyor of signals in the form of dots and dashes, or of speech or music, it then becomes a valuable means of communication. It is in this sense that we all know it. The name wireless was applied when it was first used for telegraph purposes and the name radio has been popularly accepted since speech and music have been broadcast.

In the transmitters described above the amount of energy radiated on one reversal of the potentials of the antenna and ground is not great. On the other hand, it is possible to make the current alternate a tremendous number of times each second, so the total energy radiated per second may be quite large.

The early method for obtaining high frequency alternating current and voltage for transmitting was satisfactory for sending telegraph signals but useless for telephone broadcasting because it was not steady enough. Modern broadcasting depended on the discovery of a method which produces

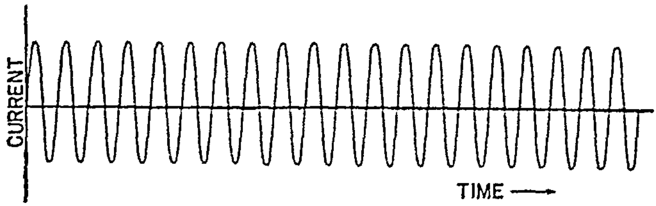


Fig. 151.—A graph to represent high frequency alternating current of steady amplitude.

alternating current at high frequencies but with steady amplitudes as illustrated in Figure 151.

4.12. Radio Telephone Transmissions

To place speech or music on the high frequency current at a transmitter it is necessary to have some electrical device which will control the current in the transmitter and which will control it in a manner corresponding to the sounds that are to be sent.

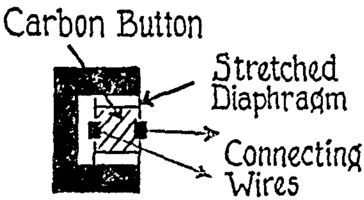


Fig. 152.—A carbon button microphone.

This problem was considered in Chapter 9 in connection with wire telephone systems. There a carbon granule microphone was described whose function it was to offer a variable resistance to the flow of direct current so as to make the amount of current vary with the sound reaching the microphone.

The only essential difference in the problem with wireless as compared to the wire telephone is that in the latter it is direct current that is to be controlled and in the former it is high frequency alternating current. A simplified drawing of the carbon microphone previously described is shown in Figure 152. When one talks into such a contrivance, the

stretched diaphragm moves in and out with the sound waves and so makes a varying pressure on the carbon granules. When they are squeezed hard they conduct electricity much better than when the pressure is light.

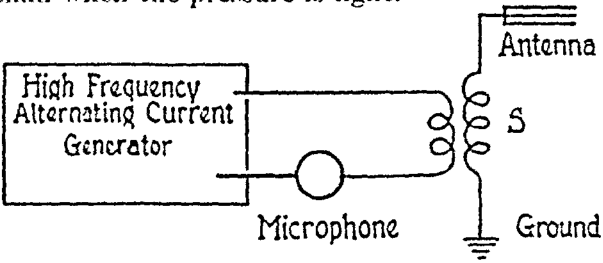


FIG. 153.—A simplified drawing to show the essential parts of a wireless telephone transmitter.

So a carbon button microphone can act like a varying resistance and when it is included in series in a circuit, as shown in Figure 153, it can control the amount of electricity that flows through it. Figure 153 illustrates in a very simplified form how a microphone can be used to control the flow of high frequency energy to an antenna in accordance with the

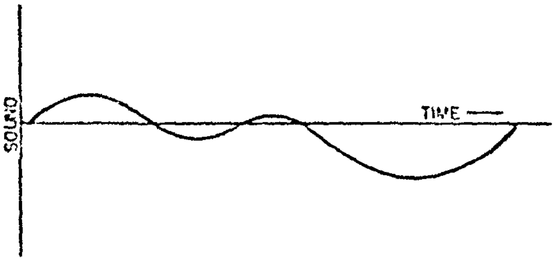


FIG. 154.—The sound wave hitting the microphone may be represented by this graph.

sounds striking the microphone. In actual practice, considerably more equipment is used than is shown in this figure, but the principle is the same.

When no sound is going into the microphone the current in the antenna might be represented by Figure 151 and " figure might also be used to represent the waves of magnetic energy flowing out from the antenna.

When the diaphragm of the microphone goes back and forth the current in the antenna might be represented as in Figure 155. Now the waves flipped off by the transmitter will be uneven in size and so the electric currents arising in some distant receiving antenna will also be irregular. In fact, they will look (except for a general reduction in amount) exactly like the currents in the transmitting antenna.

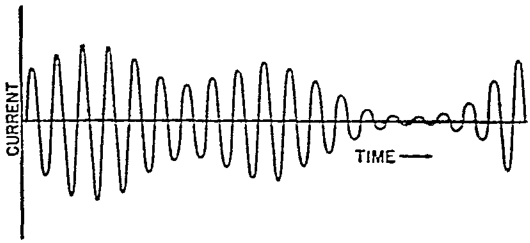


FIG. 155.—The current in the antenna as shown in Figure 151 is varied in amount by the sound which operates the microphone.

A microphone used as described above, or any other device similarly used to control an otherwise steady flow of power to a radiating system, is called a *modulator*.

5.12. Radio Telephone Receivers

Of course, if we are transmitting voice and music, we do not want a current indicator in the receiving system as shown in Figure 150. We want some kind of a gadget which will convert electrical energy back into sound.

This problem also arose in the case of the wire telephone and a device for changing electrical energy back into acoustical energy was described in Chapter 9. Such an instrument is called a telephone receiver if it handles very small amounts of power, or a loud speaker if it handles relatively large amounts of power.

However, if such a device is put into the circuit of Figure 150 in place of the ammeter, nothing at all will be heard. The rate of alternation of the high frequency waves of the wireless is too fast for the diaphragm to follow and if it did follow

them, the sound produced would be out of the range of the human ear.

Also, it is the controlling voice frequencies that we want to hear and not the high frequency signal.

Suppose that a device could be invented which would let the currents coming from the wireless signals flow in one direction in the receiving antenna-ground system but not in the other direction. Such a device is called a rectifier. Let us place this rectifier in series with telephone receivers as shown in Figure 156.

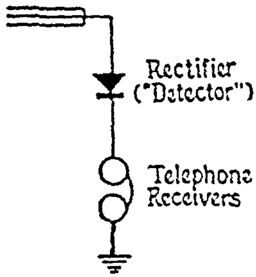


FIG. 156.—A rectifier and telephone receiver to replace the ammeter of Figure 150.

The current will now consist of a lot of high frequency pulses all in the same direction as shown in Figure 157. The *average value of these pulses is suggested* by the broken line. The diaphragm of the telephone receiver will move in and out as the average value of the pulses changes. A comparison of this average line with that of Figure 154 shows that the two are similar in shape except for amplitude.

A simple rectifier for such wireless use consists of a crystal of some mineral such as iron pyrites, or galena, against which

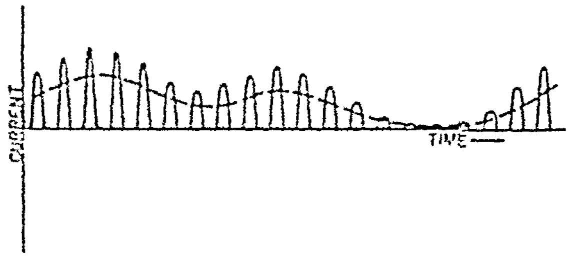


FIG. 157.—Current similar to that of Figure 155 after it has been rectified.

is placed a piece of metal. More efficient rectifiers are made with electron tubes which we will discuss in the next chapter.

Any rectifier used in a wireless circuit to obtain the signal currents from the high frequency wireless currents is called a *detector*.

6.12. Essentials of Radio Receivers and Transmitters

The radio transmitter consists essentially of (1) a device for producing steady high frequency alternating current, (2) a device for letting sound energy control the flow of this high frequency current (called a modulator) and (3) an "open" circuit for radiating the energy. (See Figure 153.)

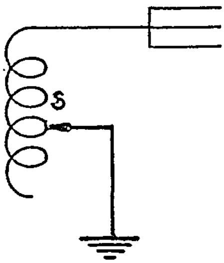


FIG. 158.—The antenna circuit of the wireless transmitter shown in Figure 153 can be tuned by making the number of turns used in the coil, *S*, adjustable.

The essential parts of the radio telephone receiver so far described are (1) an antenna-ground system, called a collector, (2) a rectifier, called a detector, and (3) a device for changing electrical power to sound (a telephone receiver or a loud speaker). (See Figure 156.)

7.12. Tuning

Both the radio transmitter and receiver can be improved tremendously by "tuning." This means that the circuits in each should be adjusted so that they will be electrically resonant (see page 520) to the frequency supplied by the high frequency alternator.

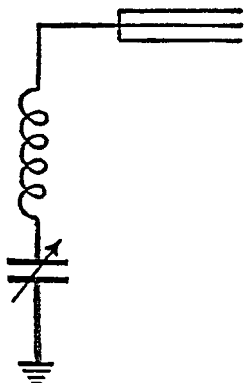


FIG. 159.—An antenna system may also be tuned by means of a variable condenser.

The antenna circuit of the transmitter (see Figure 153) can be tuned by winding the coil *S* in such a way that the number of turns used can easily be varied. Such an arrangement is indicated in Figure 158. The capacity in this circuit exists principally between the antenna and the ground. Another tuned circuit is shown in Figure 159 where the tuning is accomplished by means of a variable condenser.

The receiver circuit of Figure 156 can be greatly improved with respect to tuning by adding a high frequency transformer and a condenser as shown in Figure 160. The condenser is of the variable type

described in the last chapter. The operator can tune the receiver from one frequency to another by simply turning the condenser if he wishes to receive signals from several different stations on different frequencies.

8.12. Transformers for Radio Circuits

The high frequency transformers used in radio transmitters and receivers are wound on tubes with nothing but air for a core, and the number of turns of wire is small in comparison to those of the iron cored transformers described in a previous chapter. For a modern broadcasting station operating at a

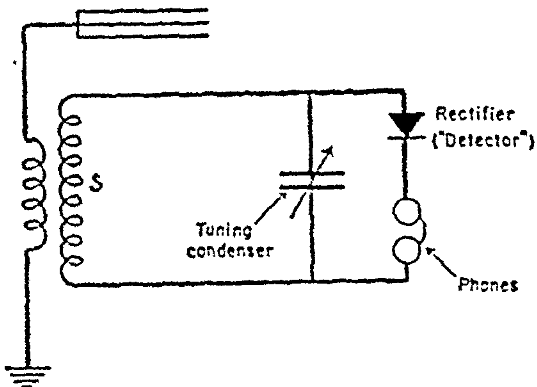


FIG. 160.—A radio receiver which can be tuned.

frequency of about 1,000,000 cycles per second the antenna coil might consist of ten to twenty turns of copper tubing used as wire and wound on a coil form of diameter 10 or 12 inches.

The corresponding coil in a radio receiver might have about 50 turns of quite small wire (say No. 26) wound on a bakelite tube 2 inches in diameter. (See coil *S* in Figure 160.)

9.12. Paths of Radio Waves

Radio waves start out from an ordinary transmitting antenna in all directions,—some parallel to the earth, some skyward, and others at all possible directions between these two extremes. Those that start out on paths parallel to the

earth are partly absorbed by the ground over which they travel. The paths also tend to curve somewhat so that they follow the earth's surface. The rate at which they are absorbed varies with the type of surface and also with the wave length. Short waves are easily absorbed in a few miles. Waves of the order of a few hundred meters, such as are used for broadcasting, are absorbed in about 100 miles and longer waves are good for greater distances.

Radio waves which start off at some angle to the horizontal, but less than straight skyward, soon reach the upper atmosphere. At heights of about 50 miles in the daytime and from 100 to 150 miles at night, the air has many of its

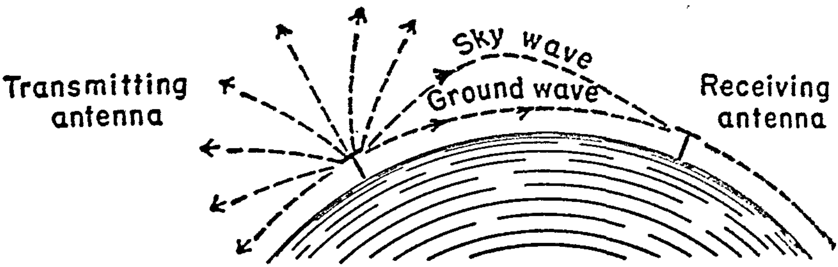


FIG. 161.—Various paths of wireless waves from a transmitter. Notice that waves may reach a receiving antenna by more than one path.

molecules in a state of ionization; that is, many of them have lost electrons. The air in this condition is in about the same electrically conducting state as the gas in the electric signs now used for advertising purposes.

When the radio waves try to pass from the ordinary air into the regions of ionization, they are bent, or gradually refracted by these conducting regions so that they start earthward again. This effect is shown in Figure 161.

At moderately nearby points, radio energy reaches the same spot by both the sky route and the path that follows close to the earth. Of course the sky wave travels a much longer distance,—its path may be several wave lengths longer than the more direct route. If these waves are in phase, that is, if both make the current in the collecting antenna move in the same direction at the same instant, much louder

signals may be received than if only one of the trains of waves were present. But if the signal from one path tries to move electricity in the antenna just opposite to the way in which the energy from the other path is moving it, interference results and the signal grows weak.

The path of the sky wave often varies, sometimes rapidly and sometimes slowly. Then the waves from the two paths may change from an aiding state to an opposing state. The result is that the signal received goes up and down in strength. We call this effect "fading."

At distances beyond the reach of the ground signal all radio reception is due to the sky wave. Sometimes the radio signal reaches a receiver by several different sky paths and fading can then take place as described above. However, the region of greatest fading is usually where the ground wave and the sky wave are about equal in strength. For most broadcasting stations this region will lie somewhere between 50 and 150 miles from the station depending on the wave length and the kind of land over which the waves pass.

Very short radio waves are quickly absorbed by the ground as indicated above. At the same time, very short waves, when sent straight up, are not reflected by the earth's upper atmosphere. For some angle less than the vertical, waves, if not too short, may be reflected sufficiently to return earthward. These come down at some distance from the transmitter. So there is a space surrounding the transmitter where no signal is heard. At some definite distance, depending on the wave length, the sky signal comes down. The distance from the transmitter to the point where the sky wave comes down is called the skip distance.

The ionized regions of the upper atmosphere that reflect the radio waves are called the "Heaviside layer,"—the name being that of a famous English physicist.

10.12. Wave Lengths and Frequency

Radio waves travel with the velocity of light which is approximately 186,000 miles per second or 300,000,000 (often written 3×10^5) meters per second.

The number of radio waves sent out per second by a transmitter is equal to the number of cycles per second of the high frequency current supplied to the antenna. If a radio station sends out 1,000,000 waves per second, the first one will have travelled 300,000,000 meters by the time the one millionth wave is emitted. At that instant 1,000,000 waves fill the distance of 300,000,000 meters. So each wave must be 300 meters long.

These statements may be summarized by saying

$$\text{Frequency} \times \text{Wave length} = \text{Velocity}$$
$$FL = V$$

This is the same relation that applies to waves in sound. The frequencies of many radio stations are listed in terms of kilocycles; that is, thousands of cycles, so that the station discussed above would be said to operate on a frequency of 1000 kilocycles. The frequencies of short wave stations are higher, as may be seen from the above equation and are often rated in terms of megacycles; that is, millions of cycles.

Some Important Facts

1. Electromagnetic energy can be radiated from an electrical circuit in which the electric fields are not closely confined.

2. Radiated electromagnetic energy can induce currents in circuits similar to that from which it was radiated.

3. Although inefficient for purposes of power transmission, electromagnetic radiation is very useful for communication.

4. A microphone somewhat similar to that used in wire telephony may be used to control the flow of high frequency alternating current in a wireless transmitter.

5. When an alternating current rectifier is used to separate the controlling audio frequencies from the high frequency wireless signals in a wireless receiver, it is called a detector.

6. The essential parts of a radio transmitter are, a high frequency power source, a control device called a modulator, and an antenna system.

7. The essential parts of a radio receiver are, a collector system (antenna), a rectifier (detector), a telephone receiver.

8. The efficiency of both wireless transmitters and receivers is greatly increased by "tuning" each circuit to the frequency of alternating current used.

9. Unless located within a "skip distance" or at a remote point, a radio receiver picks up both "ground" and "sky" waves. If they are in phase, reinforcement results; if out of phase, interference.

10. In all electromagnetic radiations including wireless waves, the product of wave length and frequency equals their velocity—approximately 186,000 miles per sec. or 3×10^8 meters per sec.

Generalization

Electromagnetic energy can be radiated from an open electrical circuit and it can produce electrical currents in similar circuits. The chief use of the effect is in the transmission of speech, music, or other intelligence.

Questions and Problems

Group A

1. What is different about the circuits of Figure 146 and Figure 145?
2. How can you detect that anything is being radiated from the transmitter of Figure 146?
3. Why would you expect the transmission of electrical power from one point to another by wireless to be inefficient?
4. What is the purpose of a microphone?
5. How can a microphone be used to control the amount of high frequency alternating current reaching an antenna?
6. Why does the source of supply of high frequency alternating current for a transmitting antenna have to give steady amplitudes of current?
7. Explain how rectifying the current in a receiving antenna makes it possible to get speech or music signals from the high frequency waves.
8. Show how a simple telephone receiver converts electric power to sound.
9. Why does tuning a radio transmitter or receiver improve its performance?
10. Describe the paths over which radio waves may travel from transmitter to receiver.
11. Explain the fading of radio signals.
12. From the daily newspaper find out the frequencies of all the broadcasting stations in your vicinity and calculate their wave lengths.

Group B

1. By means of a labelled diagram, explain Hertz's experiment which resulted in the "wireless" transmission of electric power.
2. Explain by a diagram how Marconi improved Hertz's apparatus.
3. Make a labelled diagram of a wireless transmitter capable of radiating electromagnetic waves that have been sound modulated.
4. From some source outside this text, learn what you can about Sir Oliver Lodge's famous "resonant Leyden jars"—and his other work in communication.

5. Diagram what you consider the simplest possible apparatus for "detecting" electromagnetic waves and translating these waves into telegraphic signals.

6. Diagram a simple receiver capable of detecting speech modulated electromagnetic waves and reconverting these waves into audible speech.

7. Explain by aid of wiring diagrams how sending and receiving circuits may be tuned to the same natural frequency.

8. Reconsider radio communication as presented in this chapter. Can you suggest further improvement?

9. Electric power may be transmitted through space at the speed of light. Does not this fact hold great promise of a more economical and more rapid method of delivering power for all sorts of electrically driven transportation machinery?

10. Discuss the evolution of radio communication as an outstanding example of the scientific method.

Experimental Problems

1. Construct two resonant Leyden Jar systems, and tune them so as to transmit wireless from one to the other.

2. Construct a working crystal receiving set.

ELECTRON TUBES

When a piece of metal or other material is heated, electrons come out of its surface somewhat as bubbles of water vapor come from boiling water. A filament such as is used in an electric lamp is a convenient device from which to get electrons in this way.

Such a heated filament, called a cathode, and a second piece of metal, called a plate, may easily be mounted in a glass bulb from which the air may be pumped. If a battery is connected between the filament and the plate so that the plate is positive, electrons will travel from the filament to the plate. If the battery is reversed so that the plate is negative, nothing at all will happen.

A simple tube of this type can be used to rectify alternating current, *for electricity can pass through the tube only in one direction.*

If a network of wires is placed between the filament and the plate, it can be used to control the flow of electrons from the filament to the plate. Such a network is called a grid.

A tube with a grid has the ability to cause large changes in the power flowing in the plate circuit in proportion to the power necessary to change the voltage on the grid. So the tube can be used to amplify signals.

Tubes with grids may be used for amplifying radio signals, for detecting radio signals, for creating alternating current power from direct current power, for amplifying small currents in industrial problems, and for making many kinds of laboratory measurements.

1.13. Improvements Needed in Early Radio Receivers

The simple radio receiving sets described in the last chapter obtained all of their electrical power by actual wireless transmission from the sending station to the receiving antenna. Since the waves spread out in all directions from a transmitting antenna, the amount of power that reaches a small receiving antenna falls off rapidly as the distance from the transmitter increases. What is needed to improve the receiving circuit is something which will let the small power that reaches the antenna act as a controlling device to release power in local batteries or generators. Then the loudness

of the audible signal at the receiver need not depend directly on the amount of power received by wireless.

Electron tubes have proved to be such devices and much of the development of radio as we know it is due to the invention and development of these tubes. They not only improve the radio receivers described in the previous chapter but they can also be used as generators of high frequency alternating current for use in transmitters. They have also been found useful on ordinary long distance telephone lines, on many industrial jobs, and in some types of laboratory measurements. They are so generally used that we shall study them in this chapter because of their numerous applications as well as because of their value in radio circuits.

2.13. Hot Objects Throw Out Electrons

Many objects (for example, lamp filaments) throw out electrons when they become hot. This fact was discovered in 1883 by Thomas Edison who performed a simple experiment with an ordinary electric lamp that had an extra piece of metal added to it as shown in Figure 162.

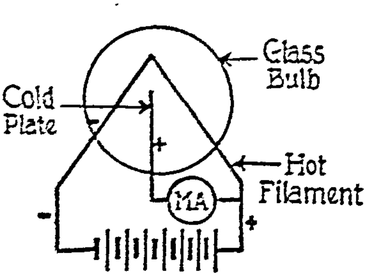


FIG. 162.—The emission of negative electricity from hot bodies was discovered by Thomas Edison.

The filament is heated with direct current and the extra plate may be connected to either the positive or negative end of the filament. The milliameter in this plate circuit reads zero when the connection is made to the negative end of the filament, but shows a current when contact is made with the positive end as indicated in the diagram.

This experiment shows that negative electricity is attracted to the plate when the plate is positive with respect to most of the filament. If the air has been well pumped from this bulb before the experiment is started we have to assume that the negative electricity comes out of the filament and moves across to the plate.

There is no record that Mr. Edison did anything with this discovery except to jot it down in his laboratory notebook. Later it was re-discovered by other scientists and its first use for radio purposes was started by an English physicist, J. A. Fleming.

3.13. A Two Element Tube Is a Rectifier of Alternating Current

In Figure 163 a tube is shown where the filament is heated by a battery of only a few volts. Electrons can travel from this hot filament to the plate; but since the plate is cold, and does not emit electrons, there are no electric charges to move back from the plate to the filament. So such a simple tube is a rectifier—electricity can move through it in one direction but not the other. Fleming thought that such a tube might be used in place of a crystal rectifier for the detector in a simple radio circuit.

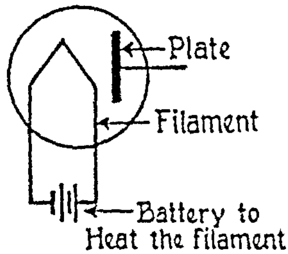


Fig. 163.—A simple two element electron tube with filament and plate.

Such a circuit is shown in Figure 164 which is similar to that of Figure 160 of Chapter 12, page 537, except for the substitution of the tube rectifier for the crystal. Such a receiver must still get all of its energy directly through the antenna by wireless and it does not turn out to be any improvement over the crystal detector.

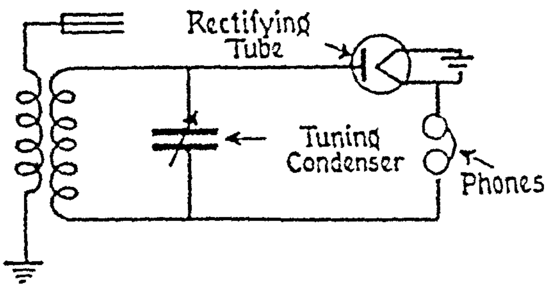


Fig. 164.—A simple radio receiver circuit using a two element electron tube for a detector in place of a crystal.

However, this kind of a tube is often used now for changing alternating current from a commercial power supply into direct current. If a tube and a direct current meter are

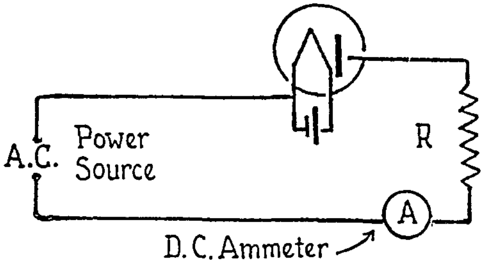


FIG. 165.—A simple two element electron tube may be used to rectify alternating current.

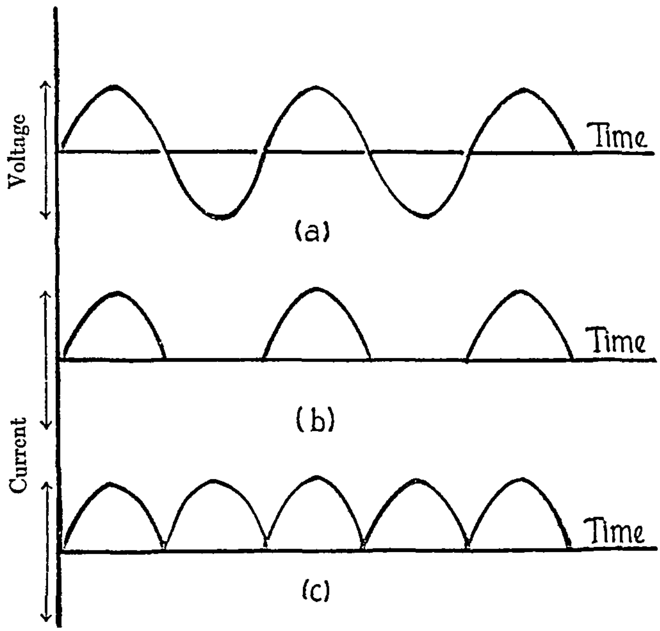


FIG. 166.—(*a*) Voltage in an alternating current power line. (*b*) Pulses of current from an alternating current source after rectification by the circuit of Figure 165. (*c*) Pulses of completely rectified alternating current.

connected in an ordinary alternating current power supply as shown in Figure 165, the meter will read, showing that the current through it is not alternating, but is always in the same direction.

The alternating voltage from the supply line may be represented by the graph in Figure 166(*a*). Current flows when this voltage makes the plate of the tube positive, but there is no current in the circuit when the plate of the tube is negative. Figure 166(*b*) shows the pulses of current that pass through the ammeter, the resistance and the tube. The blank spaces indicate that no current flows in either direction while the plate is negative and the filament positive.

More complicated arrangements of tubes may be used to invert and pass the negative parts of the alternating current cycles so that the pulses of the current in the same direction may be made to resemble those shown in Figure 166(*c*).

4.13. Experiments with Two Element Tubes

The amount of current that can be made to pass through such a simple two element tube depends somewhat on the voltage between the plate and the filament, but it depends even more on the size of the filament, the kind of material of which it is made, and at what temperature it is operated. Some experience with such tubes can be had in the laboratory by setting up the arrangement of Figure 167.

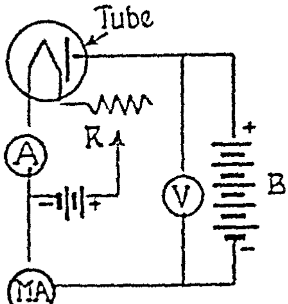


FIG. 167.—A circuit diagram to be followed in studying the action of a two element electron tube.

The filament is heated by current from a battery which can be controlled by the variable resistance, *R*. The ammeter in the filament circuit may be used as a check on the relative hotness of the filament. The filament-heating battery may be several dry cells or a storage battery depending on the type of tube used.

If only a tube with grids (which will be described below) is available, the grids should all be connected to the plate for this experiment so that the tube will behave as though it has only a filament and a plate. The grids must not be left "floating"; that is, disconnected.

The battery *B* should be several 45 volt blocks of small dry cells which can easily be bought under the name of "*B*" batteries. These batteries have several connectors so that the total voltage to the plate can be varied, and the value used can be read on the voltmeter shown in the diagram.

The milliameter reads the actual current carried in the plate circuit of the tube.

For the *first experiment* put about 100 volts on the plate and set the current in the filament to the value recommended by the manufacturer for the particular tube used. (If a tube with grids connected to the plate is used, the plate voltage for this experiment should be 50 volts or even less

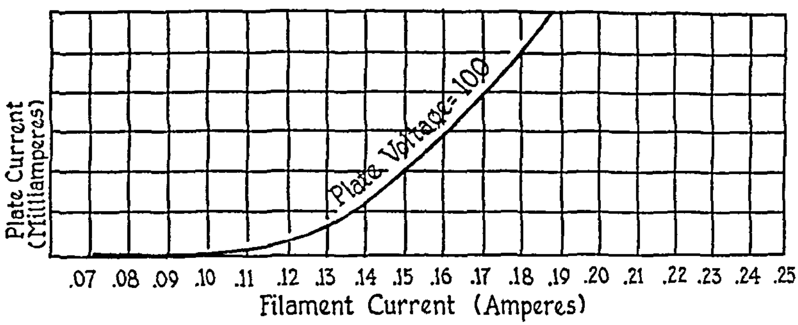


FIG. 168.—*For increasing values of filament current increasing values of plate current are obtained. This result indicates that more electrons are emitted by the filament at higher than at lower temperatures.*

instead of the 100 volts suggested here for a regular two-element tube.) Read the plate current on the milliameter, and then reduce the filament current by adjusting the resistance, *R*. This will make the filament cooler and it will emit fewer electrons. Read the current in the plate circuit under these conditions. This process should be repeated in small steps until the plate current is too small to read. The data may be plotted with filament current for one axis and plate current for the other. (See Figure 168 for a sample curve.) In this experiment the plate current depends on the number of electrons emitted per second by the filament, and the plotted curve shows the way in which the electron emission depends on the temperature of the filament.

A *second experiment* should be carried out with the filament at the manufacturer's recommended value of current for all readings. This time the plate of the tube should be placed at the smallest voltage which will give a readable current on the milliammeter. The plate voltage should then be increased in small steps and the plate current read each time. This data should be plotted with plate current for the vertical axis and plate voltage for the horizontal axis.

The data may be expected to show that with increasing plate voltage, the plate current increases up to a certain point;

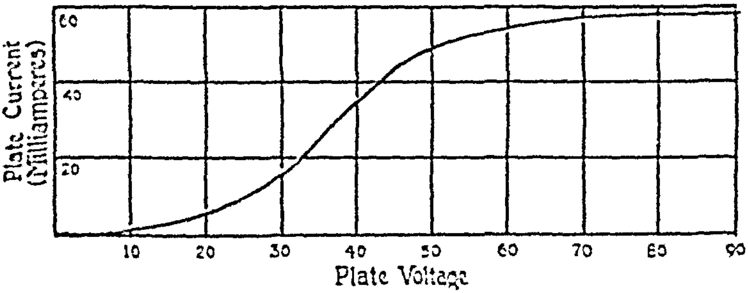


FIG. 169.—With a steady temperature of the electron emitter, plate current increases with increasing plate voltage until the latter is able to remove all the electrons as fast as they are emitted. Further increase in plate voltage cannot give an increase in current. This condition is shown by the upper right hand portion of the red curve.

but further increasing of the plate voltage does not give a larger plate current. This effect shows that all of the electrons that come out of the filament are moving directly over to the plate. A larger plate voltage cannot attract any more electrons per second because there are no more to attract. (See Figure 169 for a sample curve.)

With low values of plate voltage (less than 50 volts for the two-element tube used for the graph of Figure 169) some of the electrons emitted each second by the filament return to the filament instead of going to the plate. The lower the plate voltage the more pronounced is this effect. Under these conditions a cloud of electrons forms about the filament. It is called a "space charge."

When a tube is used to rectify alternating current the voltages are usually high enough to take the electrons to the plate as rapidly as the filament emits them. On the other hand, amplifier tubes which use grids always operate under "space charge" conditions.

5.13. The Addition of a Grid to a Simple Electronic Tube

Shortly after Fleming suggested the use of two element tubes for radio detectors, an American engineer, Lee DeForest, added a network of wires (called a grid) between the filament and the plate of the tube. This arrangement is indicated in Figure 170.

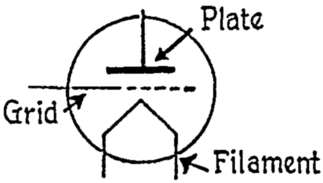


FIG. 170.—A three element electronic tube.

This three element tube may be set up in a circuit similar to that of Figure 167 except that an additional battery (called a "*C*" battery) is used to hold the grid at various potentials with respect to the filament. (See Figure 171.) An additional voltmeter, V_2, is used to read the grid voltage.

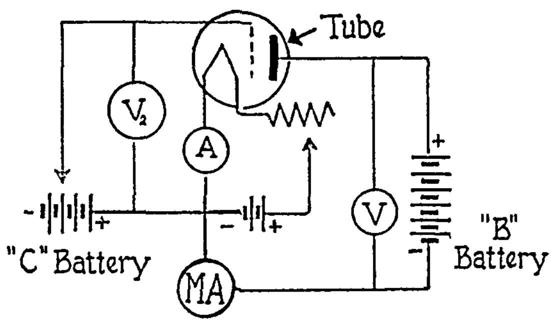


FIG. 171.—A circuit for the study of the effect of the grid in a three element tube.

The "*C*" battery should be one to which connections at 1.5 or 2 volts intervals may be made. As the connection to this battery is varied to give the grid different values of potential, both plus and minus, the plate current in the milliammeter will change even though the plate voltage and the filament current are left constant.

A typical curve is shown in Figure 172 where the plate current is plotted on the vertical axis and the grid volts on the horizontal axis.

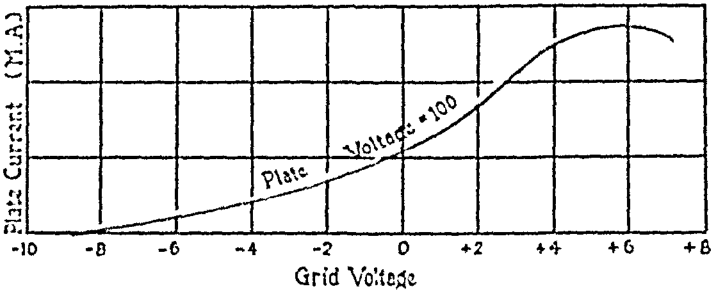


FIG. 172.—A curve to show the variation of plate current with grid voltage in a three element tube.

6.13. Experiment with a Three Element Tube

The student may perform the experiment described above using any simple three element tube (filament, grid and plate) on the radio market.

After a curve similar to that of Figure 172 is taken, the value of plate voltage should be changed and the data for

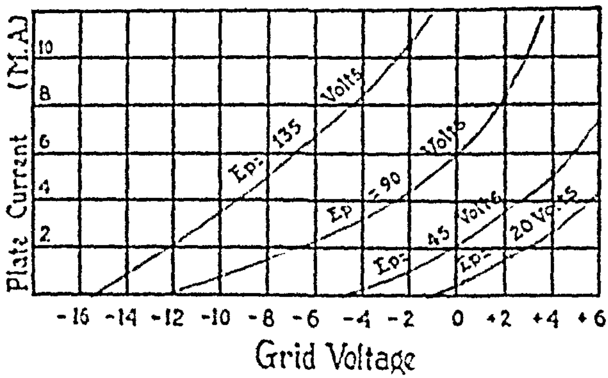


FIG. 173.—A family of curves similar to that of Figure 172, where each curve is taken for a different value of plate voltage.

various grid voltages taken again. Several values of plate voltage may be tried and all of the curves may be plotted on the same sheet of graph paper. Such a family of curves is shown in Figure 173.

One of the important things that can be learned from a study of these curves is that a few volts change of grid potential makes more variation in the plate current than a good many volts change in plate voltage. For this reason such a tube is said to be an "amplifier."

7.13. Use of Three Element Tube as an Amplifier

In Figure 174 a tube is set up in such a manner that alternating voltage E_g is placed on the grid with respect to the filament. This changing voltage on the grid will make the plate current vary so that the voltage across the resistor, R,

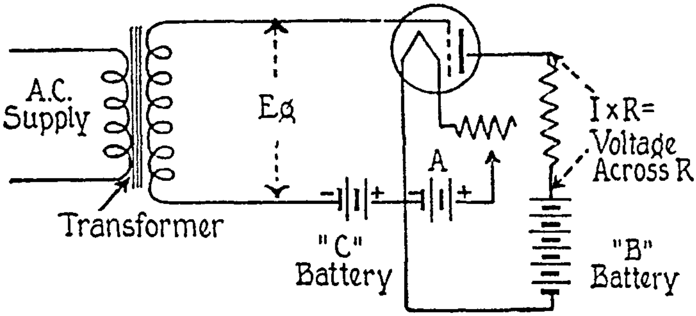


Fig. 174.—A three element tube can be used in a circuit to amplify alternating voltages.

will also vary (volts = current × resistance). This alternating voltage across R can easily be many times as large as the voltage E_g put into the tube. The voltages of the various batteries needed in the circuit of Figure 174 will depend on the tube used. Tube manufacturers supply such information for the tubes which they sell.

Figure 174 may be modified by the addition of condensers and an A.C. voltmeter as shown in Figure 175. The interested student may then actually measure the A.C. voltage developed across the resistor R and compare it with the voltage measured by placing an A.C. voltmeter between grid and filament as indicated. In this manner the actual amplification obtained from the tube can be measured. (For this type of measurement a high resistance A.C. voltmeter is required.)

Figure 174 shows an iron core transformer to handle signals of relatively low frequency. However, this type of amplifier can also be used on high frequency signals such as those created in a radio receiving antenna by wireless waves. In this case the iron core transformer of Figure 174 must be

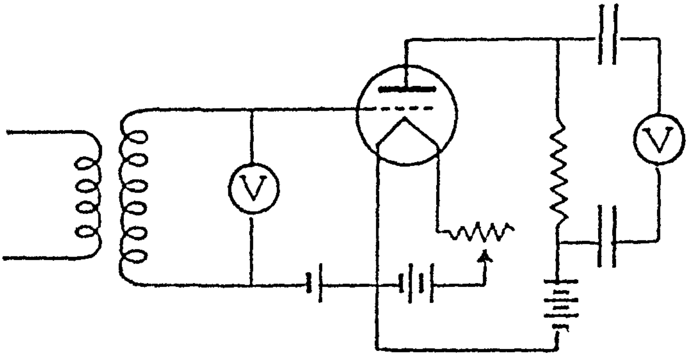


FIG. 175.—The circuit of Figure 174 modified for laboratory measurement.

replaced by an air core transformer, and for best results this transformer should be tuned with a condenser.

8.13. Three Element Tubes as Detectors in Radio Receivers

It is also possible to use a three element amplifying tube as a radio detector. A simple circuit of this type is shown in Figure 176. The student should now compare Figures 160 of Chapter 12, and Figures 164 and 176 of this chapter. The antenna and tuning circuits are identical in all three of these diagrams. In the first, a crystal rectifier is used as the detector. The second employs a simple two element tube and the third shows the manner in which the three element tube may be used.

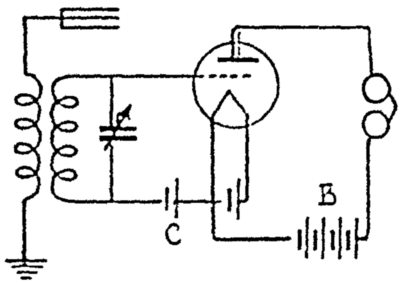


FIG. 176.—A simple radio receiver circuit using a three element tube as a detector. (Compare this circuit with that of Figure 164 where a two element tube was used as a detector.)

The first two circuits depend on the power received in the antenna by wireless transmission for the operation of the telephone receivers. The third circuit uses the wireless power to control the flow of energy from the local "*B*" battery. Louder sounds for a given strength of wireless signal may be expected from the telephones in this circuit than from those in either of the other two arrangements.

The principal difference in the circuit of Figure 176 where the tube acts as a detector and amplifier combined as compared to the cases described in this chapter where the tube acts only as an amplifier (see Figure 175) is the value of the grid potential supplied by the "*C*" battery. To make an amplifying tube behave as a detector, the negative voltage on the grid is increased until very little current flows in the plate circuit. (For example, about 6 volts for an ordinary three element tube operating as shown in Figure 176 with 45 volts for the *B* battery.) The radio signal from the antenna causes the grid to vary plus and minus about this -6 volt value and detector action instead of simple amplifying action is then produced. This detector action results from the fact that plate current can increase as the signal makes the grid less negative, but cannot decrease much as the grid goes more negative because the plate current is already almost zero.

Types of tube detector circuits, more efficient and more complicated than that of Figure 176, are often used.

Also circuits similar to that of Figure 176 are often operated at the proper grid voltage to provide amplification only as suggested in Section 7.13 above. Such a circuit is then followed by a second tube arranged as a detector. In fact, most radio receivers have several tubes arranged to amplify the signal before the detector stage is reached.

The details of the more complicated modern radio receivers are somewhat beyond the scope of the present course, and the interested student should look up the subject in any good text book on electron tubes or radio engineering.

9.13. Electron Tubes as Generators of Alternating Current

Figure 177 shows a tube and a circuit which will develop alternating current although its power supply may be entirely from direct current sources.

If any voltage change takes place on the grid, a change in plate current will occur at once. But this changing plate current flows in a coil T placed so that any changes in this current will induce voltages in the coil *S*. These voltages will at once be applied to the grid which in turn will change the plate current still more.

This effect builds up and in practice we find pulsating direct current in the coil *T* and alternating current in the cir-

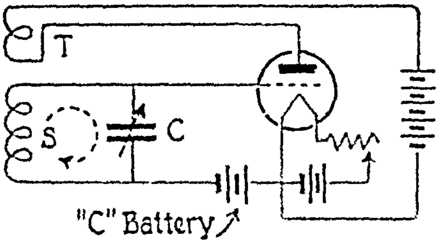


FIG. 177.—A three element electronic tube can be arranged in a circuit so as to act as a generator of alternating currents. Such a circuit combination is called an oscillator.

cuit made up of the coil *S* and the condenser *C*. Also we find that the frequency of this current is the natural frequency of this coil-condenser system. (On page 520 this frequency *f* is given by

$$f = \frac{1}{2\pi\sqrt{LC}}$$

where the capacity *C* is given in farads and the inductance *L* in henries.)

If the coil *S* consists of a great many turns wound on an iron core (like the winding of a small iron core transformer) and the condenser *C* is several microfarads, the frequency may be as low as 25 or 50 cycles per second. It is even possible to make a circuit oscillate at one or two cycles per second.

If the coil S has a few turns wound on a fibre tube and C is a small variable condenser such as is often used for radio purposes, the frequency may be of the order of one or two million cycles per second or more. This frequency can easily be changed by simply turning the condenser to a new setting or inserting a coil with a different number of turns.

There are many circuits in addition to that shown in Figure 177 which will make an alternating current generator out of an amplifying tube. All of them have some arrangement which feeds energy from the plate circuit back into the grid circuit.

The ease with which steady high frequency alternating current can be produced with such a simple arrangement has been a great boon to the development of radio telephony where any irregularity in the high frequency alternating current would introduce noise into the received signals.

High frequency power from such oscillators is also used for certain types of medical treatment now called "radiotherapy." Such circuits are also used for supplying power to induction furnaces that are used in refining some ores.

10.13. Screen Grid Tubes

The various elements of a thermionic tube such as filament, grid, and plate form condensers with one another. Of course the capacities of condensers made of these small parts are not large, but they are sufficient to affect the behavior of the tube in a circuit especially if high frequencies of current are being used. Usually the most troublesome of these "interelectrode" capacities is that existing between the grid and the plate.

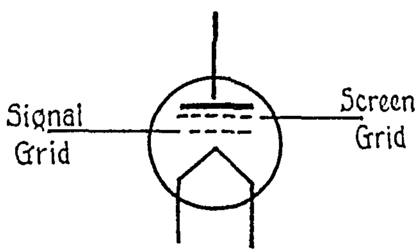


FIG. 178.—A second grid can be used to provide an electrostatic shield between the signal grid and the plate. Such an arrangement reduces the electrical capacity between the signal grid and the plate. The tube is called a tetrode.

This effect can be greatly reduced by placing a screen between the grid and the plate. A convenient arrangement

consists in winding a second grid between the signal grid and the plate, as is shown schematically in the diagram of Figure 178. It is common practice to connect this screen grid to a positive potential either equal to or less than that of the plate. The capacity of the signal grid to plate can readily be decreased to one or two percent of its former value by such an arrangement.

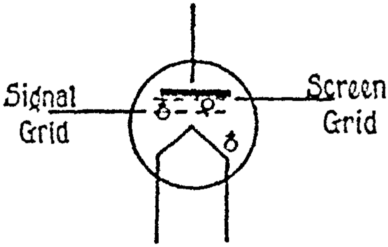


FIG. 179.—Electrons from the filament, as they strike the plate, often knock other electrons off the plate. These are called secondary electrons. In a three element tube they return to the positive plate, but in a screen grid tube they may go to the screen grid since the latter is also operated at a positive potential.

11.13. Pentode Tubes—Secondary Electrons

It frequently happens that when a fast moving electron strikes the plate of a tube other electrons are knocked from the plate as a result of the impact. Such electrons are called secondary electrons. In a simple three element radio tube such electrons are attracted back to the plate, but in a screen grid tube they are often attracted to the positive screen. This effect is illustrated in Figure 179.

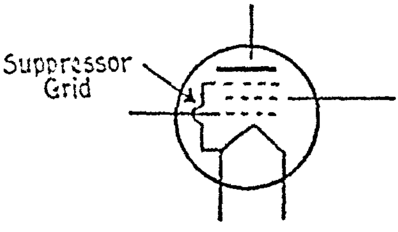


FIG. 180.—A suppressor grid at the potential of the cathode forces secondary electrons to return to the plate. The tube is called a pentode.

To prevent this loss of electrons to the screen, a third grid, called a suppressor grid, may be added to the tube. It is usually connected back to the negative side of the filament and hence repels the secondary electrons back towards the plate. The arrangement is shown diagrammatically in Figure 180.

It is practical to build pentodes with greater amplification than triodes and still keep the performance in circuits stable and reliable.

12.13. Other Uses for Thermionic Tubes

Most of the uses for thermionic tubes depend either on their ability to rectify alternating current, as in the case of the two element tubes, or on their ability to amplify signals, as in the case of tubes with grids.

Since a tube is sensitive to voltage changes on a grid, but requires almost no electrical power on the grid, it can be used to measure electrical voltages in places where the electrical power is very small. A tube with its accompanying circuit used in this manner is called a vacuum tube voltmeter. It is used in making many measurements in radio receiver designing laboratories and also in pure science laboratories.

Tubes are often used to amplify the feeble currents from light-sensitive devices (called photoelectric cells). They can control enough power to operate relays and so turn on lights, start motors, sound alarms, or do other useful things. (See Chapter 21.)

Amplifying tubes are used in connection with some types of thermometers used in science, and in some temperature control devices.

The amplification property of tubes with grids permits their use in oscillating circuits. Such arrangements convert power from direct current sources into alternating current. There are no mechanical moving parts in such an electronic generator, and the frequency of alternation can easily be varied between wide limits by using condensers and inductors of different sizes.

The purposes to which these remarkable tubes have been put are so numerous that every student of science should be expected to have some knowledge about their main principles of operation.

Some Important Facts

1. Incandescent bodies emit electrons.
2. A two-element tube, or Fleming valve, serves only as a rectifier.
3. A two-element tube is operated at such current and voltage values that practically all electrons emitted by the filament pass directly to the plate.

4. In a three-element tube, a "grid" whose voltage with respect to the filament may be controlled is inserted between the filament and plate to influence the flow of electrons from filament to plate.

5. Slight changes in grid potential may produce large changes in plate current.

6. Since small amounts of power in the grid circuit may control larger amounts of power in the plate circuit, the grid input is said to be amplified.

7. If the grid voltage is such that few electrons normally pass from filament to plate and the grid is now coupled to a receiving antenna, then the three electrode tube may serve as a detector of wireless signals.

8. An electron tube with one or more grids, suitably connected to a coil-condenser system, may serve as an oscillator.

9. Electron tubes may also serve as meters and relays for a variety of purposes.

Generalizations

1. In an electron tube, electrons emitted from an electrically heated filament pass through a vacuum to a positive plate.

2. This electron flow may be controlled by one or more grids whose potential may be varied.

3. Electron tubes may be used as rectifiers, amplifiers, and oscillators and in these capacities they have many practical applications.

Problems

Group A

1. What limits the loudness of a received radio signal when a crystal detector is all that is used?

2. In what way might a thermionic vacuum tube be expected to remove this limit?

3. What proof is given to show that hot bodies emit electrons?

4. Explain how and why a two element tube may be used as a rectifier of alternating current.

5. On the basis of electrostatic attraction and repulsion show why a grid might be expected to influence the flow of electrons from the filament to the plate of a tube.

6. What limits the total amount of electrons that may flow per second in a thermionic tube.

7. Explain the function of the tube used in a common battery charger.

8. Some tubes are said to be good oscillators. What does this mean?

9. Name and explain briefly three functions performed by electron tubes in radio reception.

Group B

1. Compare the effects on the amount of plate current in a tube when the grid voltage is changed and when the plate voltage is changed.

2. What is meant by saying that a tube can be arranged in a circuit so as to amplify signals?

3. Explain the action of a simple oscillating circuit,—with regard to power in the plate circuit and power in the grid circuit.

4. In what ways is a thermionic tube oscillator convenient as compared to a magnetic-mechanical generator of alternating current?

5. At what frequency would a thermionic tube circuit produce oscillations if the tuned circuit were made up of inductance of 0.5 henries and 8 microfarads of capacity? (Eight microfarads must be expressed in farads—0.000008 farads.) 79.6 cycles per sec.

6. At what frequency would the circuit produce oscillations if the tuned circuit were made up of an inductance of 0.0001 henries and 0.0005 microfarads? 711, 700 cycles per sec.

Experimental Problems

1. By squeezing in a machinist's vise, break the glass of several old radio tubes, and dissect them sufficiently to identify all essential parts. Account for the mechanical arrangement and spacing of parts. Preferably you should include several tubes with more than one grid.

2. Connect a two-element tube as shown in Figure 167. If only a three-element tube is available, connect the grid and the plate and so treat it as a two-element tube. Heat the filament of the tube with the current recommended by the tube manufacturer. Use dry cells or small storage batteries for the plate battery. Vary the number of cells used and read the current in the milliameter in each case. Compare your results with the graph of Figure 169.

3. Connect a three-element tube as shown in Figure 171. Change the value of the grid voltage by intervals of 1.5 volts and note the plate current in each case. Plot your results and compare with Figures 172 and 173.

4. Construct a working radio receiving set using a three-element tube as a detector.

X-RAYS

In an earlier chapter we studied the creation of waves of electromagnetic energy from a large electrical circuit. These we called wireless waves. They were quite long waves,—the shortest in general use being several meters and the longest several thousand meters.

In the present chapter we study x-rays. These, too, are electromagnetic waves which radiate through space. But they are extremely short waves—of the order of one billionth of a centimeter.

The first part of the chapter describes the method by which we might look for and discover x-rays with modern equipment and knowledge. Then the actual historical discovery of x-rays is briefly given.

In a study of the relations between the wavelengths of x-rays and the electric voltages used in producing them, a hint is given that nature uses energy in discrete little quantities, called quanta, or quants. This gives the impression that energy comes in little units just as electricity comes in the form of little particles called electrons and protons. The "quants" of radiant energy are called *photons*.

This chapter also gives some description of the operation of modern x-ray tubes and equipment, and then gives a short account of some of the uses of x-rays in medicine and in industry.

1.14. Speculations on How to Produce Short Electromagnetic Waves

In the chapter on wireless we learned that energy can be flipped off of an electrical system in pulses when electricity is moved to and fro in a circuit in which there are large open electric fields. This was the case with an antenna and ground such as is usually a part of a wireless transmitter. As the electrons are accelerated first in one direction and then in the other in such a circuit, energy is lost from the circuit and travels out through space.

This experiment might set us wondering as to whether or not there are other ways in which we could accelerate electr and produce radiant energy. **For example**, we coul

with individual free electrons such as those emitted by hot filaments as described in the last chapter. These electrons are accelerated as they leave a hot filament and travel towards a positively charged plate. When they strike this cold plate they are stopped almost instantly. We might ask ourselves if it would not be reasonable to expect some electrical energy to be torn loose from this system and radiated just as in the wireless case where large quantities of electrons in a large wire system are accelerated to and fro.

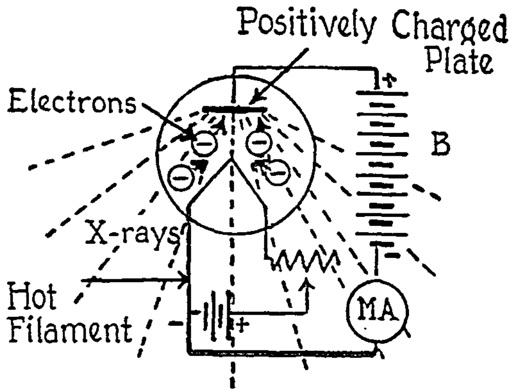


FIG. 181.—A two electrode electronic tube which might be expected to radiate electrical energy.

Figure 181 shows a simple two element tube arranged with the filament and plate at a somewhat greater distance than in the case of the tubes used for radio purposes. Such an arrangement permits the use of a large voltage between filament and plate and if any effect is produced it is reasonable to expect a large voltage to give greater results than a small one.

2.14. How to Discover Whether or Not Waves Are Being Produced

The next question that we must worry about is how we are to know that radiation is produced by this experiment, even if it does occur. Of course if long waves are produced, any of the types of detectors described for wireless waves would work.

But if the waves are extremely short we would have to look for some other effect in order to find out that they are present.

If the waves should be so short that they have a length of about the same size as that of atoms, we might expect them to do something to the electron system of an atom; such, for example, as ionizing the atom by knocking off one of its electrons. (See pages 373–379 for a description of atoms.)

If the air around a tube such as that of Figure 181 should be ionized, one of the easiest ways to discover this condition would be to place a gold leaf electroscope (see page 391) near the tube and to charge its leaves before the electric power is put on the tube. Then when the tube is started, if ions are formed in the air, they will be attracted to the gold leaf electroscope and discharge it.

If this experiment is actually carried out, we will find that the gold leaf electroscope does discharge rapidly near such a tube and so we may conclude that the tube radiates some kind of energy that has ionizing power.

3.14. How to Continue the Experiment

The next step in this organized attempt to discover new electromagnetic radiations might be to measure the wave lengths produced, or to try to find out how the radiation varies with the voltage in the tube, or to look for additional effects of the rays. In fact we would hope to examine all of these things.

The above description is an outline of how science would attack the problem and so discover x-rays today. We would call it "organized research," or the application of scientific method of discovery to a particular problem. Many problems in pure science, and nearly all inventions (that is, applications of science to practical problems) are worked on today by a planned attack of this kind.

4.14. The Actual Historical Discovery of X-rays

Actually, x-rays were discovered without such a carefully planned program, for they were discovered in 1895, many

years before the electron had been found and measured, before anything was known about the electrical nature of atoms, and before much was known about any kind of electro-magnetic radiations except wireless.

This was a period when the spectacular appearance of discharges of electricity in gas-filled tubes such as we now use for advertising purposes was first being investigated in a scientific way. In such tubes, matter seemed to be in an electrical state, and physicists were just finding out that some of the particles in the tube were positively charged and some were negatively charged.

Roentgen was one of these scientists who was trying to find out something further about discharge tubes. The tube on which he was working did not have much gas in it and so the glow was hardly visible. On the table near the tube a piece of *willemite* ore was lying, and Roentgen noticed that when the tube was turned on, the ore began to glow (or fluoresce, as the technical name is). This struck Roentgen as curious, and he investigated the effect and proved that some kind of radiation came from his tube.

We know now that the electrons in Roentgen's tube came from the ionized atoms of gas in the tube and that they produced x-rays when they were stopped by the plate of the tube.

5.14. Methods for Making Scientific Discoveries

In some ways, the discovery of x-rays might be called a scientific accident, for Roentgen was not looking in particular for radiation of this type and he had not planned his experiment with this end in view. On the other hand, he was carefully investigating the discharge tube and was alert to discover anything new or unusual that might appear. So the actual discovery of x-rays is an example of one type of scientific investigation; namely, the kind where an almost completely unknown subject is examined for whatever may be learned, as contrasted with the carefully organized type of research with something very definite in mind.

6.14. Modern X-ray Outfits

All of the early x-ray tubes depended on the ionization of a little gas left in the tube for the supply of electrons. Modern tubes all use a hot filament to emit electrons and the tubes are pumped to as good a vacuum as can be obtained.

The voltages used on the plate (called a target) are usually large, running from about 50,000 volts to a million or more. Of course x-rays can be produced with smaller voltages, but voltages of the order of several hundred thousand are the ones commonly used. To get such a voltage supply it is customary

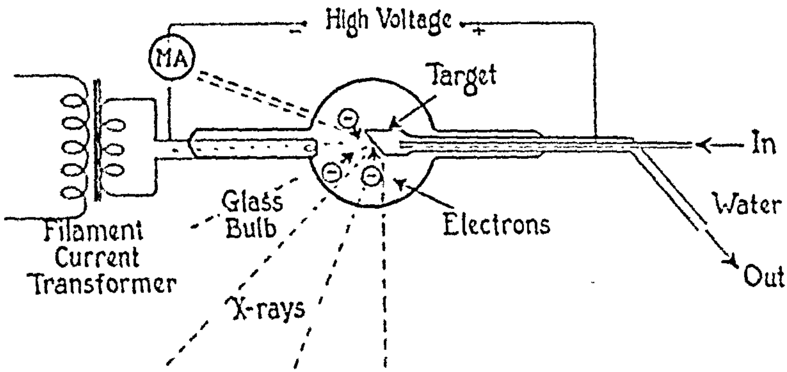


Fig. 182.—A water cooled x-ray tube of the type used for medical treatment.

to start with alternating current which can be stepped up to high voltage by means of transformers. This alternating current is then rectified either with commutator types of devices or with two element tubes as described in the last chapter.

A modern tube such as is used in hospital work is shown in the drawing of Figure 182. This particular tube has a water cooled target, for the electrons bombard the target so hard that it would become red hot and possibly even melt if it were not forcibly cooled. The potentials used on such a tube are of the order of 200,000 volts.

7.14. Controlling the Intensity of X-rays

Sometimes a storage battery is used to heat the filament of the tube and sometimes the filament is heated by alternating current supplied through a transformer as shown in the dia-

gram of Figure 182. The number of electrons produced each second will depend on the size of the filament and on how hot it is, just as in the case of any electronic tube. The amount of electron emission is measured by means of a milliameter in the plate circuit of the tube.

The intensity of the x-ray beam will depend on the number of electrons hitting the target each second and so it can easily be controlled by adjusting the amount of heating current in the filament.

8.14. Controlling the Wave Lengths of X-rays

The wave lengths of x-rays depend on the voltage that is applied between the filament and the plate. The greater the

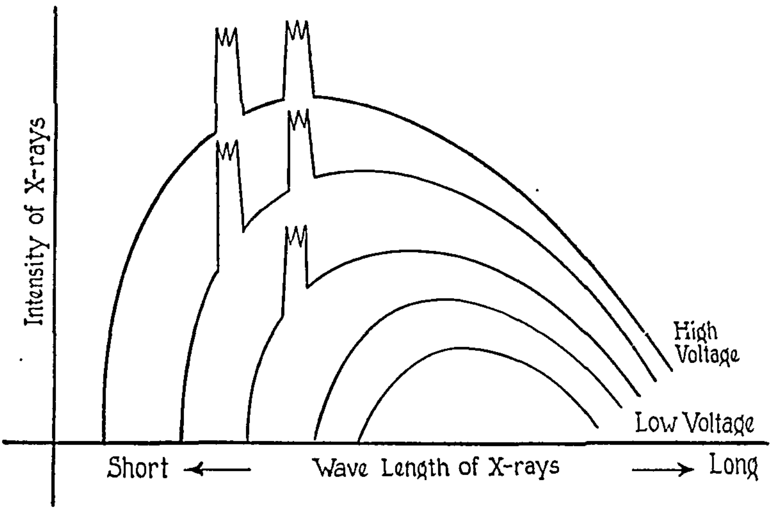


Fig. 183.—A qualitative family of curves to show the general nature of x-rays produced from a given target for various values of voltage between filament and target. Note that high voltage produces all the wave lengths obtained with lower voltages and additional x-rays at shorter wave lengths as well. The sharp increases in intensity at some points on the curves are characteristic wave lengths for the particular element of which the target happens to be made.

voltage the shorter the waves. Actually quite a wide variety of wave lengths are produced for any given voltage, but the shortest possible wave length is limited for any one voltage. When the voltage is increased all of the wave lengths produced with a lower voltage are obtained and some shorter ones also.

There appears to be a definite relation between energy in electrical or mechanical form and energy in the form of radiation. This relation was expressed by Albert Einstein. It is: Mechanical (or Electrical) energy of the electron = a constant multiplied by the frequency of the waves of the radiation; or in symbols

$$\text{Energy} = hf$$

where f is the frequency of the waves and h is a constant called Planck's constant. The value of h is approximately

$$6.547 \times 10^{-27} \text{ ergs} \times \text{seconds.}$$

The energy is then given in ergs.

If an electron falls through a large voltage, its energy is greater than if it falls through a small voltage, and so it produces radiation with higher frequency. (Higher frequency means shorter waves, for frequency multiplied by wave length equals velocity, as we have seen before.)

X-rays may be thought of as consisting of little groups of waves, each group containing the energy, hf. The total radiation of x-rays from the surface of the target is made up at any instant of millions and billions of these little groups of waves. This is a somewhat different picture than the old one where we thought of a wave spreading out from the target in all directions as water waves spread out when a pebble is dropped into the water.

9.14. Effect of X-rays

Experience shows that x-rays pass through ordinary matter fairly easily, although there is always some absorption. Substances with atoms of large atomic weight (like lead), have greater stopping power than other materials.

From the above discussion we may suppose that now and then a single group of x-ray waves (containing energy = hf) strikes an atom and loses its energy in the business of knocking an electron from the atom. So that little bit of x-ray beam is completely absorbed. But on the average, any particular

group of waves travels past a good many atoms before it hits in just the right manner to knock an electron from an atom and so lose its own energy in the process.

X-rays of short wave length have more energy in each little group than x-rays with long waves but their chance of hitting an atom in such a way as to be absorbed seems to be less. So short x-rays are more penetrating—they pass through materials better. They are often called hard x-rays while long wave x-rays, which are easily absorbed, are called soft x-rays.

Potentials of the order of a few hundred volts produce waves of a few ten-millionths of a centimeter. Potentials of the order of a hundred thousand volts produce waves of the order of a trillionth of a centimeter (one-millionth of one-millionth of a centimeter).

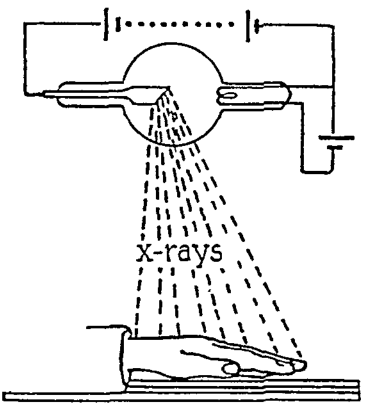


FIG. 184.—Drawing to show the method by which a shadow picture of the hand can be made with x-rays.

10.14. Effects of Ionization by X-rays

From the above description we may believe that the principal result of x-ray absorption is the ionization of atoms. This statement is probably true, but this ionization often produces additional important effects in the material acted on. For example a photographic plate will develop black after exposure to x-rays just as if it had been exposed to visible light.

If something like one's hand is held between the x-ray tube and the photographic plate, some of the x-rays will be absorbed by the flesh, and especially by the bones of the hand; so that less intensity of x-rays will reach some parts of the plate than others. When the plate is developed it will show a sort of shadow outline of the bones and flesh of the hand. (See Figures 184 and 185.)

For such work the plate is kept in a holder or envelope which protects it from ordinary visible light, but which offers little stopping power to the x-rays.

11.14. Effects of X-rays on Tissue

When x-rays are absorbed by flesh and bone, it is probable that ionization of atoms takes place just as in other cases of x-ray absorption. But these atoms belong to living cells and ionized atoms are chemically active. The result is that the cell becomes abnormal. It may die, its normal growth may be stunted, or it may be stimulated into unrestricted growth.

The apparent effect to the person exposed is that he has received some type of burn. This appearance may not be noticeable until some time after the exposure. A mild exposure does not destroy enough cells to be noticed, but a bad x-ray burn heals slowly if at all.

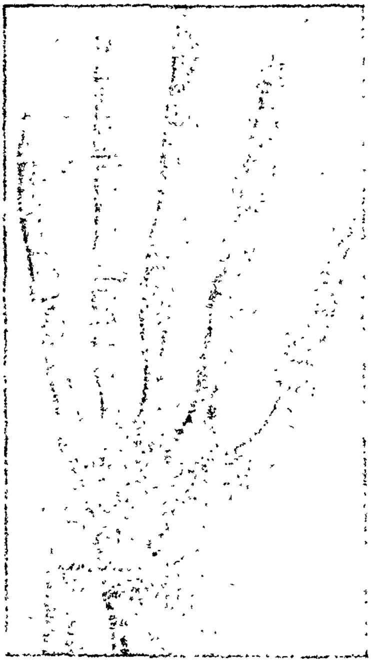


Fig. 185.—X-ray shadow picture of a hand.

12.14. X-rays for Medical Treatment

Experiments with x-rays seem to show that healthy body cells are affected less than unhealthy ones. So x-rays offer themselves as a treatment for many abnormal conditions such as malignant growths (cancers), benign growths (tumors), and some types of skin diseases.

An attempt is made to shield the part of the body not to be treated with lead sheets, but some dependence is put in the fact that the abnormal cells will be more affected than the healthy ones. Exposure is made on the diseased area and

often times the abnormal growth is partly or completely killed and in time will slough off.

Growths inside the body are treated with more penetrating rays than those near the surface, and considerable success in treating cancer of the stomach, uterus and other internal organs is reported.

Radiation of this type or combination of radiation and surgery represents the only cure for malignant growths at the time of writing this text. The percentage of cases cured is fairly good if treatment is obtained in early stages of the ailment.

13.14. X-rays for Medical Diagnosis

The taking of pictures as described above is quite generally used nowadays for checking the positions of broken bones before and after setting, and also for determining whether or not a break exists. This is common knowledge to all of us.

Sometimes a screen made of glass coated with a material which will fluoresce when irradiated with x-rays is placed in the beam of x-rays in place of a photographic plate. If a person places his hand in the beam, or stands in the beam, his shadow picture becomes visible on the screen.

Screens of this type are sometimes used so that a surgeon may see what he is doing when setting a complicated break. They are also used to watch the passage of some fluid which will absorb x-rays as it goes through the stomach and intestines. The doctor can often learn a great deal about the ailments of a person from these examinations.

Sometimes pictures are also taken so that the doctor can study the position of intestines, or look for diseased regions in the lungs or other tissues at his leisure.

14.14. X-rays in Industry

X-rays have had some applications in industry similar to those in medicine because they enable one to look through pieces of metal and so look for concealed flaws. Recently they have also been used in a study of the structure of threads,—cotton, wool, rayon—such as are used in making cloth. They

are also useful to the metallurgist, for they enable him to examine the crystal structures of many minerals.

Some Important Facts

1. It is reasonable to suppose that a two element electron tube, using very high potential difference, might emit radiant energy of short wave length.

2. These high frequency short waves should ionize the surrounding air and therefore discharge a nearby electroscope.

3. Having detected such radiations, we might reasonably try to determine:

   a. Causes for their variation in intensity and frequency.
   b. Their useful effects.

4. Actually these x-radiations were discovered accidentally—or incidentally—due to their fluorescent effect.

5. This discovery of x-rays was not accidental in the ordinary sense, since Roentgen was carefully investigating any and all effects of electrical discharge tubes.

6. Most modern x-ray tubes operate at voltages of several hundred thousand.

7. The intensity of an x-ray beam depends on the current heating the filament.

8. The frequency of x-rays emitted by a tube depends on the voltage between the electrodes.

9. Short x-rays (high frequency) are more penetrating than long x-rays.

10. A principal effect of x-ray absorption is the ionization of atoms, which depends on the frequency of the rays and the nature of the atoms involved.

11. X-rays penetrate living tissue to variable degrees.

12. One effect of this penetration is to destroy the tissue, perhaps by ionization of atoms. In general, healthy cells are less easily destroyed than unhealthy ones, which fact is utilized in the treatment of cancer.

13. The varying degree to which x-rays penetrate different tissues is also utilized in the fluoroscope and the taking of x-ray photographs.

14. This last use of x-rays is not confined to living tissue but has various industrial applications.

Generalization

When electrons travel through a vacuum tube under a pressure of several hundred to several hundred thousand volts, they strike the positive plate with sufficient energy to emit electromagnetic radiation, called x-rays.

X-rays differ from visible light rays in that their frequencies are much higher, and hence their wave lengths are much shorter.

Many common effects of x-rays are due to the varying degree to which different atoms absorb and are ionized by such x-rays as they intercept.

Problems

Group A

1. Make an outline showing how you would go about trying to discover x-rays today.
2. In what ways was Roentgen's discovery an accident?
3. In what ways was Roentgen's discovery made by scientific method?
4. Make a list of the things that could be used to detect the presence of x-rays.
5. In what way can the intensity of x-ray radiation be controlled?
6. How can the wave lengths of the x-rays produced be varied?
7. What is the effect of x-ray radiation on a photographic plate?

Group B

1. Explain what is meant by an x-ray photograph.
2. Explain the idea of x-ray radiation consisting of little groups of waves and show how this idea is used in explaining the absorption of x-rays by atoms.
3. What effects do x-ray radiations have on living tissue?
4. How are x-rays used in the treatment of abnormal growths?
5. How are x-rays used in trying to determine the ailments in the alimentary tract? In the lungs?
6. Compare the production of long electromagnetic waves (wireless) and extremely short electromagnetic waves (x-rays), pointing out similarities and differences.
7. Why might you expect the effect of wireless waves and of x-rays to be different so that different means have to be used to detect them?
8. What is meant by fluorescence and by phosphorescence? How did these phenomena feature in the discovery of x-rays?
9. From current scientific literature, discover the results of any experiments of x-rays on germ plasm.

Experimental Problem

1. Using a small x-ray tube, an induction coil and a dry cell in a darkened room, test several common substances with a view to discovering any that are phosphorescent or fluorescent.

THE NATURE OF LIGHT

One of the more useful groups of electro-magnetic waves is the one to which human eyes respond. It is called light. The eyes are sensitive only over a very narrow range of wave-lengths, but in this range we get sensations of color as well as being able to see the outlines of objects.

If the space through which light travels is a vacuum or is filled with a uniform substance that is transparent, the light moves along straight lines. This fact may be noticed in the shadows that objects cast and in the operation of the "pin hole" camera.

We see objects by the reflection of light from their surfaces unless these surfaces are perfectly smooth. In the latter case we appear to see the source of light itself in the reflection.

1.15. Visible Radiation

In an earlier chapter (see page 528) we learned that waves of an electro-magnetic nature were radiated away from "open" electric circuits when there was alternating current in these circuits. These radiations were called "wireless." The discovery, we learned, was made by Heinrich Hertz in an attempt to test a theory of the English physicist, Clerk Maxwell.

Maxwell had predicted that ordinary visible light was electric and magnetic in character. The waves obtained by Hertz were much longer than those of visible light and they produced no effect on the human eye. But these experiments did prove that Maxwell was correct in predicting that electro-magnetic energy could be radiated.

One thing of special interest in the production of wireless waves is that the energy is radiated when electricity is accelerated to and fro in an electric circuit. In the chapter on x-rays (see page 561) this fact led us to speculate on what would happen if individual electrons could be gotten up to high speeds and then stopped abruptly. We now know that electro-magnetic energy is radiated in this case also. The radiations (called x-rays) consist of very short waves as com-

pared to the waves produced by the wireless transmitter where great quantities of electrons are accelerated to and fro in large electric circuits. In fact, the x-rays have wave lengths that are short even in comparison to those of visible light and so they produce no response in the eye.

To produce electro-magnetic radiations with wave lengths between those of wireless on the one hand and those of x-rays on the other, we might hope to find some method for accelerating electricity in addition to the methods used in those cases. In the first case we had large quantities of electrons moving in large conductors. In the second case we had individual electrons traveling in a vacuum and being stopped by a massive target.

Another possible scheme would be to accelerate the electrons which are attached to atoms. In the discussion of the nature of atoms in an earlier chapter (see page 374) we learned that electrons are spaced at some distance from the nucleus of each atom. If the positions or motions of these electrons could be changed we might again expect electro-magnetic radiations. We now believe that light such as that given off by gas filled signs commonly used for advertising is produced in this manner. Light from hot solids (such as the filament of an electric lamp) involves more complicated action of the electrical parts of atoms.

These beliefs as to the nature of visible light and the methods of producing it are now generally accepted. But long before anything was known about the details of producing light—long before Maxwell's predictions concerning its nature—the ordinary behavior of light was studied because it is so useful to man.

Controversies were waged as to whether light was a wave motion or whether it was like a beam of particles. And in the meantime experimenters studied its actions—learned how to make mirrors of various shapes to reflect it—how to make prisms to separate it into colors—how to make lenses with which to make telescopes, microscopes, cameras, eye-glasses—how to measure the velocity with which light travels.

Light is so generally useful to man that, in addition to a study of its nature and causes, its behavior and applications also merit our attention. The remainder of this chapter as well as the chapters immediately following are devoted chiefly to this practical side of visible light.

2.15. Wave Lengths of Light—Color Sensations

Measurements on ordinary light show that it really is a wave motion of an electromagnetic type the same as the wireless waves and the x-rays, but lying between them in wave length.

The shortest wave of light to which the human eye responds is about 0.000038 cm. in length. The sensation produced is that of the color violet. The longest wave that gives us a sensation of sight is about 0.000078 cm. in length. The color sensation produced is that of red.

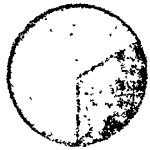


FIG. 186. When a pin wheel of the proper coloring is spun, a blending of the colors produces a white light sensation.

Wave lengths of light between these two values produce the sensations of the various well known colors of the rainbow, violet, indigo, blue, green, yellow, orange, red. Of course, there is no sharp dividing line between any of these colors as one realizes after a long argument with his neighbor over whether a piece of cloth is green or blue, for example.

When all of these colors are present in about equal proportions the combined effect is called white. A good imitation of white light for some purposes can also be made by mixing blue, green, and red in equal quantities. Those three are often called the "primary colors." A simple experiment in color blending may be carried out by mounting a piece of card board with a pin as shown in Figure 186. The card may be colored as indicated or in any manner desired. The blending of the colors is produced by simply spinning the card as a pin wheel.

3.15. Wave Length—Frequency—Velocity

All electromagnetic radiations travel through a vacuum with the same velocity; namely, 186,000 miles per second or

3×10^{10} cm. per second. The velocity is slowed down slightly by air—to greater extents by glass and other transparent substances. The amount of slowing down depends on the wave-lengths and is different not only for large changes in wave-length such as are found in comparing wireless waves with visible waves, but is slightly different for even such small changes in wave-length as exist among the different colors of visible light.

However, for most practical purposes these changes may be neglected for light traveling through air and the velocity given for a vacuum may be considered to apply approximately for any wave length of light in air.

If we multiply the number of waves of light produced by a source in one second, by the length of each wave, the product should be the distance traveled by the first wave. In other words

$$\text{Frequency} \times \text{Wave length} = \text{Velocity}$$
$$fL = V = 3 \times 10^{10} \text{ cm. per sec.}$$

Hence if a wave length of light is given, it is a simple matter to compute its frequency and vice-versa.

4.15. The Color of Objects

The above description of the relations between wave length and color sensations assumes that we can produce light of whatever wave length is desired. This is more or less true so far as making lamps is concerned. For example, table salt in a flame produces yellow light (from the sodium in the salt). Yellow is also obtained from helium in a gas-filled sign. The predominant color from a neon sign is red, etc. But we know that when white light shines on various objects they may appear red, or blue, or some other color. So we must distinguish between sources of light and objects seen by reflected light or transmitted light.

When light strikes an object it may pass through it almost completely. In this case the object is called *transparent.* An object through which light penetrates almost not at all is

called *opaque.* There are some substances through which light passes to a fair extent, but in a more or less irregular fashion. Milk and porcelain are good examples. They are called *translucent.*

When light strikes an object we may expect it to be reflected, transmitted, or absorbed. In many cases all three things happen.

The exact proportions between transmitted light, absorbed light and reflected light will differ from one object to another; and for the same object they may be quite different for one wave length of light than for another. For example, one group of waves may be transmitted much better than the others and another group may be reflected unusually well, while the remainder is absorbed. In this case, the substance will appear to have one color when it is placed between the observer and a source of white light and another color when it is looked at by reflected light. The student may easily experiment with colored solutions in clear bottles to find this effect.

Any object when viewed by reflected light, will appear to have some definite color if it reflects that color well and the others extremely poorly. So a red book is one whose cover reflects red light. It will look red when white light shines on it and also when red light is used to illuminate it. But if a pure blue light is used, the book will appear black since it can not reflect the wave lengths that produce the blue sensation.

Spectacular experiments can be performed by placing objects of various colors on a table covered with a black cloth and then using lights of different colors for illumination. For such an experiment the light from a Bunsen burner with a little sodium compound (such as ordinary table salt) placed on it may be used for a yellow source. A neon tube will give chiefly red light. A mixture of yellow and green may be obtained from a mercury vapor lamp.

5.15. The Path of Light Rays

If the surroundings through which a beam of light travels are uniform, (for example, a vacuum, or ordinary air) the

light moves in a straight line from one point to another. For example, in Figure 187 we picture waves of light radiating from a source, *S*. The path of that part of the wave that has travelled from *S* to *B* is a straight line and the light travelling along this line is called a ray.

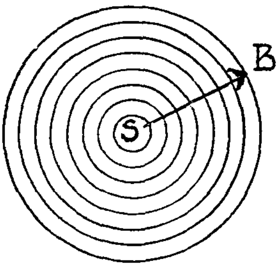


FIG. 187.—Waves of light spread out in all directions from *S*. Any point on a wave moves out in a straight line. We can say that a ray of light travels on a straight line from *S* to *B*.

Of course a line has no dimensions except length and if we want to talk about a real quantity of light moving from *S* to *B* we shall probably call it a beam or a pencil of light. In cases where we are interested in directions and positions of objects we usually refer only to "rays."

6.15. How We See Objects

If one looks directly at a lamp or other source of light, it is obvious that the light travels directly from the source to the eye. However, if we look at an ordinary opaque object the light which reaches the eye must have come from some other source since the object itself is not creating light. So we see such an object by means of the light which it reflects.

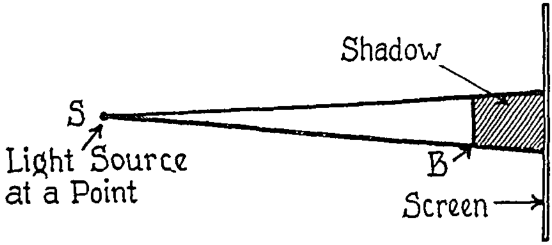


FIG. 188.—The object, *B*, causes a shadow on the screen from a point source of light.

All objects that we look at are large in comparison to the waves of light to which our eyes respond. Even with the aid of a microscope (which we shall study later) we cannot expect to see atoms or other objects that are smaller than the waves which affect our eyes. For if the objects are smaller than the

waves, we cannot expect the waves to be reflected from them in such a manner as to show up their details of outline.

We may study this effect in water waves as well as with light. If a large ocean steamer stood crosswise to water waves, we could get some idea of the size of the steamer by observing the effect on the waves on the leeward side of the ship. But, if a small iron pipe stood upright in water its effect on large waves would not be sufficient for one to notice that the pipe was present. Similarly, with light, an object cannot be clearly outlined unless it is large as compared to the light waves.

7.15. Simple Shadows

The fact that light travels in straight lines has many useful applications. For example, in Figure 188, light from a point

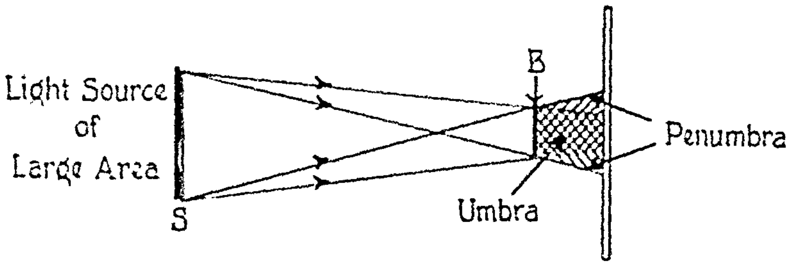


FIG. 189.—The nature of the shadow on the screen changes when the source of light has large area.

source *S* is stopped by the opaque object *B* and so casts a moderately sharp shadow on the screen. If *S* were a light source of large area. (see Figure 189) the shadow would consist of two parts, the central one where the shadow is complete and an outer region where only part of the light from *S* is stopped by the opaque object. The region of complete shadow is called the "umbra" and that of partial shadow is called the "penumbra."

An effect of this kind is responsible for eclipses between the sun and the moon and earth. One such possible eclipse is shown in Figure 190. (In order to show this effect in a small drawing, the sizes of the objects are greatly exaggerated in

proportion to the distances, the moon is too large with respect to the earth, and the earth is too large with respect to the sun.)

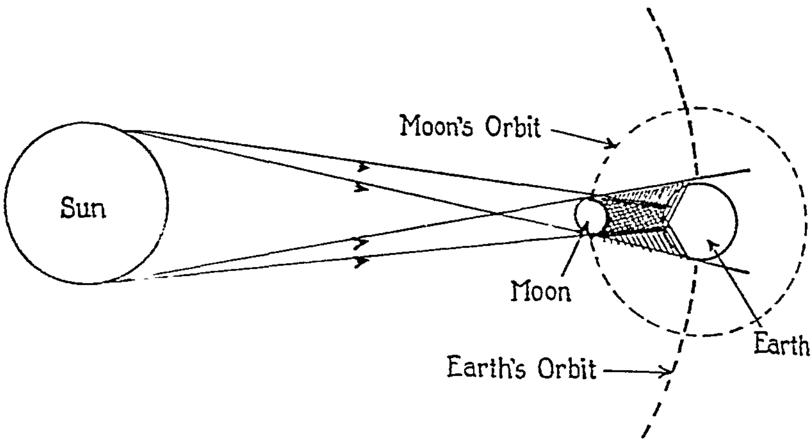


FIG. 190.—An eclipse of the sun on the earth caused by the moon.

8.15. The Pinhole Camera

Another effect of light travelling in straight lines is illustrated in Figure 191. L is a good black curtain in a window It has a very small hole at A. The arrow O represents a tree.

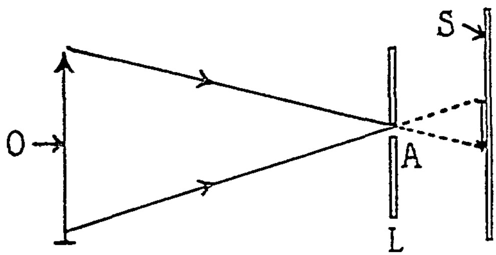


FIG. 191.—An inverted image of the object at O is found on a screen after the light rays pass through a small opening. The small opening and screen combination is called a pinhole camera.

or other out-door object. Rays of light reflected from the object pass through the hole A and produce an image of the object on a white screen placed as shown at S.

This experiment may be carried out on a large scale as suggested here, or it may actually be used for taking pictures. A very tiny hole in a piece of opaque material is substituted for

a lens in an ordinary camera. Because the hole is so small, exposures of several minutes may be required in comparison to the small fraction of a second necessary with the proper lens.

However, this type of camera does not have to be focused *as does an ordinary lens camera, and a larger or smaller field* of view may be included in the picture by having the hole close to or far from the plate. The device is called a "pinhole" camera.

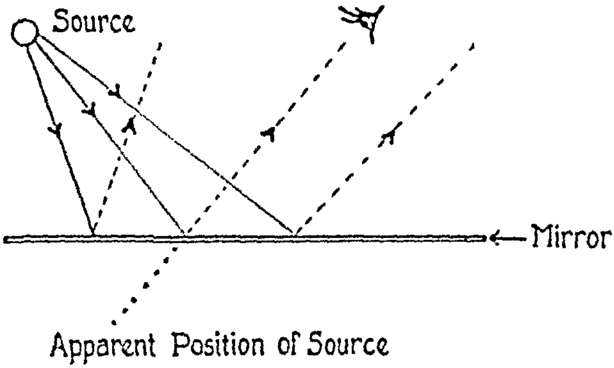


FIG. 192.—Reflection of light from a smooth surface.

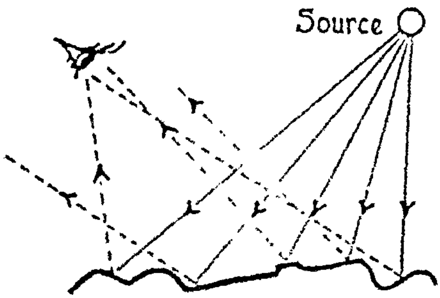


FIG. 193.—Reflection of light from a rough surface.

9.15. Reflection from Smooth and Rough Surfaces

If light is reflected from a perfectly smooth surface (such as we call a mirror), we appear to see the source of light rather than the mirror itself. If the mirror is perfect we do not see its surface at all. The source of light appears to be in a different location than it really is. (See Figure 192.)

If the surface is slightly rough (as most surfaces are) the light will be reflected in various directions from nearby parts of the surface and light will reach the eye from so many parts of the surface that it will seem to come from all parts. In this case we have the impression of "seeing" the surface rather than the source of light. This effect is shown in exaggerated form in Figure 193.

Some Important Facts

1. Visible light comprises a small band (about one octave) of electromagnetic radiations. Light waves are much longer than the longest X-rays, but much shorter than the shortest radio waves.

2. The colors of the visible spectrum (red-orange-yellow-green-blue-indigo-violet) vary in wave length from about .00008 cm. to .00004 cm.

3. The color of self-luminous bodies is determined by the wave lengths they emit. The color of opaque bodies is determined by the wave lengths they reflect, all other wave lengths being absorbed. The color of transparent and translucent bodies as seen by transmitted light is determined by the wave lengths they transmit, all other wave lengths being absorbed or reflected.

4. Light travels in straight lines through uniform media.

5. All objects seen in reflected light must be very large in comparison to the length of the waves they reflect, otherwise there could be no definite reflection pattern.

6. The portion of a shadow from which all light from a given source is excluded is called the umbra; the portion from which only part is excluded, the penumbra.

7. When light from an object passes through a very small opening, it can form an inverted image on any plane surface.

8. Regularly reflected light from a polished surface causes us to see an image of the object from which the light came. Irregular reflection from a rough surface causes us to see the reflector itself.

Generalization

The various electromagnetic wave-motions differ in wave length and frequency, the product of which is their common velocity, 186,000 mi. or 300,000,000 m. per sec. Such electromagnetic wave lengths within the approximate range, .00004 cm. to .00008 cm., constitute visible light.

Problems

Group A

1. What physical similarities and differences exist between visible light and other radiations that we have studied in previous chapters?

2. What physical differences exist among various colors of light?

3. Explain how an object that is seen by light which it reflects gets its color.

4. How does an object that is seen by transmitted light get the appearance of color?

5. Suppose that a substance actually reflects only green and red light. What color will it appear to have when illuminated with light as follows: white, red, green, yellow, violet?

6. Illustrate by labelled diagram: (1) an eclipse of the moon; (2) a total eclipse of the sun; (3) a partial eclipse of the sun.

7. Make a drawing to show what would happen to the image in a pinhole camera if the pinhole were larger.

8. Can you "see" a perfect reflecting surface?

9. Distinguish by labelled diagrams regular and irregular reflection.

Group B

1. Arrange the electromagnetic wave series in a table giving the approximate wave length range, the frequency range and the velocity for each band.

2. In terms of the electro-magnetic wave theory explain what is meant by complementary colors.

3. What limits the smallness of an object that can be visible to the human eye even with the aid of a microscope?

4. Illustrate by shadow diagram the phases of the moon.

Experimental Problems

1. Construct a color disk which when spun rapidly appears to be white or nearly so.

2. Construct a symmetrical shutter that will spin with the color disk and cover about half its area. Using the finger as a break, cause the shutter and the color disk to speed at unequal rates. Approximately how many revolutions per second must the color disk make to cause the colors to appear to blend?

3. Construct a pinhole camera which will produce a visible image on a screen of ground glass or oiled paper, or, if possible, take an actual picture on a sensitized plate.

4. Determine the colors of smoke as viewed with reflected light and transmitted light.

Chapter 16

ILLUMINATION

The term *brightness* is used to refer to the amount of light radiated by a light source. The term *illumination* refers to the amount of light which falls on a surface per unit area.

Light radiated from a small but concentrated source usually produces both glares and deep shadows. The same amount of light radiated from a diffuse source is more satisfactory for most cases of illumination.

The brightness of lamps is usually measured in terms of a "standard" candle and the measurement is usually made as a comparison with a lamp of known candle power.

Illumination is measured in terms of foot-candles. The measurement may be made by comparison with the illumination from a lamp of known brightness, or by means of direct reading meters that have previously been calibrated with known amounts of illumination.

1.16. Brightness of a Light Source

There are many occasions where it is desirable either to measure the ability of a lamp to produce light, or to measure the amount of light reaching a given surface per unit of surface. Before we can measure anything, it is always necessary to agree on a unit by which the measurement should be made. The need for such units in light was felt many years ago and the standard unit chosen for a light source at that time was one which may seem a bit foolish to the present generation. But any unit is one of arbitrary choice, and so the old unit of brightness has been kept and is still in use.

Candles were the standard means of artificial illumination when the unit for brightness of a lamp was first defined, and so the light from a standard candle became that unit. Of course, there was a lot of difficulty because of the fact that some candles are brighter than others and the specifications for the "standard" candle were settled in 1909 by an international agreement.

Electric lamps such as are now sold for general use are sometimes rated in terms of candles (*candle power* is the tech-

nical term), but more often they are simply marked for the amount of electrical power they consume. Ordinary tungsten filament lamps give about one candle-power for each watt of electric power; but some of the newer lamps are more efficient.

Any object that emits light is said to be *luminous*.

2.16. Illumination

The term *brightness* as used above refers only to sources of light. The term *illumination* is used to refer to the amount of light actually reaching any particular spot per unit area. For example, at a distance of one foot from a standard candle, the illumination is called one *foot-candle* (light flux in *lumens* per square foot of surface perpendicular to the direction of travel of the light). The foot-candle is an accepted unit of illumination.

In Figure 194 is shown a point source of light with spheres of radii of 1 and 2 feet respectively drawn concentrically around it. We will suppose that the inner sphere is made of something that is perfectly transparent. Then all of the light that strikes this sphere must also strike the second one. But the second sphere has much more surface and hence the intensity of the illumination must be less than at the first sphere. The area of a sphere depends on the square of the radius. So the illumination on the two spheres will vary inversely as the squares of the two distances. So we can say that the intensity of light from a point source falls off with distance in the relation of the inverse square of the distance.

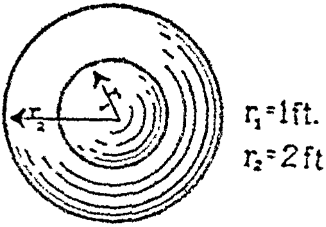


Fig. 194.—Light from a point source spreads uniformly in all directions. Above, two spheres are indicated. All of the light crossing the surface of the smaller sphere crosses the surface of the larger sphere. The intensity of the light must be greater at the surface of the smaller sphere than at the surface of the larger sphere.

For example, the illumination at two feet from a point source of light is only one-fourth the value at one foot,—at three feet the value is one-ninth the value at one foot.

If the lamp in Figure 194 has a brightness of 10 candle power, the illumination at the first sphere is 10 foot-candles (since the

distance is one foot). The illumination at the second sphere is 2.5 foot-candles since the distance is 2 feet and the illumination falls off as the inverse square of the distance.

$$\frac{10}{(2)^2} = \frac{10}{4} = 2.5 \text{ foot-candles}$$

3.16. Practical Lighting

The amount of illumination that one wants at any position varies with the kind of thing that is to be done there. For example, from 10 to 30 foot-candles is usually wanted for reading in classrooms, offices and similar places. In machine shops, and especially in shops where fine work is done, 30 to 50 foot-candles or even more are better values.

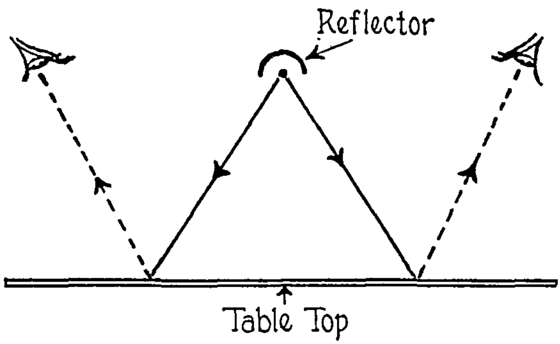


FIG. 195.—When a source of light is located over the center of a table, objectionable glare is often reflected directly into the eyes of a reader.

It is now accepted practice to have light come from a source of large area or from many small sources instead of having a single lamp of great brightness but small surface. The latter type of lamp is blinding if one looks directly at it, and it tends to produce glaring reflections from any object that is nearly smooth or that has small sections that are smooth. If the source of light is more diffuse, such glares are reduced or eliminated, shadows are less pronounced, and eye strain is greatly reduced.

An interesting example of how not to use lights may be found in almost any library where it is still common practice to place lamps in the middle of the table. The lights are

covered with a reflector which also cuts off direct radiation to the eyes of the reader, but the glaring reflection from the almost smooth sheets of paper which one uses takes place directly into the eyes. (See Figure 195.) If a concentrated source of light must be used for such work it should be located to the side or slightly back of the reader as indicated in Figure 196.

It is still better to use light of the indirect type, where it is reflected to a ceiling and walls and comes down to the table from all directions. Artificial sky lights where the lamps are concealed and the light appears to come from the entire area are also successful. Where such elaborate lighting fixtures

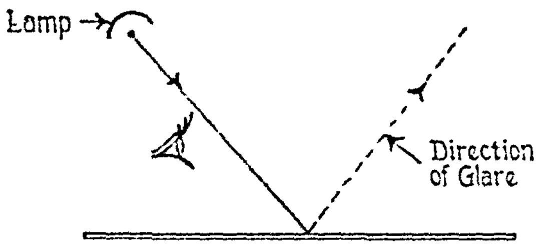


FIG. 196.—When lamps are properly located the direction of glare type reflection can be kept from the eyes of a reader.

cannot be had, lamp bulbs should be surrounded with large globes of translucent material such as ground glass or porcelain. These globes make the light appear to come from the entire surface instead of from the relatively small lamp.

4.16. Comparison of the Brightness of Lamps—Shadow Method

In recent years an international agreement was made in which the standard candle was defined in terms of the light emitted by certain standard electric lamps operated under specified conditions of current, etc. Other lamps are compared with these standards and are called secondary standards. It is possible to buy an electric lamp which has been compared with these standards and so it can be used as a standard in our own laboratory.

Since we have adjusted these illuminations to be equal we may write

$$\frac{C_1}{d_1^2} = \frac{C_2}{d_2^2}$$

This equation may be solved to give

$$C_2 = C_1 \frac{d_2^2}{d_1^2}$$

5.16. Comparison of the Brightness of Lamps—Photometer Method

Another way to compare the brightness of two lamps is shown in Figures 198 and 199. The screen S is a sheet of white

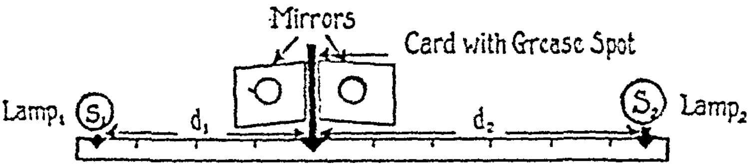


FIG. 198.—A "grease spot" photometer for comparing the brightness of two light sources.

paper or cardboard in which one spot has been made partially transparent by the application of a little grease of almost any kind.

If one looks at such a paper by reflected light from one source only, the grease spot will appear darker than the rest of the paper, for it transmits much of the light and hence can reflect less. (See Figure 199(*a*).) But if the paper is placed between the observer and a light source, only the grease spot will appear bright, while all the remainder of the paper is dark. (See Figure 199(*b*).)

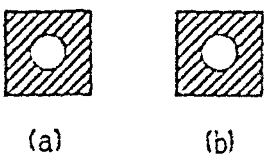


FIG. 199.—Surface view of a card with a grease spot in the center. (*a*) As seen by reflected light. (*b*) As seen by transmitted light.

If such a paper is placed between two lamps, as shown in Figure 198, it will appear uniformly lighted if the amount of light reaching it from the two lamps is equal. This must be true, for what is lost by transmission of the

light through the grease spot from one lamp is largely made up by what is transmitted through in the opposite direction by the other lamp.

A little light may be absorbed in the greased part of the paper and so when there is a perfect match in the appearance of the grease spot and the rest of the paper on one side, the other side may show the grease spot to be darker than the border. To avoid this trouble mirrors may be arranged as indicated so that the observer may adjust the position of the screen until the two sides give the same appearance.

Since the screen is now located so that the illumination from the two lamps is equal, we may use the same equations as those given for the shadow method, $C_2 = C_1 \frac{d_2^2}{d_1^2}$. The distances are measured from each lamp to the screen as shown in Figure 198.

Devices for comparing light sources in this manner are called photometers, the above type being called a "Bunsen" photometer and sometimes simply a "grease spot" photometer. Several more complicated types are used in some laboratories and are described in more advanced texts.

6.16. Measuring Illumination

A clever modification of the grease spot idea of the photometer results in a very handy type of illumination meter. A piece of cardboard has a series of holes cut in it and the holes covered with translucent paper. The cardboard is mounted as the top of a box which contains a small electric lamp near one end. A side view of the box showing the location of the lamp is given in Figure 200 and a view of the top of the box in Figure 201.

A small lamp such as would be used in a flash lamp is satisfactory. An ammeter is included in the outfit so that the current may be adjusted through the lamp by means of a small rheostat.

Obviously the holes near the lamp will appear much brighter than those at the far end of the box. It is possible to

calibrate the holes in terms of the amount of illumination that gets through them. Then, when light falls on the outside surface of the card with these holes in it, a lightness match will be seen at the hole where the illumination striking the card is the same as that coming through the hole.

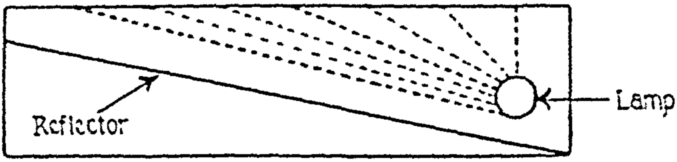


FIG. 200.—Sectional view of a foot-candle meter. The light indicated here by red dotted lines is white light of a quality obtained from a standard incandescent lamp.

A different form of illumination meter has recently been placed on the market and is rapidly coming into general use. It is built around the discovery that certain substances behave like miniature electric batteries when light falls on them. They are known as voltaic type photoelectric cells. (Later we will study about photoelectric cells in general.)

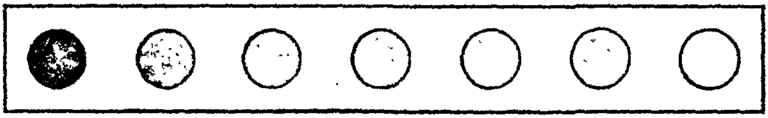


FIG. 201.—Top view of the foot-candle meter of Figure 200.

A cell of this type is simply connected to a portable micro-ammeter whose scale is calibrated in foot-candles instead of microamperes. It then reads the illumination directly.

Some Important Facts

1. The power of a light source may be rated in candle power of light output. However, ordinary electric lights are commonly rated in watts of electric power input.

2. The amount of illumination intercepted by a square foot of surface at a distance of 1 foot from a standard candle is called a foot-candle, and is the unit for measuring intensity of illumination.

3. In general, satisfactory illumination requires 20 to 50 foot-candles, emanating from an indirect or diffuse light source.

4. When two light sources cause a rod to cast equally illuminated shadows, then the candle power divided by the square of the distance in feet is the same for both lights.

5. More generally, when two light sources cause equal illumination, C_1/D_1^2 for the first equals C_2/D_2^2 for the second.

6. Some illumination meters operate on the photometer principle; more modern ones, on the photoelectric effect.

Generalization

In practical illumination, the light source itself should not ordinarily be directly visible, all glare due to regular reflection should be eliminated and the foot candle intensity on any surface should be adequate to the particular visual needs.

Problems

Group A

1. What is meant by the brightness of a light source?

2. What is meant by illumination? In what units is it measured?

3. A point source of light rated at 32 candle power produces how much illumination on a surface facing the light at a distance of 5 feet?

1.28 ft.-candles.

4. Using the same distance and position as in No. 3, find the strength of light source needed to produce 15 foot-candles of illumination.

375 candle power.

5. In an ordinary room the actual illumination is found to be greater than that calculated as in cases of problems No. 3 and No. 4. Why should this be true?

6. What advantages are claimed for illumination from diffuse sources as compared to concentrated sources?

7. Compare a simple desk lamp with an indirect lamp for illuminating a desk or table, as to dimensions, brightness, cost of operation, ease on the eyes.

8. Would you expect the illumination needed for easy reading to be more or less with diffused light as compared to a concentrated source so arranged as to produce glare?

9. A standard lamp rated at 16 candle power is used in the simple arrangement of Figure 197. It is placed 50 cm. from the wall. A lamp of unknown candle power appears to give equal illumination at a distance of 75 cm. Find the brightness of the second lamp. 36 candle power.

10. Show from a diagram how a simple illumination meter works.

11. A grease spot photometer has a standard lamp of 28 candle power at one end of a 2 meter light bench and an unknown lamp at the other end.

Equal illumination is shown on the screen for a distance of 75 cm. from the standard lamp. Find the candle power of the unknown lamp.

77.8 candle power.

Group B

1. Account for the use of the candle as a unit for measuring the power of a light source. Look up the history of the "standard candle" and give its quantitative definition.

2. Does the Law of Inverse Squares apply to any forms of energy transmission other than light? Illustrate your answer by labelled diagram.

3. Make a table of 10 to 15 common occupations with the foot-candles of illumination desirable in each case.

4. Beginning with Edison's incandescent lamp, list in order the main improvements in electric lamp efficiency. Plotting candle power per watt against years, show this increasing efficiency by a graph.

5. Select five actual cases of illumination known to you. In each case tabulate in parallel columns the desirable and undesirable features.

Experimental Problems

1. By means of a meter stick, a photometer and a light source of known candle power, determine the candle power of an unknown source.

2. If an illumination meter is available, measure the foot-candles of illumination in various places where you read, study or do other visual work. Compare the observed values with the ideally correct ones.

Chapter 17

REFLECTION OF LIGHT

In Chapter 15 we discussed the reflection of light from smooth and rough surfaces. In that chapter we were more concerned with objects sufficiently rough so that they could be seen than with smooth surfaces which reflect regularly. In the one example given of reflection from a smooth surface we learned that when a person looks at the surface he has the impression of seeing the source of light rather than the surface.

The present chapter continues the subject of reflection from smooth surfaces and describes what is to be expected when they are plane or when they are curved in some simple shapes.

For each important case the reflection is first studied by means of waves of light spreading out from a point source. Then the same subject is studied by following rays of light from the source.

1.17. Reflection from a Plane—Wave Method

Figure 202 shows light waves spreading out from a small lamp, S, until they strike a smooth plane reflector, RR. The center of the section of a wave strikes surface, RR, and starts back on itself while the tips of this section of the wave continue to move forward to the reflector. By the time they touch the reflector the wave will be in the position shown by the dashed line.

The radius of curvature of this dashed line has the same value as the dotted line which represents the location of the incoming wave if it had not been reflected, but it is in just the opposite direction. This makes the reflected light appear to come from a source, S', as far behind the surface as the real source, S, is in front of it.

2.17. Reflection from a Plane—Ray Method

This simple problem in reflection can also be solved by following the paths of several rays of light from the source, S. Such a ray may be represented by the radius of a wave

front. For example, *SB* represents a light ray in Figure 202. Experience shows that when a ray of light strikes a plane surface it is reflected along a path that makes an angle with a perpendicular to the surface at that point equal to that made

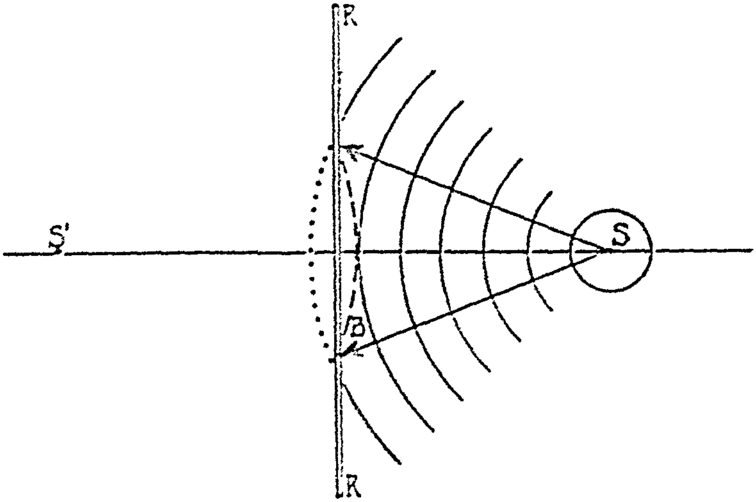


FIG. 202.—Light reflected from a plane mirror makes the lamp *S* appear to be at *S'*.

by the incoming ray. This is usually stated as follows: The angle of *incidence* is equal to the angle of *reflection*. These angles are marked *i* and *r*, respectively in Figure 203.

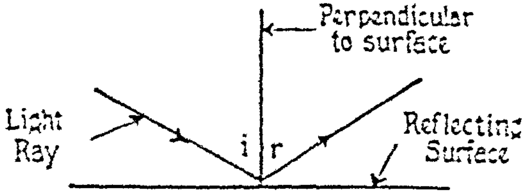


FIG. 203.—The angle of reflection, *r*, is equal to the angle of incidence, *i*.

The source of light and the plane reflector of Figure 202 are re-drawn in Figure 204. Here four rays of light are shown although as many more as one desires may be added. One ray is drawn perpendicular to the plane and so must be reflected back on itself.

A perpendicular to the surface at the point where the ray strikes is drawn for each of the others and the directions of the reflected rays are found by making each angle of reflection equal to its corresponding angle of incidence.

The actual reflected rays of light, travelling to the right, keep diverging from one another. Extensions of the directions of these rays to the rear of the mirror (shown by dotted lines) show that they all meet in a point. By a little manipulation of plane geometry one can show that this point has the

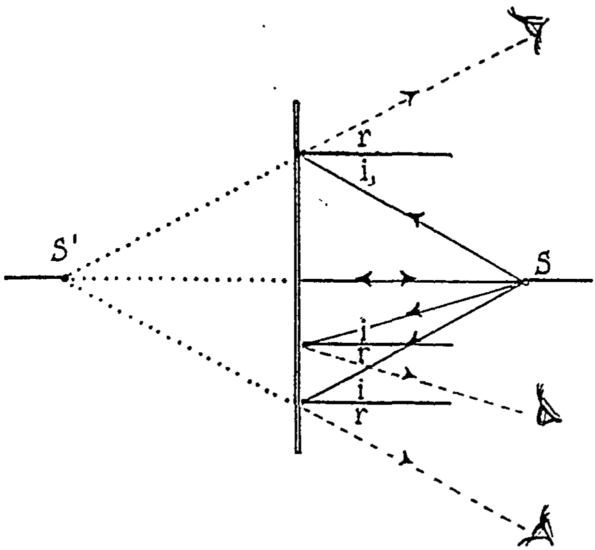


Fig. 204.—Locating the image, S', of the object, S, by the ray method. (See Figure 202 for the wave method.)

same relations as to position and distance with respect to the rear of the mirror as the source has to the front of the mirror. These conclusions are the same as we obtained in the previous section by studying the waves of light.

Any one standing in front of the mirror and looking at the reflected light (as from the positions shown for an eye in Figure 204) would think that the light was in the position S'.

The source from which light travels to a reflector is called the "object" and the reflection of the object seen by an observer is called the "image" of the object. In the case just

studied, no light actually exists behind the mirror where the image appears to be. So this image is called "virtual" to distinguish it from real images.

All four dotted lines meet in the same point in Figure 204, so it seems hardly necessary to draw even this many rays, for any two of them drawn from a common point on the object will locate the apparent position of the image of this point just as well. And since any two will do, we may pick out the easier ones to draw. One is the ray that is perpendicular to the surface. There is not much choice among the others and so any one will do.

3.17. Size of Plane Mirror for Viewing One's Own Image

Plane mirrors are used by all of us in order to look at ourselves. From the fore-going discussion we may expect to see

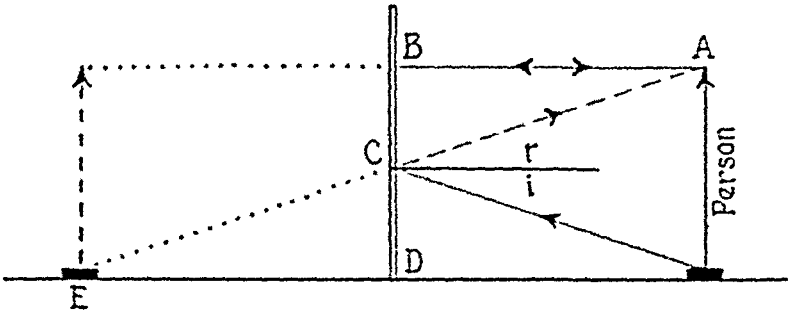


FIG. 205.—Viewing one's self in a plane mirror.

an image of ourselves at the same distance behind the mirror as we actually are in front of it.

The problem as to how large a mirror is required for a person to see a full length image of himself is easily settled. In Figure 205 we represent a person by an arrow and for simplicity we assume eyes at the top of the arrow.

Of course the eyes see the image of themselves by looking along a perpendicular to the mirror, AB. Light from the feet strikes the mirror in some point C so that it is reflected into the eyes at A but appears to come from a point E behind the mirror. The angles i and r must be equal as we have seen.

5.17. Reflection from a Concave Surface—Wave Method

Figure 208 is somewhat similar to Figure 202 except that a small section of the surface from a sphere is shown as a reflect-

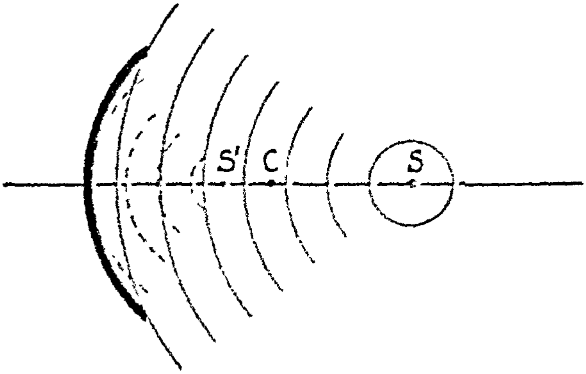
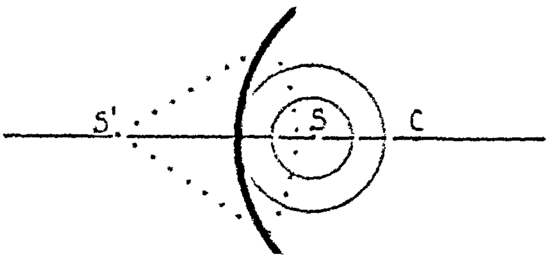


FIG. 208.—Waves radiating from S are reflected with changed curvature from a concave surface. The dotted lines indicate the reflected waves.

ing surface instead of a plane reflector. Light travelling from a small lamp S is again shown in wave form. In this figure the section of a wave about to strike the mirror meets the edges of the mirror and starts back while the center of the



mirror. The image of the source, S, appears at S', and since the light actually reaches this point the image is called *real.*

However, if the source of light S is moved very close to the mirror, it is possible to have the center of the wave section strike the mirror before the outer part reaches it. In this case the curvature of the reflected ray is reversed so that the center of the curve lies behind the mirror as in the case of the plane mirror. (See Figure 209.) The image would be virtual since the light does not actually reach the spot where the image S' seems to be.

6.17. Shape of Reflecting Surface

The reflected wave in Figure 208 will not be truly spherical if the reflector is a very large section of a spherical surface.

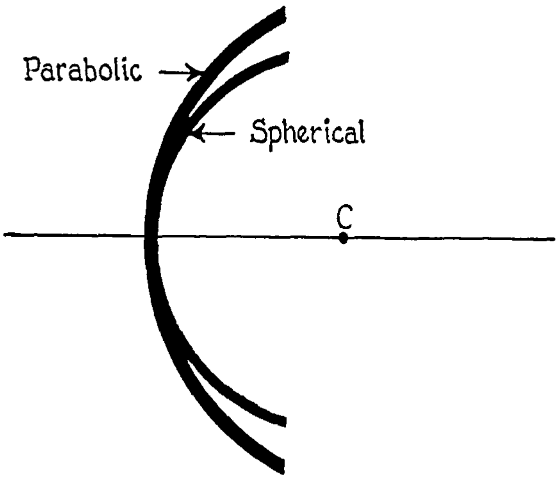


FIG. 210.—Reflectors of parabolic and of spherical shape may be compared in this diagram.

Or, in terms of the line drawings in the figures used this far, the line representing the reflected wave will not be truly a section of the circumference of a circle when the line representing the curvature of the mirror is truly circular.

This defect in reflection (called spherical aberration) will cause the image of an object to be blurred. The error can be corrected by giving the reflector a parabolic instead of a

spherical shape. There is not much difference in these two shapes as long as the sections are small, but when they are large the difference is great. (See Figure 210.)

7.17. Reflection from a Concave Surface—Ray Method

We can now apply the ray method to the case of a concave reflector just as was done in the case of a plane surface. The light source and mirror of Figure 208 are re-drawn in Figure 211 and the paths of several rays of light are shown. The light source in both of these drawings is placed on a line which is drawn through the center of the surface and the center of curvature of the spherical surface used. This line is called the *principal axis*.

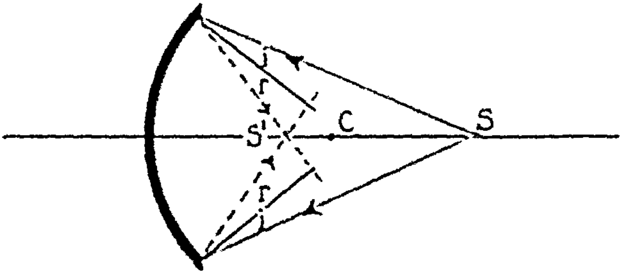


FIG. 211.—The conditions of Figure 208 illustrated with rays instead of waves.

The ray of light passing to the mirror along this path must be reflected back on itself. For the other rays, a perpendicular to the surface is drawn at each point of contact and the path of each reflected ray is found by making its angle of reflection equal to the angle of incidence. If the section of spherical surface is relatively small (or if the surface is parabolic in shape), all the reflected rays will meet in a point S'. This is the same location of the image S' as was found by the wave method illustrated in Figure 208.

8.17. Real Image with a Concave Mirror

The ray method is easy to apply in a case where the object has appreciable dimensions instead of being a point source. For example: In Figure 212 we see an object represented by an

arrow in an optical arrangement otherwise similar to that of Figure 211.

If a great many rays are drawn from the head of the arrow, they will all meet at a common point after reflection and will thus show the location of the image of the head of the arrow. But, if all the rays meet in the same point we need draw only two of them, and since any two will do we may as well draw the easier ones.

The easiest ray to draw will be the one which is normal to the surface since it will be reflected back over itself. It will have to pass through C, the center of curvature of the

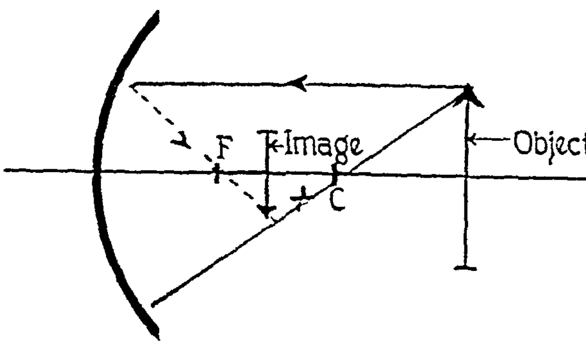


FIG. 212.—Ray method for locating the position of the image of an object.

mirror, and this makes the actual drawing of the line still more simple.

For convenience we will draw the second ray parallel to the principal axis. After determining the direction of the reflected ray in the usual manner we find that it crosses the principal axis just half way between the mirror and its center of curvature. This point is called the *principal focus*. In future drawings of this type we can omit the careful measuring of the angle of incidence and angle of reflection and simply draw a ray parallel to the principal axis and then take back the reflected ray through the principal focus.

The intersection of the two reflected rays locates the image of the tip of the arrow head. Other points on the arrow may be located in the same manner and the whole image will be found in the position shown in the figure.

The student may now show by construction that if the object is placed in the position of the image in Figure 212, the image will be formed in the place formerly occupied by the object. In other words their positions are inter-changeable.

9.17. The Principal Focus—Objects at This Point

Suppose that the object is placed at the position of the principal focus as shown in Figure 213. A ray of light follow-

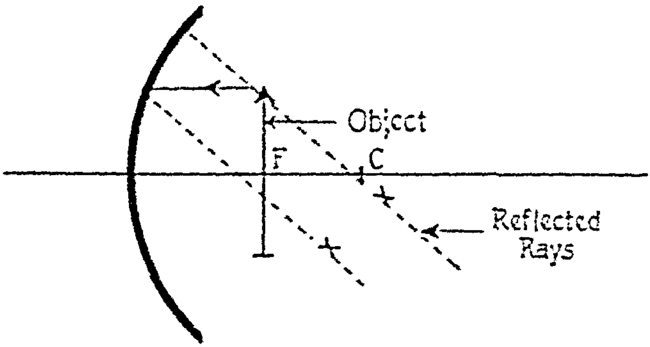


FIG. 213.—Reflected rays from a point on an object at the distance of the principal focus are parallel to one another.

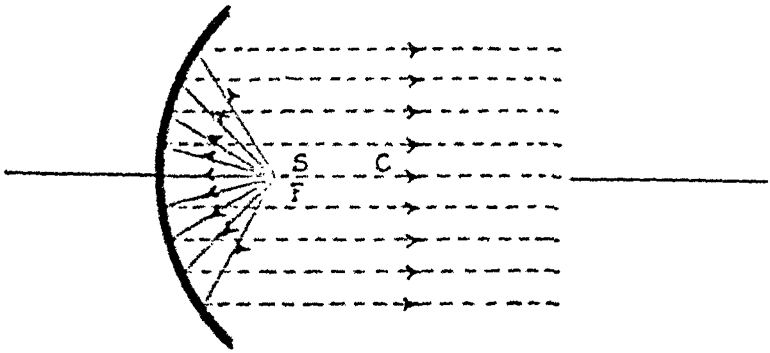


FIG. 214.—A concentrated light source at the principal focus produces a reflected beam in which the rays are all parallel to one another and to the principal axis.

ing a radius of the spherical surface strikes the surface perpendicular to a tangent of the surface, is reflected back on itself and passes through the center of curvature. Another ray, drawn parallel to the principal axis, passes back through

the focus after reflection. These two rays turn out to be parallel and hence they pass out into space indefinitely without forming a sharp image.

If we take a point on the object of Figure 213, which is also on the principal axis (see Figure 214), and draw a number of rays by careful construction, we find that they are all parallel to the principal axis. Hence a beam of light is thrown out into space.

Search lights and head lights on automobiles are built in this manner. A lamp bulb, or an arc lamp, is placed at the principal focus so that a powerful beam of light is radiated.

In the case of the automobile head light, the bulb is usually placed just a little in front of the principal focus so that the beam is not quite parallel but tends to focus at a point about 20 feet from the lamp. The light is spread to some extent by glass lenses of various designs placed in front of the lamp and reflector.

10.17. Principal Focus—Image at This Point

If light comes from a very distant source, all the rays which reach the mirror will be nearly parallel to one another,

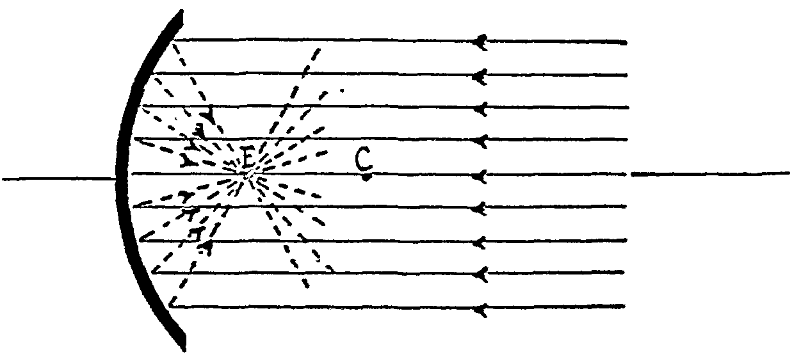


FIG. 215.—Light from a distant source consists of rays parallel to one another. They will be focused at the position of the principal focus if they are parallel to the principal axis.

and if the mirror is properly turned will be parallel to the principal axis. (See Figure 215.) The rays of light, after reflection by the mirror will now pass through f, the principal focus.

An easy method for finding the principal focus of a concave mirror is to reflect the image of a distant tree or building on a small piece of white paper. If the paper is small in comparison to the size of the mirror, it can be moved to and fro directly in front of the mirror without cutting off a very large percentage of the rays travelling toward it from the distant object. The position of the paper where the image is sharply focused is found in this manner and so the distance of the principal focus from the mirror can easily be measured.

11.17. Locations of Object, Image, and Principal Focus

In more advanced texts a relation between the object, image, principal focus distances and the radius of curvature as measured from the reflector is derived either from the ray diagrams or from the wave pictures. If these distances are respectively indicated by D_o, D_i, f, and r the relation is

$$\frac{2}{r} = \frac{1}{f} = \frac{1}{D_o} + \frac{1}{D_i} \tag{1}$$

In Figure 212, if the object is 50 cm. from the mirror and the principal focus is 10 cm., we may find the position of the image very simply, for

$$\frac{1}{10} = \frac{1}{50} + \frac{1}{D_i}$$

From which

$$\frac{1}{D_i} = \frac{1}{10} - \frac{1}{50} = \frac{4}{50}$$

And

$$D_i = \frac{50}{4} = 12.5 \text{ cm.}$$

12.17. Virtual Image with a Concave Mirror

The same methods used above will also apply in case the object is inside the principal focus as shown in the wave picture of Figure 209. A case of this kind is given in Figure 216, where an arrow is used for the object and rays instead of wave fronts are shown.

Two rays are drawn just as in the previous cases. The ray drawn perpendicular to the surface passes back through the center of curvature, C. The ray drawn parallel to the principal axis goes back through the principal focus. The actual reflected rays diverge as they leave the mirror, but their projections back of the mirror intersect. So the image of the arrow seems to be behind the mirror and, in this case, it is greatly enlarged.

When equation (1) is used on this problem one must remember that if we call distances positive when measured

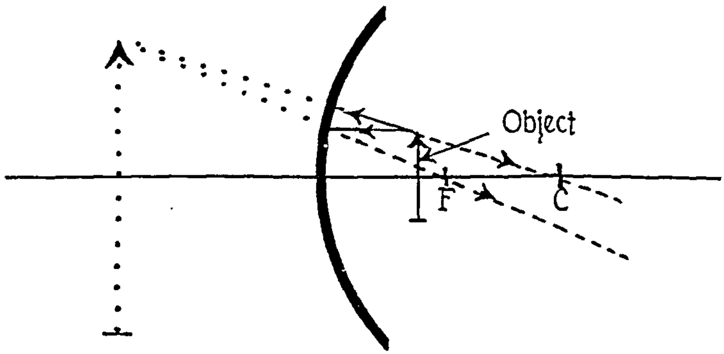


FIG. 216.—If an object is located within the principal focus of a concave mirror, the image will appear to be back of the mirror. (See Figure 209 for a wave view of this situation.)

from the face of the mirror they will be negative when measured in the opposite direction from the back of the mirror.

Suppose that in the above case the object is placed 8 cm. from the mirror and that the principal focus is 10 cm. Then, in equation (1) we write

$$\frac{1}{f} = \frac{1}{D_o} + \frac{1}{D_i}$$

$$\frac{1}{10} = \frac{1}{8} + \frac{1}{D_i}$$

or

$$\frac{1}{D_i} = \frac{1}{10} - \frac{1}{8} = -\frac{1}{40}$$

and

$$D_i = -40 \text{ cm.}$$

The minus sign in front of the 40 cm. indicates that the image is back of the mirror.

13.17. Sizes of Image and Object

An examination of Figures 212 and 216 shows that the image is larger or smaller than the object depending on which is farther from the mirror. A student looking for an extra assignment may easily prove that the relative sizes of object

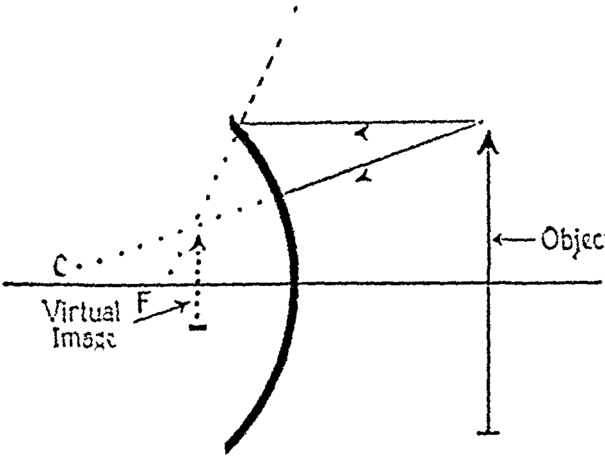


FIG. 217.—Ray diagrams for convex mirrors show that the image of any real object is always behind the mirror.

and image are the same as the relative distances. For example: if a linear dimension of the object is S_o, the corresponding dimension of the image is S_i and the distance of object and image from the mirror D_o and D_i we may write

$$\frac{S_o}{S_i} = \frac{D_o}{D_i}$$

In the example above, with a virtual image at a distance of 40 cm. behind the mirror and the object 8 cm. in front of it, any linear dimension of the image (such as the length of the arrow) would be 5 times the corresponding dimension of the object.

14.17. Convex Mirrors

Occasionally one sees mirrors silvered on the convex side. An optical set-up of mirror, object, and image of this type is shown in Figure 217. The center of curvature and also the principal focus lie behind the mirror in this case and so when a numerical value for f is inserted in equation (1), page 605, it must carry a negative sign. Of course no light actually passes through the position of the principal focus, but some rays of light on reflection from the mirror appear to come through this point.

In Figure 217 the ray of light perpendicular to the surface appears to come through the center of curvature, and the ray drawn parallel to the principal axis appears to come through the focus. These rays diverge, but their projections behind the mirror meet and show the location of the virtual image.

For an actual case, suppose that the principal focal length of the mirror is -10 cm. and the distance of the object 20 cm. Then in equation (1)

$$\frac{1}{f} = \frac{1}{D_o} + \frac{1}{D_i}$$

we may write

$$-\frac{1}{10} = \frac{1}{20} + \frac{1}{D_i}$$

And

$$\frac{1}{D_i} = -\frac{1}{10} - \frac{1}{20} = -\frac{3}{20}$$

Or

$$D_i = -\frac{20}{3} = -6.67 \text{ cm.}$$

The minus sign shows that the image is behind the mirror as we already know from the ray construction in Figure 217.

Some Important Facts

1. When a ray of light is reflected by a plane mirror, the angle which the incident ray makes with the perpendicular to the mirror equals the angle made by the reflected ray with the same perpendicular.

2. The image formed by a plane mirror is upright, virtual, laterally reversed, the same size as the object and as far behind the mirror as the object is in front of the mirror.

3. With a plane mirror, the intersection of any two rays of light that started from the same point on the object determine the location of that point of the image.

4. The smallest plane mirror in which a person can see his full length image must be half as tall as the person.

5. In a transparent reflector, reflected images are merged with objects seen directly through the reflector.

6. A concave reflector causes the reflected rays to converge or, at least to be less divergent than the incident rays.

7. A concave reflector may produce either real or virtual images.

8. To eliminate spherical aberration and thus secure a reasonably accurate focus, concave reflectors are made parabolic rather than spherical.

9. If a point of an object lies on the principal axis of a concave mirror, the image of that point is formed where any and all of the reflected rays from that point intersect the principal axis.

10. In locating the image formed by a concave mirror, draw from each object point two rays, one, parallel to the principal axis, which will be reflected through the principal focus; another, through the center of curvature, which will be reflected back upon itself. The intersection of these two reflected rays is the corresponding image point.

11. Rays approaching a concave mirror parallel to the principal axis are brought together at a point called the principal focus. Rays approaching a concave mirror from a light source at the principal focus are rendered parallel on reflection.

12. The principal focus of a concave mirror may be determined approximately as the point where images of distant objects are formed.

13. In a concave mirror, the reciprocal of the focal length equals the sum of the reciprocals of the object distance and the image distance, i.e. $\frac{1}{F} = \frac{1}{D_o} + \frac{1}{D_i}$.

14. In case the object is located within the focal length of a concave mirror, the image is virtual and enlarged, and the image distance, D_i, is a negative quantity.

15. The size of the image with respect to the object varies as the image distance to the object distance or $\frac{S_i}{S_o} = \frac{D_i}{D_o}$.

16. In a convex mirror the image is always virtual, upright and smaller than the object; and F and D_i are always negative quantities.

Generalization

Images formed by plane mirrors are always upright, virtual, laterally reversed, the same size as the object and as far behind the mirror as the object is in front of the mirror.

a. A plane mirror.
b. A concave mirror.
c. A convex mirror.

In each case tell whether the image is real or virtual, erect or inverted, larger or smaller than yourself.

2. Draw a concave mirror whose radius of curvature is 3 inches. Place an object arrow at a distance of one inch from the mirror and locate the image by drawing rays.

3. Locate the image of problem 2 by means of the equation on page 605 and compare the result with that of problem 2. —3 inches.

4. A concave mirror with a radius curvature of 24 inches is used by a person to look at his own face. If he holds the glass 6 inches from his face where does the image appear to be? —12 inches.

5. Compare the size of the image and that of the object in problem 4. 2 times.

6. A concave mirror with radius of curvature of 30 inches is used as in problem 4. Where should the glass be held in order to get an image of the face 12 inches behind the mirror? 6.66 inches.

7. Make several ray diagrams to find out whether or not a convex mirror can form a real image of a simple object.

8. If you were making a reflector for a search light would you use a small or a large section of a parabolic surface?

9. What kind of an image would you expect to get from a mirror made from a section of a cylindrical surface? Where are such mirrors used?

10. From the candle and jar of water illusion (see Figure 206) invent a system whereby an audience may see an elephant vanished from a cage on a theater stage.

Experimental Problems

1. Use a small plane mirror, sheet of paper, and three pins, and stake out with the pins a triangle in front of the mirror. By taking two lines of sight at the image of each pin, construct the image of the triangle in the back of the mirror. Rearrange the pins to form a different shaped triangle and repeat.

Using your knowledge of the nature, size, and location of the image formed by a plane mirror, check the accuracy of your results. Account for any inaccuracies.

2. Using a concave mirror, candle, small screen and meter stick, place the candle (1) within *F*; (2) at *F*; (3) between *F* and *C*; (4) at *C*; (5) beyond *C*. Determine the nature, size, and relative distance to the image in each case.

Chapter 18

REFRACTION OF LIGHT

In the previous chapter on the reflection of light, it was not very important whether we thought of light as a motion of waves or whether we considered it to be made up of little particles shot out by the source of light. Nor were we much concerned with the rate at which light travels from one place to another.

In this chapter we are to study the speed at which light travels, and especially the difference in speed in various substances. We are also interested in variations of speed with the color of light in some substances. Explanations for some of these variations are most readily made on the belief that light is really a wave motion instead of the motion of particles.

When light passes obliquely through a surface separating two substances in which the speed of light is different, the direction of the rays is changed. This effect has useful applications in the making and using of prisms and lenses.

1.18. The Speed of Light

The actual measurement of the speed of light is difficult because light travels so rapidly that it is hard to time the short periods that it takes to get from one place to another. For this reason all of the first experiments in measuring the speed of light in laboratories were failures.

However, as early as 1676, Römer, an astronomer, observed that the time taken for one of Jupiter's moons to go around once in its orbit seemed less when the earth was near Jupiter than several months later when the earth had moved around in its own orbit so as to be at its greatest distance from Jupiter. From the time difference which he observed he estimated the speed of light and obtained a value quite close to what is now considered correct.

The present measurements which science has accepted were made by laboratory methods by Professor Albert Michelson working in California over a period of years from 1924 to 1931. Some of his data was taken between two

mountain tops about 22 miles apart and some was taken in a large pipe about one mile long. The air was removed from this pipe by pumps, so that the speed of light was measured in a vacuum.

The details of these experiments are given in many text books and the interested student should consult them. The accepted value for the speed of light in a vacuum is:

29,979,500,000 cm. per sec.

or

186,284 miles per second

For many practical calculations it is sufficiently accurate to take the velocity of light in a vacuum, or in air, as

30,000,000,000 cm. per second

or

300,000,000 meters per sec.

These numbers may be written as:

3×10^{10} cm. per second

or

3×10^{8} meters per second

The approximate value for the speed of light in English units is ordinarily taken as:

186,000 miles per second

Even with this enormous speed, it takes light about eight minutes to reach the earth from the sun. The fixed stars are so much further away that years are needed for light to travel from them to the earth. Astronomers frequently specify distances to the stars in terms of the number of years required for light to make the journey. So, for example, the star Sirius is 8.8 light years from the earth; Polaris is 40 light years away; the stars in the Big Dipper, about 70 light years away.

2.18. Variation of Speed of Light in Various Substances

Even though a substance may be considered transparent, —such as air, glass, water,—the speed of light is reduced in it as compared to a vacuum. The reduction of speed in ordinary air is very slight, but the velocity of light in water is only about three-fourths that in a vacuum, and in glass two-thirds to three-fifths of the vacuum value. A substance in which the velocity of light is reduced more than in some other substance is said to have greater *optical density* than the other substance.

The ratio of the speed in a vacuum to that in any other substance is called the "index of refraction" of the substance. The following table shows the indices for a few common substances.

Air (at normal pressure and temperature)	1.00029
Water (at 20°C.)	1.333
Alcohol (at 20°C.)	1.360
Glass	
Light crown (for yellow light)	1.517
Heavy flint (for yellow light)	1.890
Carbon bisulphide	1.625

The difference of the speed of light in air and in vacuum is so small that for most practical purposes these speeds are considered the same.

3.18. Variation of Speed with Wave Length

All of these substances which slow down the speed of light have more effect on the short waves than on the long ones. The difference in the range from deep red to extreme violet light amounts to several per cent in some cases. For example, the "Handbook of Chemistry and Physics" gives an index for heavy flint glass of 1.867 for deep red and a value of 1.945 for violet light.

4.18. Effects of Variations of Speed of Light

If the speed of light is as hard to measure as was indicated in the early part of this chapter one might at first wonder whether the differences in speed in various substances could

possibly have any noticeable effect in ordinary life. Curiously enough these differences account for many, perhaps a majority, of observed optical effects.

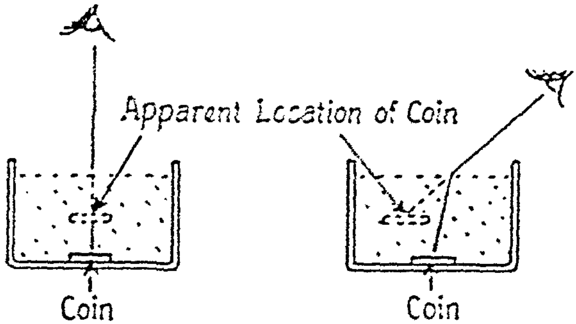
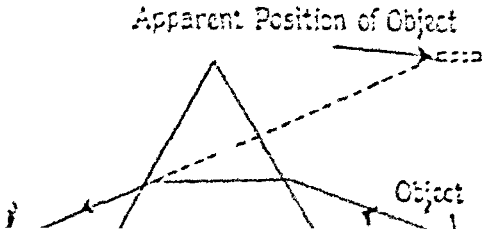


FIG. 218.—Viewed from the air, objects submerged in water appear to be closer to the observer than they really are. This effect is due to the fact that light travels slower in water than in air.

For one thing, if a person looks straight down in water, the bottom of the vessel holding the water seems much closer than it really is. Also, if you look at a spot on the bottom of the vessel from some position not directly over it, it will appear to be displaced sideways from its true position. This



in which he is looking. In other words, you can see around a corner if you use a prism. (See Figure 219.) Also a prism may be used to spread out ordinary white light into all the colors of the rainbow.

Experience shows that a ray of light is bent when it passes obliquely across the bounding surface between two substances in which the speed of light differs. Specially shaped pieces of glass (or other transparent substances) can make use of these bending properties in such a way that light can be focused in somewhat the same manner as is done with concave mirrors.

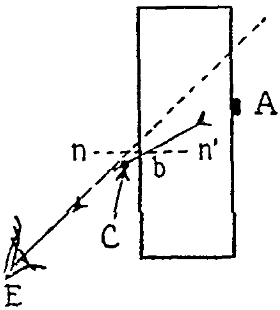


FIG. 220.—Light entering the glass at A may emerge at *b*. It suffers a change in direction at *b* so that the angle of the ray with the perpendicular, *n n'*, is greater in the air than in the glass.

Perhaps the best way to study these effects is to experiment with them in the laboratory, and so a number of projects that are simple to carry out are described here.

5.18. Simple Bending of Light Rays

Figure 220 shows a block of glass lying on a sheet of paper on the laboratory table. A pin is placed at *A* touching the glass and is looked at by placing one's eye in position *E*. The pin will appear to be somewhere along the line *Ecb*. A second pin should be placed at the spot, *b*, and a third pin at *c*. Evidently a ray of light passes through the glass from *A* to *b*, is bent at *b* and proceeds through *c* to *E*. The pins, *c* and *b* determine the line of sight and the pins *b* and *A*, the direction of the light beam through the glass.

The line *nn'* is drawn perpendicular to the surface of the glass at *b*. The angle which the ray makes with this normal to the glass is greater in air than in the glass where the speed of light is lower.

This experiment may now be repeated with the first pin moved to a position such as *O*. (See Figure 221.) The points *c* and *b*, may be located as they were in the first experiment

described above. The perpendiculars to the surfaces of the glass may be drawn and the angles noted. This experiment may be repeated with the eye at position E' and a new set of angles at the point b' will be found.

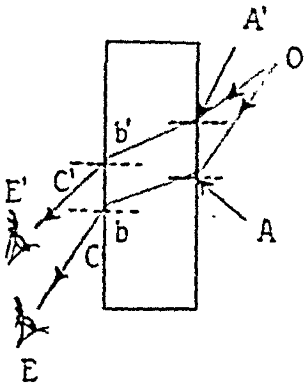


FIG. 221.—The amount of bending of a light ray as it crosses the surface from glass to air depends on the angle of incidence.

The entering angle in each case is called the angle of incidence and the other angle the angle of refraction. These angles may be measured with a protractor and if they are both small it will be noticed that the larger divided by the smaller gives about the same answer as the position of the eye is varied fom E to E'. This ratio is approximately equal to the index of refraction of the glass.

For large values of the angles this ratio will not give the true index. In more advanced texts we learn that it is the ratio of the "sines" of these angles that gives the true index of refraction. Students who have had trigonometry may make the computations using this relation.

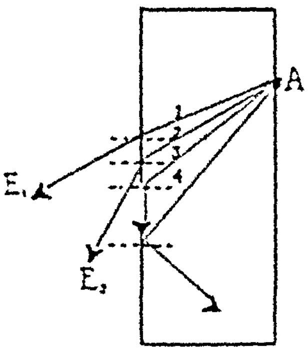


FIG. 222.—When the angle incidence is sufficiently great a ray of light in the glass does not pass through the surface at all, but is totally reflected.

These experiments may be repeated with water in a tank having parallel glass sides. If the glass is thin in comparison to the width of the tank, the effect of the glass may be neglected.

6.18. Total Reflection

An experiment similar to those described above is illustrated in Figure 222 where the glass is somewhat longer than that shown in Figure 220. Here, as in the case of Figure 221, we move the position of the eye to various points such as E_1, E_2, this time making the line of sight more and more

nearly parallel to the surface of the glass as we change from one position to the next.

Rays 1, 2, and 3 show greater and greater bending as they emerge from the glass. If ray 3 is just parallel to the surface

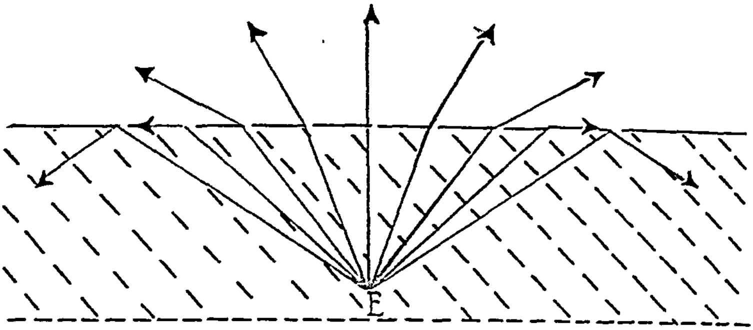


FIG. 223.—Possible directions of vision for a fish in water located at *E* and trying to look through the water to air surface.

of the glass, it follows that another ray further moved in the same direction (such as 4) can not escape from the glass at all, but will be entirely reflected back into the glass. Of course some of the light that reaches the surface on the other rays is also reflected, but total reflection occurs for values of the angle greater than the one which makes the escaping beam parallel to the surface. The minimum angle of incidence for which total reflection occurs is called the critical angle.

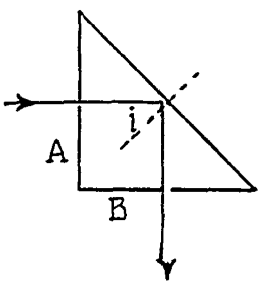


FIG. 224.—Total reflection may be used to form a good mirror.

From this experiment we learn that if a fish looks upward through the surface of water, he can see through only a limited region of the surface, for if a line from his eye to the surface makes too glancing an angle he will only see a reflection of things below the surface of the water. (See Figure 223.)

Total reflection is sometimes used with a prism to form a good mirror. Such a prism is shown in Figure 224, where a beam of light enters a prism perpendicularly to one face, *A*,

strikes a face internally at the angle *i* which is greater than the critical angle, and is totally reflected through another face, *B*, which is perpendicular to the new direction of the beam.

7.18. Bending of Light by a Prism

Experiments similar to those described in connection with Figures 220, 221, and 222 may now be repeated with a triangular prism such as is shown in Figure 225.

A pin is placed at *A* and is looked at from positions 1, 2, 3 and 4. With the aid of other pins it is easy to locate points *a*, *a′*, *a″* along the line of sight while keeping the eye steady

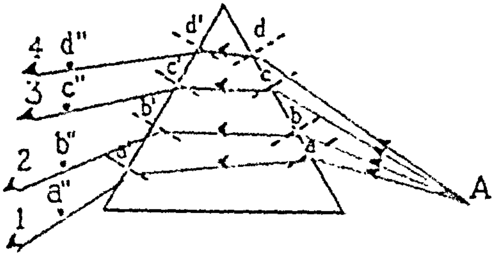


FIG. 225.—Refraction of light by a prism.

at position 1. Similarly the other points, *b*, *b′*, *b″*, *c*, *c′*, *c″*, *d*, *d′*, *d″*, may be found. When these points are connected, the lines from *A* through the prism show the directions of these various rays of light.

8.18. Dispersion by a Prism

If the light reflected towards the prism by the pin is of just one color the effects would be exactly as illustrated in Figure 225. When the experiment is done with the pin illuminated with white light, a color effect is noticed as one observes the pin from the various positions. This effect is due to the fact that light of different wave lengths has slightly different velocities in glass.

Actually the appearance of a white streak of light from *A* would be spread out into all the colors of the rainbow when viewed from any of the positions indicated in Figure 225. A somewhat exaggerated view of this state of affairs is shown in

Figure 226. The variation of refraction among the different wave lengths of light is called *dispersion.*

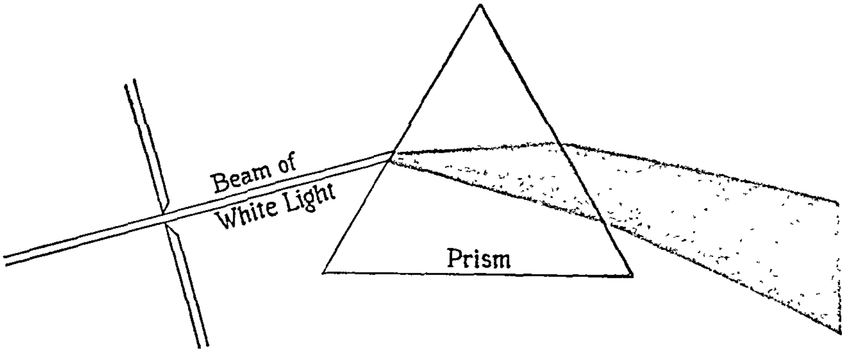


FIG. 226.—While light is broken into its component colors when passing through a prism.

9.18. "Prism" with Curved Faces

A prism-like object can be made with curved faces as illustrated in Figure 227, and these faces can be shaped so that various rays of light such as those in Figure 225, all starting

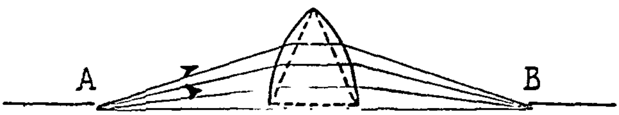


FIG. 227.—A modified prism with curved faces. The rays of light shown leaving A in various directions are refracted different amounts so as to bring them together at B.

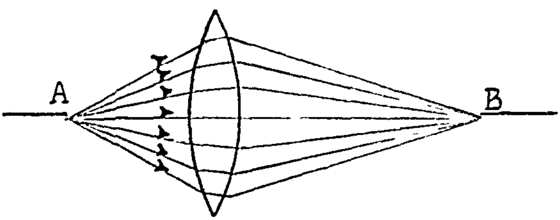


FIG. 228.—Passage of light rays from point, A, through a simple lens.

from the source A, would be seen from a single position B instead of from different positions.

The shape of these prism faces might now be rotated to form a glass piece similar to that shown in Figure 228. Then light from a point source A would pass through some common

point B no matter what part of the glass it went through. A piece of transparent material with such a shape as this is called a lens.

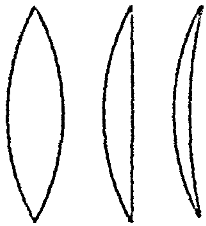


FIG. 229.—Converging lenses may be of various shapes.

If the lens is small, it will be found that the shape of either face is nearly spherical. But if the diameter of the lens is not small in comparison to the radius of the curvature of the face, these surfaces should be changed slightly from the spherical shape.

Experience shows that the curvature of the two sides of the lens need not be the same and a number of possible shapes are shown in Figure 229.

All of these lenses cause rays of light which are diverging from a point such as A in Figure 228 to converge towards some

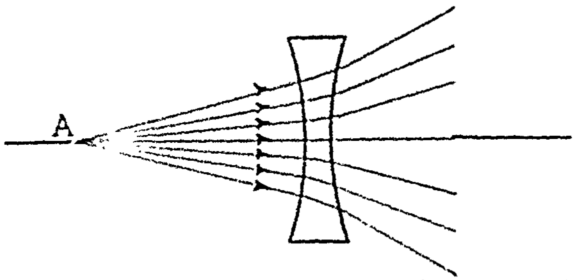


FIG. 230.—Rays of light diverge on passing through some types of lenses.

point B or at least the tendency to diverge is reduced. Lenses of this type are called *converging*.

If a lens is shaped as shown in Figure 230, rays of light tend to diverge on passing through it instead of converging as in the previous types. These are called diverging lenses. A number of possible shapes are shown in Figure 231.

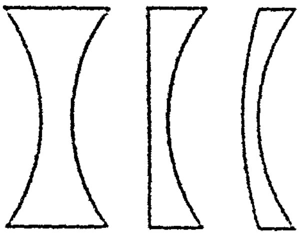


FIG. 231.—Diverging lenses may be of various shapes.

10.18. Focal Length of a Converging Lens

A line through the center of a lens and through the center of curvature of the faces of the lens is called the *principal axis*.

This corresponds to the similar axis in the case of a spherical or parabolic mirror (see page 601).

Suppose that light from a distant source, for example the sun, approaches a lens parallel to the principal axis. All the rays which enter the lens will of course be approximately parallel to one another as well as to the principal axis as shown in Figure 232. These rays will intersect at some point *f* after

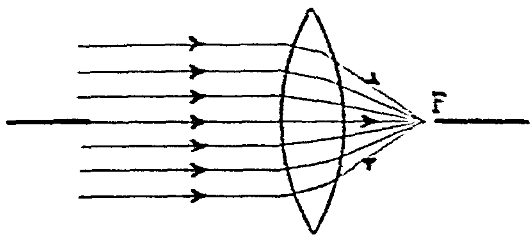


FIG. 232.—Rays of light parallel to the principal axis pass through a common point after going through a converging lens. This point is called the principal focus and its distance from the center of the lens is called the focal length of the lens.

being refracted by the lens. If the lens is turned about, this point will be found at the same distance from the lens as before. The distance from the center of the lens to this point is called the focal length of the lens, and the point is called the *principal focus* of the lens.

11.18. Positions of Objects and Images—Ray Method

In Figure 233 an arrow represents an object placed beyond the focal length of a converging lens. We can attempt to

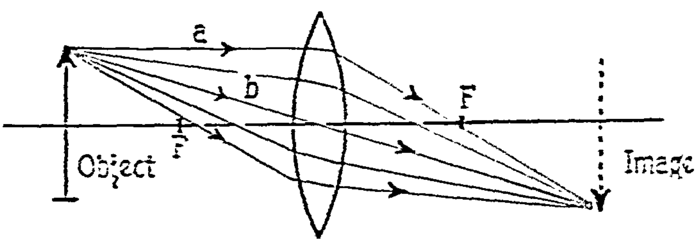


FIG. 233.—The image of an object formed by a converging lens can be located by following the light rays.

locate the image of the head of the arrow by drawing a number of rays from the arrow head and locating their intersection

on the other side of the lens as was done in the case of spherical mirrors in the preceding chapter.

This method is illustrated by the rays in Figure 233, but actually the method is difficult because one must know the index of refraction of the glass and carefully draw the proper angles of incidence and refraction for each ray at each face of the prism.

In any case, if all these rays meet in a common point, drawing any two of them will be satisfactory and we may as well draw the two that are most easy to locate.

In Figure 233 the ray *a* is parallel to the principal axis and must therefore pass through the principal focus *f* after going

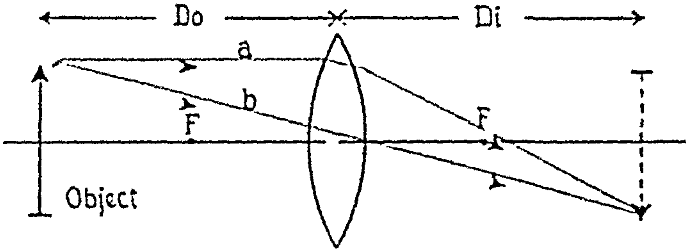


FIG. 234.—Only two of the rays shown in Figure 233 are required to locate the image.

through the lens. The ray *b* passes through the center of the lens where the two faces of the prism are parallel, and so, neglecting a slight sideways shift, (see Figure 221) it is not bent at all by the action of the lens. Figure 234 is a copy of Figure 233 showing just the rays *a* and *b*. Their intersection locates the image of the head of the arrow just as well as all the rays shown in Figure 233. The entire image is drawn in to match the location of the image of the head of the arrow.

12.18. Positions of Objects and Images by Calculation

In more advanced texts a method is given for finding the focal length of a lens in terms of the index of refraction of the glass and the radius of curvature of the two faces. In the course of this development a simple relation among object distance, image distance and principal focal length is found. It is:

$$\frac{1}{\text{Focal length}} = \frac{1}{\text{Object distance}} + \frac{1}{\text{Image distance}}$$

If we use as symbols for these distances f, D_o, and D_i we may write

$$\frac{1}{f} = \frac{1}{D_o} + \frac{1}{D_i} \qquad (1)$$

The student will recognize this equation as being similar to that for mirrors. (See page 605.) The only difference is that the actual numbers for object distance, D_o, and image distance, D_i, are considered positive when measured to opposite sides of the lens for simple cases as shown in Figure 234 while these distances for the mirror were considered positive when both were measured from the front of the mirror.

Suppose that in Figure 234 the object is 15 cm. from the lens, and that the focal length of the lens is 10 cm. We can locate the image by using equation (1).

$$\frac{1}{f} = \frac{1}{D_o} + \frac{1}{D_i}$$

$$\frac{1}{10} = \frac{1}{15} + \frac{1}{D_i} \qquad \begin{aligned} f &= 10 \\ D_o &= 15 \end{aligned}$$

$$\frac{1}{D_i} = \frac{1}{10} - \frac{1}{15} = \frac{1}{30}$$

and

$$D_i = 30 \text{ cm.}$$

13.18. Parallel Rays of Light

If a source of light is placed directly at the principal focus of a lens we would expect a beam of parallel rays of light to be sent out through the lens. The ray picture would be similar to that of Figure 232 where parallel rays come in and are focused at this same point. If the source of light is placed at the focus the light would travel in the direction opposite to that in this figure.

Lenses may be used in this fashion for search lights, but usually reflecting mirrors as described in the previous chapter

are preferred for this purpose. (Why would you expect this to be true?)

14.18. Virtual Image with a Converging Lens

In Figure 235 an object is placed closer to a converging lens than the position of the principal focus. We can attempt to locate the image of the arrow by drawing the rays *a* and *b* suggested on page 623. One ray, *a*, is drawn parallel to the principal axis. This ray passes from the lens through the principal focus. The second ray, *b*, is drawn undeviated through the center of the lens.

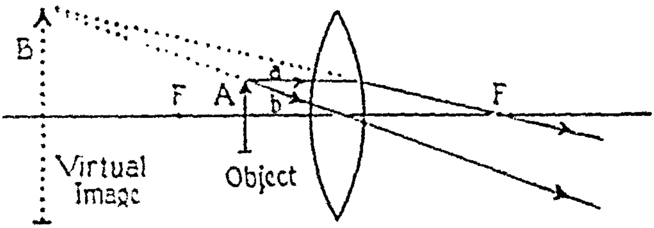


FIG. 235.—When an object is located between the principal focus and the lens, the rays that pass through the lens continue to diverge. The rays appear to come from a position on the same side of the lens as the object but further removed from the lens than the object. The image is called virtual to distinguish it from a real image.

These rays do not actually meet but rather tend to diverge from one another. If we project lines representing these rays back through the lens, we find that the projections meet and so the rays of light which diverge on the right hand side of the lens appear to come from a point *B* on the same side of the lens as the object *A*.

If one looks through the lens at the object he will appear to see an image of the object at the position shown by the dotted arrow. Actually the light rays are not present in the position *B* but only appear to be there. The image is called *virtual* to distinguish it from real images where actual light rays are focused.

The location of the image in a case of this kind can easily be found by using equation (1) on page 624. If the object

distance is 8 cm. and the focal length is 10 cm. we may write

$$\frac{1}{f} = \frac{1}{D_o} + \frac{1}{D_i}$$

$$\frac{1}{10} = \frac{1}{8} + \frac{1}{D_i} \qquad \begin{aligned} f &= 10 \\ D_o &= 8 \end{aligned}$$

From which

$$\frac{1}{D_i} = \frac{1}{10} - \frac{1}{8} = -\frac{1}{40}$$

or

$$D_i = -40 \text{ cm.}$$

The minus sign in front of the 40 shows that the image is on the same side of the lens as the object instead of on the side where a real image could be formed.

15.18. Image with a Diverging Lens

Figure 236 shows an arrow for an object with a diverging lens. A ray of light, *a*, proceeding parallel to the principal

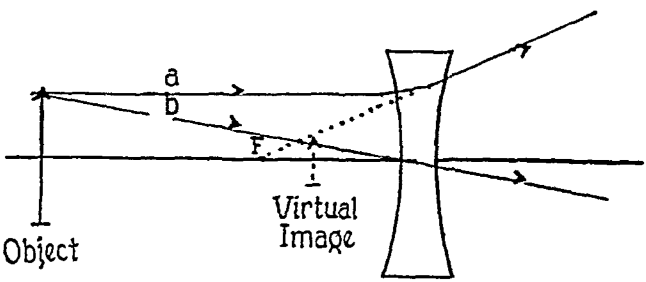


FIG. 236.—Virtual images of real objects are always produced by a diverging lens.

axis, is bent away from this line on passing through the lens. It will appear to come from the principal focus, *f*, on the same side of the lens as the object, as indicated by the dotted line.

A ray of light, *b*, passing through the center of the lens, proceeds undeviated.

The actual rays of light do not intersect, but a projection backwards along the direction of the ray *a* gives an intersection with *b* behind the lens: that is, on the same side as the object. The image appears in the dotted position when one looks

through the lens towards the object. The image does not really exist, but is virtual.

This image can easily be found by calculation if we remember that the focal length of a diverging lens is considered negative. Suppose that the object distance in the case of Figure 236 is 15 cm. and the focal length is -10. Then we may write

$$\frac{1}{f} = \frac{1}{D_o} + \frac{1}{D_i}$$

$$-\frac{1}{10} = \frac{1}{15} + \frac{1}{D_i} \qquad \begin{aligned} f &= -10 \text{ cm.} \\ D_o &= 15 \text{ cm.} \end{aligned}$$

From which

$$\frac{1}{D_i} = -\frac{1}{10} - \frac{1}{15} = -\frac{1}{6}$$

Or

$$D_i = -6 \text{ cm.}$$

16.18. Effects of Lenses on Light—Wave Method

Figure 237 shows waves spreading out from a point source, *A*, and moving in the direction of a converging lens. The

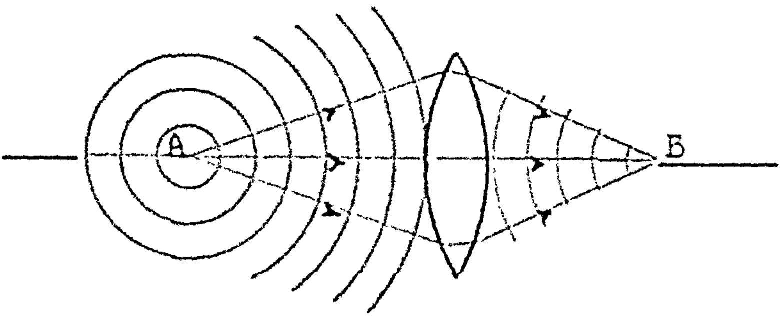


Fig. 237.—The action of a converging lens may be studied by observing the action on the waves of light.

shown in this figure, so that the light moves towards a point *B*. This effect was shown by means of rays in Figure 228.

In Figure 238 we see waves spreading out from *A* and moving towards a diverging lens. Here the center of the

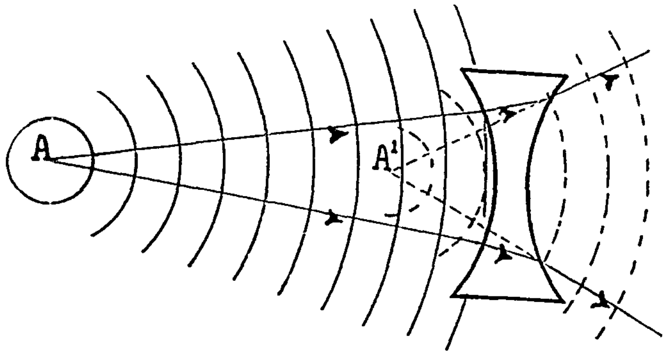


FIG. 238.—The action of diverging lens may also be studied by observing the action on waves of light.

wave gains distance on the outer portions because there is less retarding glass to move through in the center than near the outside edge of the lens. The curvature of the wave is increased so that it looks as if the rays come from some point *A′*.

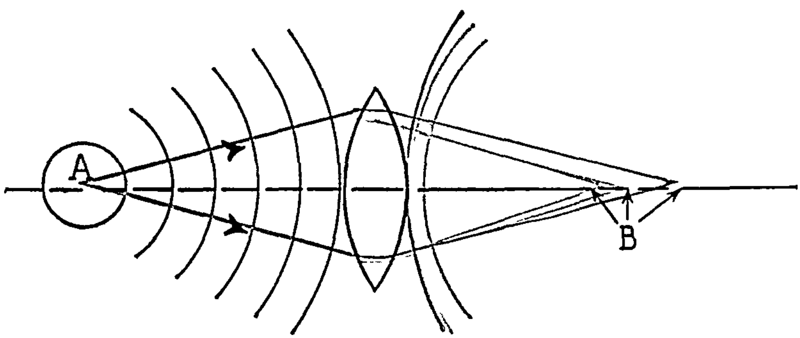


FIG. 239.—The curvature of light waves by a lens varies with the color of the light. Action on waves at the red end of the spectrum is less than on waves at the violet end.

If white light is used either for the converging or diverging lens above, the curvature of the refracted rays after passing

through the lens will be different for one wave length of light than for another. This effect is shown in *exaggerated form* in Figure 239 for the case of the converging lens. Light of different colors (different wave lengths) will come to a focus at different positions of *B* and if one looks at the light through such a lens, he will observe some color effect.

Lenses can be corrected for this defect by making them of *several slabs of glass having different indices of refraction and* various dispersion properties. In the finished lens, the pieces of glass are cemented together so perfectly that the lens appears to be made of a single piece of glass.

17.18. Sizes of Image and Object

As in the case of mirrors (see page 607) the relative size of image and object with a lens is the same as the corre-

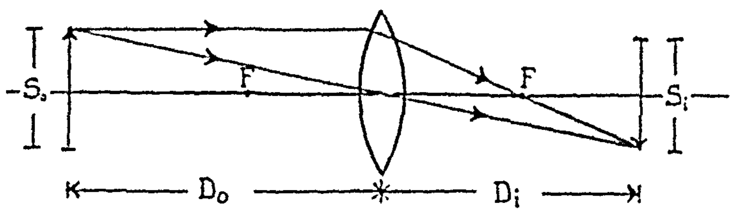


FIG. 240.—The size of the image compares to that of the object as the respective distances from the lens.

sponding distances from the lens. If the object distance is D_o, the image distance D_i and the lengths of two corresponding lines on object and image are S_o and S_i respectively, we may write

$$\frac{S_o}{S_i} = \frac{D_o}{D_i}$$

This relation is shown in Figure 240.

FIG. 241.—Six Possible Object Positions with Reference to Convex Lenses, which also apply to Concave Mirrors. (Note: Position 1 and 5; 2 and 4; and 3, Illustrate Conjugate Foci.)

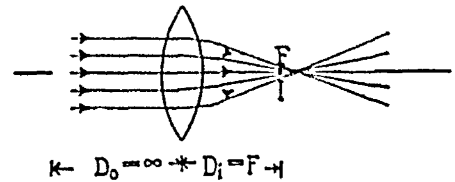
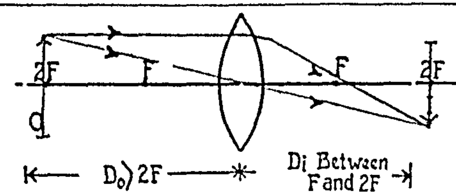
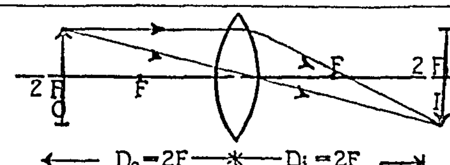
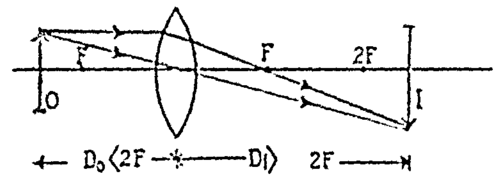
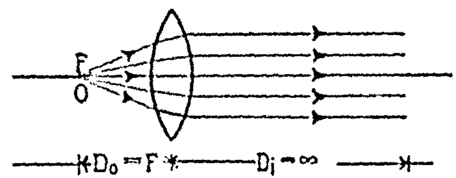
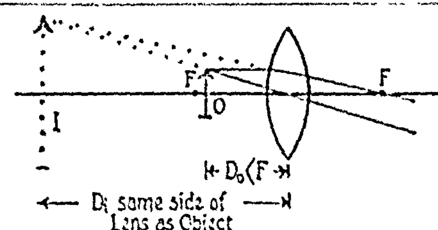
No.	Position of Object	Position of Image	Characteristics of Images	Construction of Images	Applications
1	Very great Distance (Infinity) from lens so that the incident rays are parallel	At *F*, principal focus.	Very small, real, and inverted.		Telescope (astronomical), Burning glass.
2	Between *F* and Infinity	Between *F* and *2F*	Smaller than object, real and inverted.		Camera, Terrestrial Telescope, Binoculars, Human eye, Opera glasses.
3	On *2F*	On *2F*	Same size as object, real and inverted.		Duplicating Camera, Portrait (life size) and Finger-print Camera.

FIG. 211—(*Continued*)

No.	Position of Object	Position of Image	Character-istics of Images	Construction of Images	Applications
4	Between *2F* and *F*	Between *2F* and Infinity	Larger than object, real and inverted.		Motion Picture Projections, (Film and slide Projections).
5	On *F*	At Infinity, but strictly speaking no image.	Rays that emerge are parallel.		Light House Searchlight, Spot light, Locomotive Searchlight.
6	Between *F* and the lens	On the same side of the lens as the object.	Larger than object, erect virtual.		Magnifying glass, Eyepiece of Microscope, & Telescope, Traffic-signal light.

Some Important Facts

1. In a vacuum or air, the velocity of light is approximately 186,000 mi. or 300,000,000 meters per second.

2. The ratio of the speed of light in a vacuum to its speed in any given substance is called the index of refraction of that substance.

3. In most substances the index of refraction is greater for short wave lengths than for long ones.

4. When light passes obliquely from one medium to another in which its velocity is changed, it is bent and also broken up into colors.

5. As light passes from one medium to another, it is bent toward the normal if its velocity is decreased, but away from the normal if its velocity is increased.

6. The angle of incidence at which total reflection occurs is called the critical angle.

7. When a beam of white light obliquely enters one side of a triangular prism, leaving through a second side, it is broken up into a spectrum with the violet part of the beam experiencing the greatest bending at the surfaces.

8. All light rays parallel to the principal axis of a converging lens are, after passing through the lens, brought together at a point called the principal focus. The distance of the principal focus from the center of the lens is called the focal length.

9. To locate the image formed by a convex lens, we draw from each object point two lines, one passing through the center of the lens, the other parallel to its principal axis. The first passes through the lens undeviated and the second is bent at the lens so that it passes through the principal focus. The intersection of these two lines locates the corresponding image point.

10. The reciprocal of the focal length of a convex lens equals the sum of the reciprocals of the object distance and the image distance, that is, $\frac{1}{f} = \frac{1}{D_o} + \frac{1}{D_i}$. When D_i is a negative quantity, the image is virtual.

11. If a light source is placed at the principal focus, rays which it sends through a converging lens are rendered parallel.

12. When an object is placed within the focal length of a converging lens, the image is enlarged, erect, and appears on the same side of the lens as the object, that is, it is virtual.

13. The image formed by a concave, or diverging, lens is always smaller than the object, upright, virtual and located within the focal length on the same side of the lens as the object. It may be found by the formula, $\frac{1}{f} = \frac{1}{D_o} + \frac{1}{D_i}$, but D_i is always a negative quantity.

14. Since light rays may be considered as radii of concentric wave sh
all light diagrams may be constructed using curves representing w
fronts together with, or in place of, lines representing the radii of tl
curves.

15. The size of the object bears the same ratio to the size of the im
as their respective distances from the lens, that is, $S_o/S_i = D_o/D_i$.

Generalization

Light, which travels at a velocity of approximately 186,000 mi. (3
000,000 meters) per second in a vacuum, is slowed up somewhat in
other transparent media. Important results of this varying velocity
light are refraction and dispersion, which find application in a variety
optical instruments.

Questions and Problems

Group A

1. If light could travel in a circular path around the earth at
equator, how long would it take to go around once? 0.134 se

2. Use the table on page 614 and figure the velocity of light in wa
in centimeters per second. 2.251×10^{10} cm. per se

3. Use the data on page 614 and figure the velocity of red light an
violet light in heavy flint glass.
1.607×10^{10} cm. per sec. 1.542×10^{10} cm. per se

4. (*a*) What happens to a ray of light as it enters obliquely a medi
of greater optical density?

(*b*) Suppose a ray of light leaves a medium of higher optical den
obliquely and enters one of lesser optical density. What will be the ef
on the light ray? Make diagrams for both cases.

5. Make a drawing showing the path of a ray of light from a st
on the bottom of the lake to the eye of a person standing on the sh
Show where the stone appears to be.

6. What is meant by "total reflection"?

7. Compare the terms dispersion and refraction.

8. Show by a ray diagram of light from one point what is meant b
"converging" lens.

9. How may the principal focal length of a converging lens be m
easily found?

Group B

1. Find the size of the image in problems 10 and 11 in Group *A*. 2 cm.

2. Use the lens of problem 10 in Group *A* and the same object. The object is placed 3 cm. from the lens. Draw two rays to scale and locate the image. What kind of an image is it?

3. Calculate the position of the image in problem 2 by using the equation on page 624. Compare the result with that obtained by drawing. -12 cm.

4. Find the size of the virtual image in problems 2 and 3. 4 cm.

5. An object is located 25 cm. from a converging lens and the image of this object is found to be 50 cm. from the other side of the lens. Use the relation on page 624 to find the focal length of the lens. 16.7 cm.

6. An object one centimeter long is placed 6 cm. from a diverging lens that has a focal length of -4 cm. Make a ray drawing to scale and locate the virtual image.

7. Compute the position of the image in problem 6 by using the equation on page 624 and compare the result with that obtained in problem 6. -2.4 cm.

8. Find the size of the virtual image in problems 6 and 7. 0.4 cm.

9. An object is placed 10 cm. from a diverging lens and the virtual image is observed to be -5 cm. from the lens. Find the focal length of the lens. -10 cm.

10. A point source of light is located on the principal axis of a converging lens at a distance less than the focal length. Follow the general plan of wave diagrams in Figures 237, 238, and 239 and make a drawing for this case.

11. In comparing the object and image as to relative sizes and distances from the lens in many of the problems of both the *A* and *B* groups both ray diagrams and formulas were used. How did the results obtained by these two methods compare? How do you account for any discrepancies?

Experimental Problems

1. Using a rectangular glass prism and a pin or other small object placed on a sheet of paper, plot the path of light from the pin to the eye when the pin is seen obliquely through the prism.

2. Repeat No. 1 using a triangular glass prism.

3. Pass a source of white light through a triangular glass prism. Intercept the emergent light by a white screen. Diagram the result using colors where necessary.

4. Using a right-triangular glass prism, determine the relative positions of the incident and emergent rays when total internal reflection occurs. Illustrate the result by a labelled diagram. Also, illustrate by diagram any practical uses to which this relationship might be applied.

5. Devise two methods for determining the focal length of a convex lens. Use both experimentally on the same lens, illustrating your results by a scale diagram. Account for any discrepancies in the results as determined by the two methods.

6. Mount on a meter stick, a screen, a convex lens and a light source, such as a candle. With reference to the lens, place the light source in each of the following five positions:

(1) Within *F*.
(2) At *F*.
(3) Between *F* and 2*F*.
(4) At 2*F*.
(5) Beyond 2*F*.

Determine and diagram the nature, location and size of the image in each case.

What other relationship might obtain between the object, lens and image? Why is it impossible to determine this relationship with the apparatus used in this experiment?

Chapter 19

SOME APPLICATIONS OF MIRRORS AND LENSES

In this chapter a number of uses for lenses and mirrors are described. Many of these uses are well known to every one.

The human eye. the camera, the projector, the simple and compound microscope, the reflecting and refracting telescope, are some of the common optical instruments that will be discussed.

1.19. The Eye

The most generally used type of lens is the converging, for such a lens is in the eye of every person and every being that has the ability to see.

The image is always focused on the inner surface of the back of the eye (see Figure 242) and this sensitive inner surface

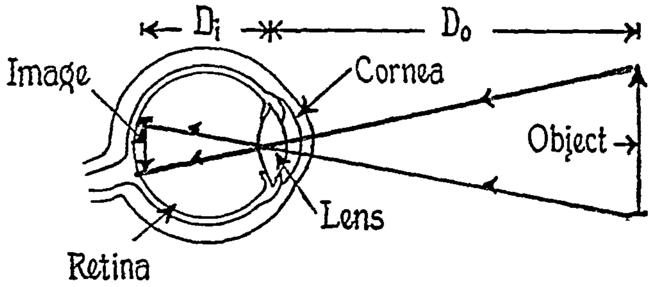


FIG. 242.—The optics of seeing.

of the eye is called the retina. The action of the light on the retina is probably of the type known to chemists as photochemical. The stimulation of this activity causes signals to be carried along the optic nerve to the brain where they are interpreted into what we call the sensation of sight.

The image distance D_i is always the same in the case of any one eye, while the object distance D_o varies with the posi-

tion of the thing at which one is looking. In order to keep the image in focus, the lens of the eye is built in such a manner that its curvature can be changed by muscles. In this way the length of the principal focus, f, is varied so that the relations among these distances remain in agreement with the equation found for ordinary lenses. (See page 624.)

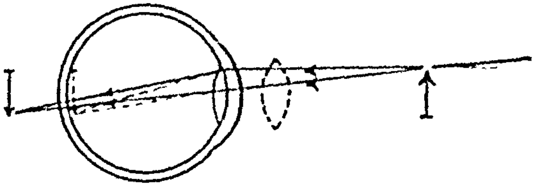


Fig. 243.—Farsighted eye corrected by a converging lens.

Sometimes an eye is too short in a horizontal direction for the lens, so that the image distance (distance from lens to retina) is too short for easy focusing and a constant muscular strain must be exerted on the lens to keep an image sharply focused on the retina. A person with eyes of this type is said to be farsighted because he can see things more easily at a distance than when they are close.

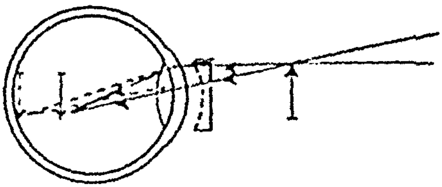


Fig. 244.—Nearsighted eye corrected by a diverging lens.

If an eye is too elongated in a horizontal direction for the lens of the eye, the opposite effect to the one described above is experienced. The person is said to be nearsighted because of the relative ease with which he sees near-by objects and the difficulty that he has in focusing on distant objects.

Both of the eye defects here described are easily compensated for by additional lenses of glass worn on the face in front of the eyes and commonly called just "glasses."

2.19. The Camera

Lenses are used in cameras in a way that is similar to their use in eyes. Here a photographic plate is substituted for the retina of the eye and a glass lens of the converging type takes the place of the lens in the eye. (See Figure 245.) In most cameras the lens is made movable with respect to the plate so that the image distance (distance from lens to plate) may be varied in order to focus the images of objects at different distances.

In some inexpensive cameras this distance is not variable. These are called fixed focus cameras. Such cameras use very

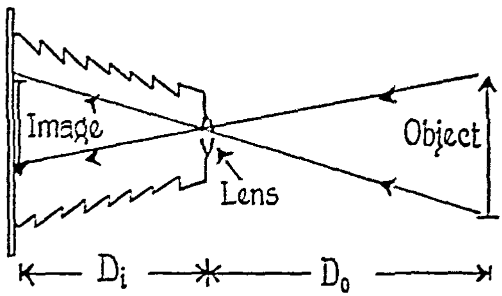


FIG. 245.—A lens on a movable mount is used to get accurate focus with a good camera.

small lenses (small opening) and the focal length and the image distance (lens to plate) are kept small. The amount of motion of the lens needed for sharp focusing is not very great provided one does not try to take pictures of objects closer than eight or ten feet. What little fuzziness exists due to the image being slightly out of focus for some distances of the object one must just take for granted with such cameras.

Lenses having large areas in proportion to their focal lengths must always be movable for focusing in photographic work if sharp images are to be obtained.

3.19. Projection Machines

Projection machines are essentially like cameras working backwards. Suppose that a photographic plate that has been developed is placed in the normal position of a plate in a

camera. The back of the camera is then removed and a powerful light permitted to shine through the plate. Light rays will pass through the plate and through the lens and be focused at the original position of the object. (See Figure 246.)

If a white or light colored screen is placed in this position a picture of the original object is easily seen. We now call the plate the object, and the picture on the screen the image.

The plates ordinarily used in such machines are called slides and a standard size of slide in this country is $3\frac{1}{4}$ by 4 inches. Tape around the edges of the finished slide reduces the useful area to about $2\frac{3}{4}$ by $3\frac{1}{2}$ inches.

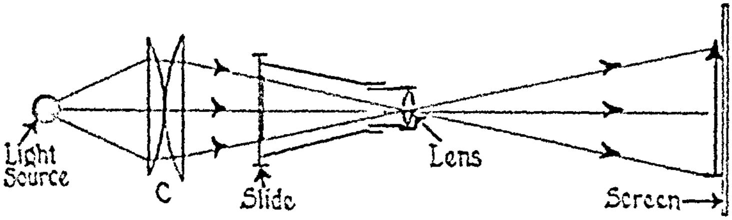


FIG. 246.—Optical arrangement for the projection of pictures from a slide. An additional lens, *C*, is usually employed to bring more light from the source to the slide.

When projection machines are to be purchased one must decide on the size of screen that he wishes to use and on the distance of the lens from this screen. From these two numbers, (and the size of the slides), one can compute the principal focal length of the lens that will be required.

Motion picture projectors are very much like slide projectors except that the standard size of picture on moving picture film is much smaller, and the machine is arranged to move the film into position for a brief instant and then follow it with the next picture. Successive pictures up to 24 per second are shown on many modern moving picture projectors.

4.19. Simple Microscope

When a simple converging lens is placed closer to an object than the principal focal length of the lens, a virtual image is

formed. (See page 625.) This image is always larger than the object and is in the same relative position; that is, it is not inverted.

So a converging lens may be used to get a somewhat larger image of any small object that one wishes to see. Converging lenses with large areas are also often used to aid people with poor eyesight to read ordinary books or papers. When a converging lens is used in this manner it is called a "reading glass."

A person usually holds a reading glass at such a distance that the virtual image appears to be in a position with respect to his eye where he can see most easily. A person with normal vision ordinarily reads with a page between 10 and 14 inches from his eyes.

5.19. Combinations of Lenses

When an image is formed by a lens or mirror, it is quite permissible to look at this image with the aid of another lens

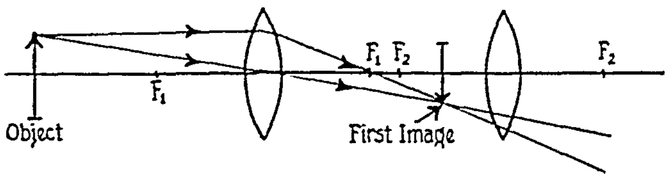


FIG. 247.—An arrangement of two convex lenses showing only the object and the position of the image from the first lens.

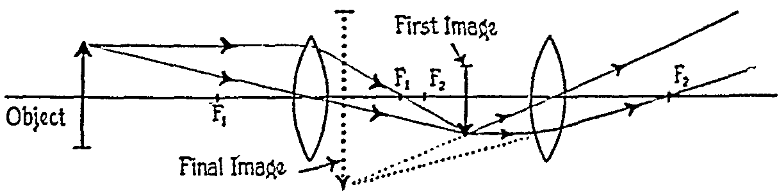


FIG. 248.—The arrangement of Figure 247 showing how the first image may be considered as the object for the second lens and showing also the final image.

or mirror. In Figure 247 two converging lenses are shown and the image of an object formed by the first lens is indicated by the inverted arrow.

In this case the image comes just inside the principal focus of the second lens. This image can be treated as the object

for the second lens as is done in Figure 248. The location of the final image can then be determined with ray diagrams as shown in Figure 248 or by the equations given in Chapter 18.

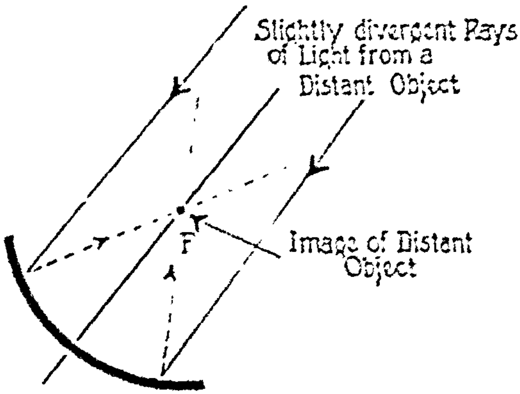


FIG. 249.—A concave mirror may be used in place of the object lens for a telescope.

In the case shown, the second image is virtual and greatly enlarged. This arrangement of two lenses is used in both telescopes and microscopes. In telescopes the first lens, called the object glass, has a long focal length and in microscopes this first lens has a short focal length. A microscope using two lenses as described here is called a compound microscope.

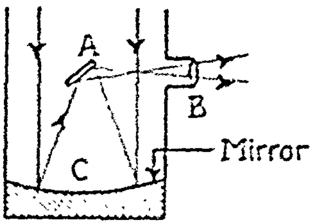


FIG. 250.—A reflecting telescope. Parallel rays of light are focused by the large concave mirror, *C*. The rays are reflected by the plane mirror, *A*, to eye piece, *B*.

6.19. Mirror Telescopes

Mirrors as well as lenses may be used to make telescopes and it is the accepted custom to use a mirror when a large telescope is to be made. At Mount Wilson Observatory a concave mirror 100 inches in diameter has been used for several years to view distant bodies in the heavens. More recently a mirror 200 inches in diameter has been prepared for similar use and has been installed in an observatory on Mount Palomar, in California.

A real image of the distant object is formed by such a concave mirror just as described in a previous chapter. This image is then examined by means of other mirrors or lenses or both.

Some Important Facts

1. In the eye, the image distance is constant, although the object distance varies. Therefore, the focal length of the converging lens must be varied by muscular effort.

2. In the camera, an accurate focus is secured by moving the lens with respect to the film and hence the image distance is adjustable.

3. A projector is essentially a camera in reverse, the relative positions of the object and image being interchanged.

4. A simple microscope or reading glass is a converging lens through which the object is viewed, the image being virtual, erect, enlarged and on the same side of the lens as the object.

5. In the compound microscope, telescope and many other optical instruments, one views through the eye-piece an enlarged, virtual image of the real image formed by the objective.

6. A concave, converging mirror, rather than a convex, converging lens is often used where large, light-intercepting areas are necessary, as in reflecting telescopes.

Generalization

Image forming devices such as lenses and mirrors, singly and in combinations, are used in the construction of optical apparatus.

Problems

Group A

1. Is the image formed by the lens of the eye real or virtual?

2. A farsighted person and a person with normal vision look at a moderately close object. Compare the action of the muscles of the eye on the shape of the eye lenses in the two cases.

3. Why are distant objects seen more easily than near objects with farsighted eyes?

4. Repeat problem 2 for a nearsighted person in comparison to a person with normal vision.

5. Why is a lens with a large opening desirable for most photographic work?

6. Can you think of any reasons why a lens with a very small opening does not have to be carefully adjusted for the correct position in order that sharp images may be obtained?

INFRA-RED AND ULTRA-VIOLET

Infra-red and ultra-violet radiations are so near visible light in wave-length that they have many of the same properties and can be produced and handled in somewhat the same fashion. However, the wave-lengths of these rays, and hence their frequencies, are different enough so that they produce some effects which visible light does not. This chapter gives a brief account of some of the unusual effects which can be obtained with these radiations.

1.20. General Nature of Infra-red and Ultra-violet

Ordinary sunlight contains a great deal of energy in waves both too long and too short to give a sight sensation in human

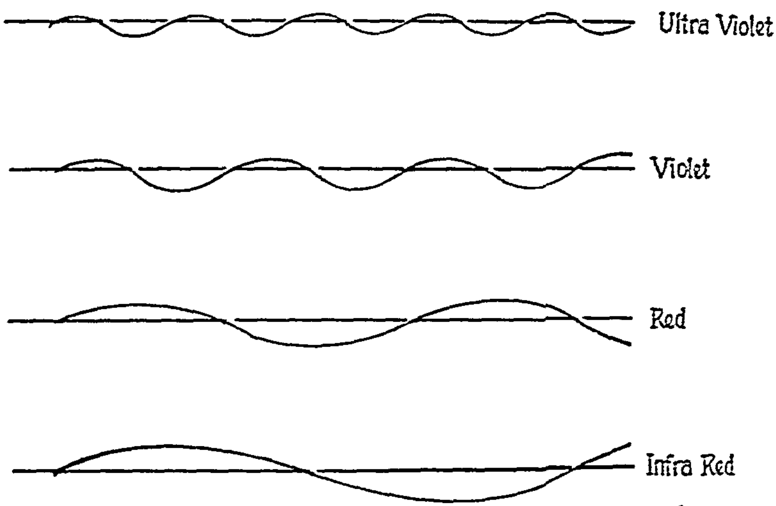


FIG. 251.—Some typical wave lengths multiplied about 50,000 times.

eyes. The waves just slightly shorter than the visible are called ultra-violet and those longer are called infra-red.

Tan and ordinary sunburn are due almost entirely to ultra-violet in the sunshine, while the simple sensation of warmth

is due mostly to the infra-red. In fact, the latter is often called heat radiation.

Artificial sources of visible light all produce relatively large quantities of heat radiation. Most of the electrical energy put into an ordinary electric lamp is radiated away as heat. From the point of view of visible light, all of our lamps are inefficient devices. The invention of "cold" light, such as is produced by the lightning bug, would probably be heralded as the greatest technical discovery of the age.

Electric lamps of the hot filament type and lamps of the flame type, where illuminating gas or other combustible material is burned, produce almost no ultra-violet. On the other hand, extremely hot types of lamps, such as arc lamps, and also many lamps of the gas discharge type, do produce ultra-violet.

Both infra-red and ultra-violet may be handled much the same as visible light. They can be refracted by lenses and reflected by mirrors. But ordinary glass, although transparent to visible light, is opaque to ultra-violet. So lenses for ultra-violet must be made of quartz or some other substance which will be transparent to it.

Recently microscopes have been built with quartz lenses so that photographs of very small objects such as living cells may be taken with ultra-violet light instead of visible light. In this way we get around the difficulty pointed out on page 578 about not being able to see things unless they are large in comparison to the wave lengths of light to which the eye responds. Ultra-violet waves, being considerably shorter than visible light, permit one to take pictures of very small structures in greater detail than may be observed with the human eye.

Infra-red is also being used in connection with photography to extend ordinary vision. Dust particles in air which reflect and scatter the shorter wave lengths of ultra-violet and visible light have relatively little effect on heat radiation. Recently, photographic plates have been made which are sensitive to heat rays. They make it possible to take bird's eye views of

distant points even when a haze almost obscures the place from ordinary sight.

Both ultra-violet and infra-red are used in various kinds of medical treatment. Infra-red is used chiefly as a baking process to get heat into muscles and joints. Ultra-violet of wave lengths shorter than those in sunlight is used to treat various types of skin diseases.

2.20. Sunlight—Natural and Artificial

It is generally considered that exposure to sunlight in moderate quantities has the effect of improving the health of people. It seems to build up the ability of the body to resist disease. It is not a direct cure for ailments already existing,

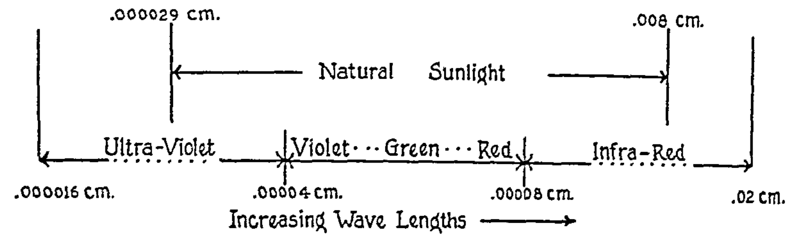


FIG. 252.—A chart of wave lengths in the visible and near visible regions.

but it does seem to have value in preventing the acquiring of disease.

It was at one time thought that all of the radiations of the sun from far infra-red through the visible and ultra-violet were required for these beneficial effects. But recent experimental work shows that at least for some of the desirable results, most, and possibly all, of the healthful ray region exists in a small part of the ultra-violet, lying in wave length from 0.000031 cm. to 0.000029 cm. This latter value is the short wave length limit of the sunlight which penetrates the earth's atmosphere.

Mercury vapor lamps and most other artificial sources of ultra-violet emit not only rays in this desired wave length region but also at much shorter wave lengths. Ultra-violet a little shorter than that in natural sunlight seems to have healthful properties, but radiation with waves as short as

0.000025 cm. or less seems to inflict bad burns without having any desirable effect on the general health of the person. (See Figure 252.)

Recently a type of glass has been developed which is transparent to ultra-violet down about as far as natural sunlight or a little further. However, it is opaque to the shorter waves which produce harmful burns.

Mercury vapor lamps may be made by using this new type of glass for the bulbs. They are then quite safe to use in the home without any more precaution than one would take in exposing himself to natural sunlight.

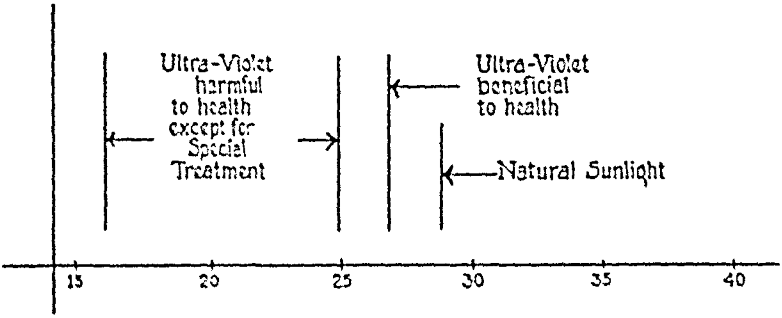


FIG. 253.—A chart of wave lengths in the ultra-violet region. (See also Figure 252.) The wave lengths are given in millionths of centimeters.

Some of these mercury vapor lamps also have filaments in them which not only heat the mercury and aid the arc to start, but they also provide visible light which prevents the flicker of an ordinary mercury arc lamp from bothering the eyes of a person who might wish to read under the lamp.

Lack of sunshine is often supposed to be responsible for rickets, a disease of the bones quite common in small growing children. It is a condition where the proper amount of calcium does not go into the bone structure to make the bones as hard and firm as they should normally be.

The proper absorption of calcium seems to take place if the child is exposed to a moderate amount of sunshine. In many places the amount of sunshine available in the winter is

too small or the temperatures may be too cold to permit children to play in the sun. In such cases the starting of rickets may sometimes be prevented by the moderate use of a good artificial daylight lamp. Other substitutes for beneficial effects of natural sunlight seem to be certain kinds of foods that contain vitamin D.

Glass of a type similar to that used in the special lamp described above is now being used to make windowpanes, and many of our newer buildings have windows of this material. This use is an attempt to get ultra-violet from natural sunlight into our houses.

This is a worthy cause, but the use of such glass for windows is probably not as valuable as one would at first think. Even if we took all the windowpanes out, so that there would be nothing at all to stop ultra-violet from coming in, the amount that would reach the average individual in a room with only a normal number of windows, would be extremely small in comparison to what he would get out-of-doors in the sunshine.

Of course if we lived on the top floor of a house which was equipped with a large skylight equipped with the new glass, the effect would be quite pronounced.

Good artificial daylight lamps available at this time are moderately expensive, but most of the cheaper ones either do not contain any ultra-violet, or else they emit the shorter burning rays as well as the rays that are desired.

Some Important Facts

1. Light waves shorter than the shortest visible violets are called ultra-violet, and are useful in microphotography.

Light waves longer than the longest visible red are called infra-red, and are used in photographing objects through fog.

Both ultra-violet and infra-red rays penetrate many substances opaque to visible light and have special therapeutic values.

2. The beneficial effects of sunlight appear to be largely due to a narrow ultra-violet wave band (0.000029 cm. to 0.000031 cm.). Part of this band is also emitted by the mercury vapor lamp.

Ordinary glass is opaque to these wave lengths.

Generalization

In intrinsic nature, ultra-violet and infra-red radiations differ from visible light only in wave length and frequency. However, their effects, physiological, chemical, etc., are quite different.

Problems

Group A

1. What is the chief objection to having a lamp produce heat?

2. In what way does ultra-violet light extend the use of the microscope?

3. Explain one use of infra-red radiation in photography.

4. What effect would radiation from a mercury vapor lamp in a quartz bulb have on a person?

5. A person sits in the middle of a room that is 10 feet high and has walls 10 feet long on each side. There are two windows each 3 ft. by 6 ft. in one wall. They are equipped with glass that is transparent to ultra-violet. There are no windows in the other walls. Find the total wall and ceiling area and compare the window area with this value.

500 sq. ft. 36 sq. ft. $^{36}/_{500} = 0.072$.

Group B

1. Chart all the types of electromagnetic wave motions which you have studied in the order of their increasing wave lengths, giving the names and approximate wave lengths of each band.

2. Why is vitamin "*D*" sometimes called the sunshine vitamin?

3. Collect and evaluate available experimental results on the therapeutic value of sunlight in treating diseases, such as tuberculosis, rickets, etc.

Experimental Problem

1. Repeat Experimental Problem No. 1 of Chapter 14 but use an ultra-violet lamp to discover substances that will give visible fluorescence or phosphorescence.

Chapter 21

PHOTOELECTRICITY

In an earlier chapter we learned that when electrons fall on a solid object such as a metal target, x-rays are emitted. In that chapter we also learned that when x-rays strike atoms they may knock electrons from the atoms, thus ionizing them.

If we let the x-rays fall on a solid, such as a metal plate, we find that electrons come off of the surface of the plate with appreciable velocities.

Other forms of electromagnetic radiations, such as ultra-violet, ordinary visible light and infra-red, liberate electrons from some substances.

The emission of electrons from a substance when it is subjected to any form of electromagnetic radiation is known as the *photoelectric effect*.

The effect can be applied to various devices, such as illumination meters, burglar alarms, and television.

Definite relations seem to exist between the maximum speed with which the electrons can leave the solid and the wave length of the light which is used. The discovery of these facts greatly aids us in developing ideas concerning the structure of atoms.

1.21. X-rays Remove Electrons from Atoms

In Chapter 14 we learned that x-rays, a very short wave length type of electromagnetic radiations, are produced when speeding electrons are stopped abruptly by striking a heavy object such as the target in an x-ray tube.

This is an experiment that can also be tried in the reverse direction. That is, we may let a beam of x-rays strike an object and we can investigate to find out whether or not electrons are knocked from this target.

This experiment was, in fact, suggested in the x-ray chapter as a means for finding out whether or not x-rays were present. One of the more important effects of x-rays on atoms was to ionize them, that is, to knock one or more electrons from an atom. The electrons thus removed from

atoms move away at various speeds, some very slowly, some very rapidly, and in all directions.

If we let x-rays fall on an electrically conducting solid object, such as a piece of metal, we will find that electrons come flying out of the surface of the metal. These will have various speeds, but the fastest speed will be nearly equal to the speed of the electrons that produce the x-rays. If we increase the speed of the electrons in the x-ray tube, thus getting shorter wave length x-rays, we will get a higher speed

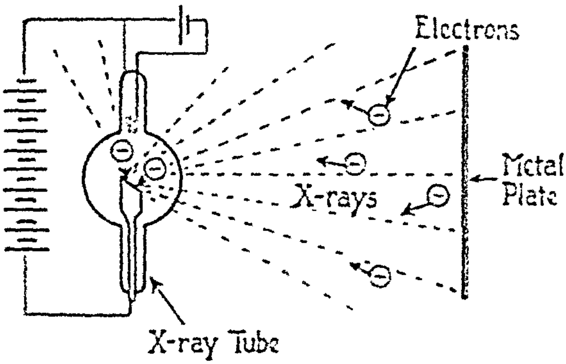


FIG. 254.—High speed electrons strike the target of the x-ray tube and produce x-rays. The x-rays strike a metal plate and knock electrons from it.

for the fastest electrons coming from the plate of metal on which the x-rays fall.

2.21. Light Can Free Electrons

We need not restrict ourselves to the use of such short wave length radiations as x-rays, but can try ultra-violet light, visible light, and infra-red. With varying degrees of success we will find that electron emission does take place from many substances for all of these types of radiation. The whole phenomenon, which is just the reverse of the production of x-rays, is called the PHOTOELECTRIC EFFECT.

The number of electrons emitted per second with radiation of any one wave length varies enormously with the substance of which the plate is made. There is also a variation in the number of electrons emitted from the same substance when

radiations of different wave lengths are used. Nearly all conducting substances emit electrons fairly well when very short waves, such as x-rays, are used. Very few substances emit very well for such long wave length radiation as infra-red.

3.21. Construction of Photoelectric Cells

One of the earliest substances used with much success in the ultra-violet and visible light regions was selenium. During the past few years such metals as pure sodium have been used with more success. Çesium shows some response all the way from ultra-violet through visible light to infra-red and is a favorite for use with ordinary visible light.

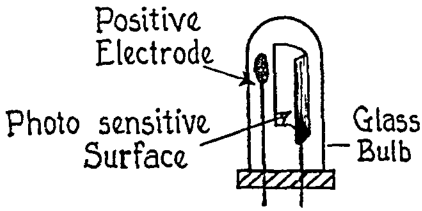


FIG. 255.—A photoelectric cell showing the sensitive plate and the positive electrode in a glass tube.

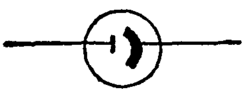


FIG. 256.—Schematic representation of a photoelectric cell.

Simple photoelectric cells are usually made by sealing a plate of metal in a glass tube as indicated in Figure 255. In some cases the tube is exhausted to a high vacuum and in other cases a small amount of gas is left in the tube. The second electrode shown is small in size so that it does not stop much of the light that enters the tube. In schematic wiring diagrams the photocell is usually indicated as in Figure 256.

4.21. A Simple Experiment with a Photoelectric Cell

It is a good exercise to connect a photocell in series with a resistance, a battery, and a sensitive galvanometer as shown in Figure 257. The resistance is used to protect the galvanometer or the photocell against excess current in case the battery voltage is too great for the photocell used.

As the lamp is moved toward or away from the photocell, variations in the reading of the galvanometer will occur. Of course, the current shown by the galvanometer is due to the number of electrons per second that leave the photoelectric surface, move through the tube, and enter the second electrode of the tube which is fastened to the positive side of the battery.

For more accurate results with this experiment it would be advisable to repeat it in a darkened room so that the light from the lamp is the only light that reaches the photocell. Experiment should then show a definite relation between the galvanometer readings (which correspond to the electron

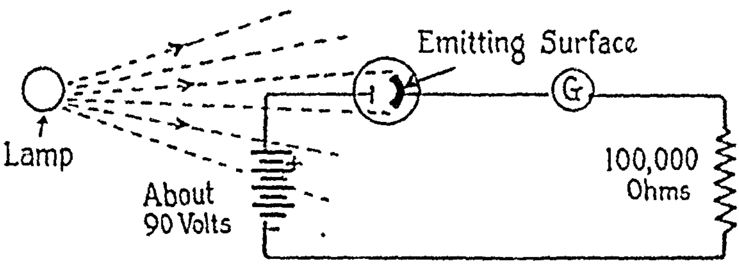


FIG. 257.—A simple circuit for measuring the electron current from a photoelectric cell.

emission per second in the tube) and the various relative distances of the lamp from the tube.

Moving the lamp to different distances changes the intensity of illumination of the photo-sensitive surface of the cell. The interested student may refer to the section on illumination in Chapter 16.

5.21. Voltaic Photocells

A somewhat more involved case of photoelectricity is found in what is known as a photovoltaic cell. Such a cell is shown in Figure 258 where certain crystals rest against a piece of material of different structure. Some combinations of two substances have the peculiar property of having electrons forced from one material to the other when light shines on the combination. In other words, the effect of light is to make the combination behave like a miniature battery. The

actual e.m.f. developed is small, but quite appreciable numbers of electrons are transferred per second by a very moderate amount of light. This current is sufficient to operate a sensitive relay; and, of course, no battery is needed with this type of cell.

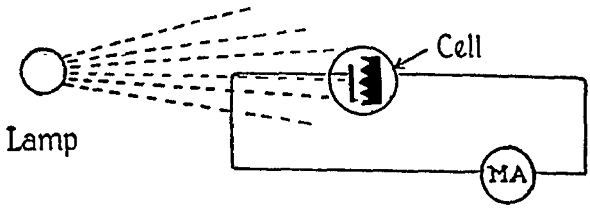


FIG. 258.—A simple circuit for measuring current from a voltaic photoelectric cell.

6.21. An Experiment with the Voltaic Photocell

The experiment suggested with the first type of photoelectric cell may now be repeated with the voltaic type cell. See the diagram shown in Figure 258. Again we will find that the readings of the meter are greater with the greater illumination and weaker as we move the lamp away from the cell. The galvanometer required with this cell will not need to be so sensitive as the one used with the first photocell described in this chapter.

7.21. A Photocell Illumination Meter

The above experiment suggests a practical use for the photovoltaic cell. Instead of calibrating the galvanometer in fractions of amperes, we might calibrate it in terms of the illumination on the photo cell for any given position of the needle on the meter. This combination of a photovoltaic cell and a galvanometer would then make a very fine illumination meter. This type of meter was mentioned on page 591. It is now in common use by illumination engineers. It is also built up into compact form and used by photographers to determine how long they should make exposures when taking pictures.

8.21. Other Uses for Photocells

Figure 259 shows a photovoltaic cell connected to a relay. The relay is arranged to control current to an electric bell.

The arrangement is such that when light shines on the photocell the relay is held open. When the light is cut off, the relay closes and the bell rings.

In place of the bell, an electric motor might be placed in the relay circuit. Such an arrangement could be used to open a door. For example, a beam of light could be focused across the path to a door and let fall on a photovoltaic cell. As a person walked through the beam of light, he would interrupt it and the photocell would cause the relay to close. The motor could be connected in such a manner that the door would open just as the person got to it. The device is particularly useful in restaurants to open doors to the kitchens

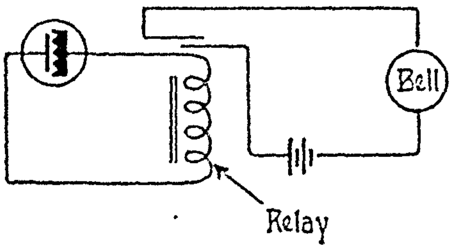


FIG. 259.—The bell at the right is controlled by a photovoltaic cell operating through a relay.

and in railway stations to open doors through which people walk with their hands full of baggage. It is used in reverse fashion on some modern elevators to keep the doors from being closed or the elevator from starting if any person is standing in the doorway.

Many other uses will occur to the student: for example, such things as burglar and fire alarms, and devices for turning on lights in buildings depending on the amount of daylight outdoors.

9.21. Amplifiers and Photoelectric Currents

Photoelectric cells of the first type may also be used for all of these purposes, but the current supplied is not great enough to operate a simple relay directly. So it is necessary to amplify the effects of these cells with the aid of electron tubes and circuits such as are described in Chapter 13.

Figure 260 shows a simple arrangement of this type. A photocell is connected in series with a large resistor (20 megohms) and a 90 volt battery. This circuit is shown to the left of the dotted line in the figure. When light strikes the photosensitive cell a small current will exist in the resistor, so there will be a voltage across this resistor according to Ohm's law (volts = amperes × ohms, $E = IR$).

The voltage developed across this resistor is applied between the grid and filament of the tube shown to the right of the dotted line. Any variation in the illumination on the photocell will change the current and hence the voltage across the resistor. This will change the voltage on the grid

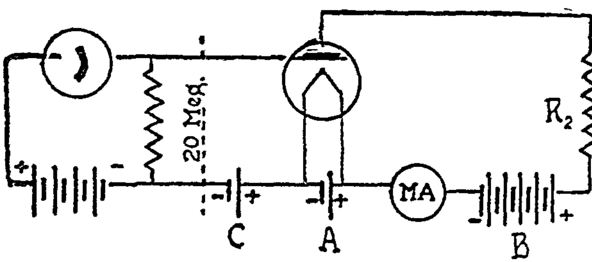


FIG. 260.—A photoelectric cell connected to an electron tube amplifier.

of the tube. The ambitious student might try this arrangement. The tube will amplify the effect so that the voltage changes across the resistor R_2 will be greater than those across R_1 or so that the current changes shown on the milliammeter in the tube circuit will be greater than current changes in the circuit of the photoelectric cell. A bell or relay may be substituted for the resistor R_2.

10.21. Photocells for Television

This general method of amplification of feeble photocell currents has many applications, one of the more important of which is in the field of television. Light from small sections of the scene that is to be transmitted falls on the cell and the effect is amplified through many tubes until it controls the entire output to a radio transmitter. Light from successive sections of the scene follow one another on the photocell and

so varying amounts of energy are released to the transmitter depending on the illumination of the various parts of the scene.

11.21. Photoelectric Data Aid Pure Science

Applications of the photoelectric effect are numerous and many of them spectacular; but of equally great importance are the clues that photoelectric experiments give us about the structure of atoms and the way nature acts.

Early in the chapter we noted that the highest speed with which electrons can be shot off from a surface irradiated with light depended not on the intensity of the light but on the

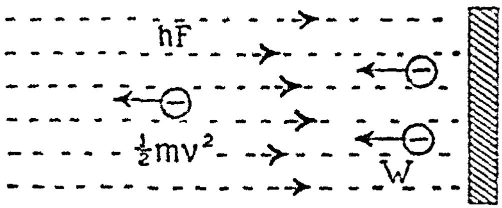


FIG. 261.—The kinetic energy of the photoelectrons is less than that in the radiant energy by a small amount which represents the work done in tearing electrons away from the solid substance.

color of the light—or, in terms of physics, the wave length of the light. The shorter the wave length, the higher the electron speeds.

This effect is similar to the effect in the production of x-rays where we saw that the higher the electron speeds, the shorter the wave lengths of x-rays produced. There we learned that a simple relation described this effect. (See page 567.) It was: Energy of impacting electrons = a constant term multiplied by the frequency of the waves. In symbols we wrote:

$$\text{Energy} = hf$$

where h is the constant number that relates the frequency of the waves to the energy of the impacting electrons, and f is the frequency of the waves (number per second).

For the photoelectric case we find that the energy of the highest speed electrons is always just a little less than would be predicted by this equation. Apparently it takes a little work to remove an electron from the surface of a solid and so a small part of the energy absorbed from the radiations that fall on the solid is used in getting the electrons away from the solid. So the kinetic energy of the electrons ($\frac{1}{2}mv^2$) must be a little less than was put into them by the radiation.

We can re-write the x-ray equation for the photoelectric effect thus:

Energy of electron speeding from surface = energy put in by radiation minus the work required to remove the electron from the solid.

Now the energy put in by the radiation is hf and so we may write

$$\text{Energy of electron} = hf - W$$

where W is the work required to remove the electron from the solid.

12.21. Energy Comes in Parcels

One of the more important things to learn from these effects is that the amount of energy that any one electron can absorb from electromagnetic radiation is hf. Suppose that we observe that the highest speed with which an electron leaves the surface of a piece of metal illuminated with blue light is a certain number of centimeters per second. If now we change the light to red instead of blue we find that the maximum speed for the electrons is lower. No matter how long the red light shines on the surface no electrons ever leave with as great speed as they did when we used the blue light.

Changing the intensity of illumination changes the number of electrons that leave the surface each second, but it does not change the top speed.

These things lead us to think that electrons do not absorb energy slowly over a long period of time, but rather that they absorb a definite amount (hf) at one time depending only on the frequency of the waves in the radiation.

This state of affairs gives us the impression that light itself is not continuous wave motion, but that it comes in little spurts, called "quanta" by scientists. The fact that radiant energy, such as light, seems to exist in definite little quantities instead of coming along continuously is one of the more important ideas of present day science, and the experiments on the photoelectric effect give us some of the best evidence in favor of this view.

The equation relating the energies in photoelectric emission of electrons was developed by Albert Einstein whom most people think of only in connection with theories of gravitation and relativity.

Under ordinary circumstances we do not get the impression of light being made up of little individual bundles of energy because so tremendously many of these little quantities are required to make up any amount of light such as we ordinarily use. So for most purposes the light appears to be continuous.

Some Important Facts

1. When x-rays strike the surface of a good conductor, electrons are emitted from the surface at velocities which depend on the frequency of the x-rays.

2. To a lesser degree, ultra-violet, visible and infra-red light have this same photoelectric effect.

3. A photoelectric cell is essentially a small positive electrode partially surrounded by some light-sensitive metal, such as *cesium*. Both electrodes are in a suitable housing, such as a vacuum tube.

4. If a photoelectric cell, battery, galvanometer and a suitable resistance are connected in series, the galvanometer deflection is seen to increase directly with the light intensity.

5. When light falls on some combinations of two conductors, the combination behaves as a miniature voltaic cell whose current intensity varies directly as the light intensity.

6. Since the photovoltaic cell furnishes its own current, it may be used without a battery.

7. A compact and convenient form of illumination meter is a photovoltaic cell in series with a galvanometer.

8. Since the power output of a photoelectric cell, or even a photovoltaic cell, is very small, these cells are commonly used to operate relays to switch on or off a power source of any desired magnitude.

9. Frequently photoelectric currents are amplified by vacuum tubes, variations in the feeble photocell-grid circuit causing corresponding variations in the relatively powerful plate output.

10. The photoelectric cell is an essential part of all television and wire-photo apparatus.

11. The direct ratio between radiation frequency and the kinetic energy of emitted electrons serves as a clue in solving many problems concerning the nature of matter and energy.

12. An important implication of the photoelectric effect is that energy exists in little spurts, called "quanta."

Generalization

The kinetic energy of high-speed electrons may be transformed into radiant energy; conversely, radiant energy may be transformed into the kinetic energy of moving electrons. In either case, the radiation frequency varies directly with the kinetic energy of the electrons.

Questions

Group A

1. What causes x-rays to be produced?

2. What is the effect of x-rays when they fall on free atoms?

3. What happens when x-rays strike a piece of solid material such as a metal plate?

4. What are the differences between x-rays, ultra-violet light, visible light and infra-red radiation?

5. What differences are there in the emission of electrons from a metal plate for light of various wave lengths?

6. What variations are there among different substances as regards their photoelectric emission of electrons?

7. What change in the photoelectric emission of electrons occurs with variation in the intensity of illumination?

8. Make a list of services that could be performed by light-controlled relays operating from a photovoltaic cell.

Group B

1. How can the effect of Question 7, Group A, be used for making an illumination meter?

2. Compare the construction of a simple photoelectric cell and a photovoltaic cell.

3. What advantages for some practical applications does the photovoltaic type of cell have?

4. If a resistor, battery, and photoelectric cell are connected in series, a voltage is found across the resistor and its value changes with the illumina-

tion of the cell. Why should there be a voltage across this resistor and why should it change?

5. If the photoelectric effect is the reverse of the x-ray effect, why does not the same energy equation hold?

6. What important idea in modern science comes from an interpretation of the photoelectric effect?

Experimental Problems

1. Arrange a photoelectric cell, galvanometer, battery and suitable resistance in series so as to form a simple illumination meter.

2. Repeat No. 1, using a photovoltaic cell.

3. Using a photocell, light source, electric bell, batteries, relays (and suitable connections) construct a burglar alarm so that intercepting a beam of light will ring a bell.

4. Alter the connection in No. 3 so that the bell will ring when the light is turned on.

MORE ABOUT WAVES

This chapter gathers together many facts about wave motions of various kinds, and indicates some important similarities and differences in the different types.

Some emphasis is placed on the fact that electromagnetic waves, unlike all other types, can travel through a vacuum.

In order to detect the presence of radiations of any kind it is necessary to use some device which will absorb a little of the radiated energy and give an observable indication.

The importance of electromagnetic radiations both to man's general welfare and to the progress of science is briefly suggested in the latter part of the chapter.

1.22. The Media through Which Waves Travel

One of the more important things in the physical world is the radiation of energy by wave motion from one point to another. Of the various wave motions with which we are familiar, water waves, sound waves in air, and some electromagnetic waves are perhaps among the more commonly observed. For the first of these, the medium that does the "waving" is obviously the water. For the sound waves we again have a material medium, this time the air. But no material medium at all seems to be necessary for electromagnetic waves. They travel through space apparently without the need of any substance to do the waving. Air has but little effect on their travelling and is never any necessary aid to them.

2.22. Types of Waves

The general appearance of water waves is an up and down motion as the waves travel in a horizontal direction over the surface of any body of water. Closer observations show that any particle of water actually performs a more involved motion than simply moving up and down. It moves forward when on top of the wave and backward in the trough. (See Figure 262.)

In sound waves, molecules in air move to and fro in the direction in which the sound waves of compression and rarefraction are travelling. Such waves are sometimes called by the descriptive name "compressional waves." Sometimes they are called "longitudinal waves" to indicate that the particles move to and fro along the same line in which the wave is progressing. (See Figure 263.)

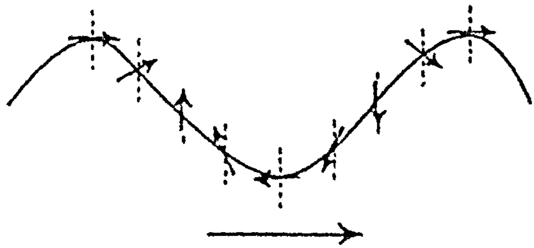


FIG. 262.—Cross section of water waves travelling from left to right. The small arrows indicate the motions of particles of water at various points on the wave.

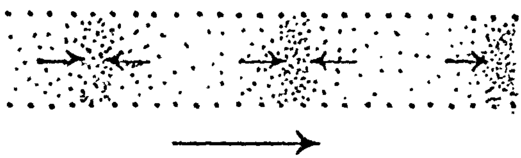


FIG. 263.—Sound waves travelling in air from left to right. The density of the air is indicated by the arrangement of the dots. The actual motion of the air molecules is suggested by the short arrows. A moment later the molecules will be rebounding away from the dense regions. The timing of this to and fro motion of the molecules is such as to make the locations of the dense and rarified regions progress towards the right in this case.

Electromagnetic waves consist of moving electric fields. The electric field vibrates crosswise to the direction in which the wave is travelling. The motion of this electric field gives rise to a magnetic field which vibrates at right angles to the electric vibration and also at right angles to the direction of travel of the wave. Electromagnetic radiation is said to consist of "transverse" waves, which term indicates that the directions of vibration of the electric and magnetic fields are crosswise to the direction of travel of the waves. (See Figure 264.)

3.22. Polarized Waves

In the case of wireless waves radiated from a simple horizontal antenna, the electric vibrations in the waves are up and

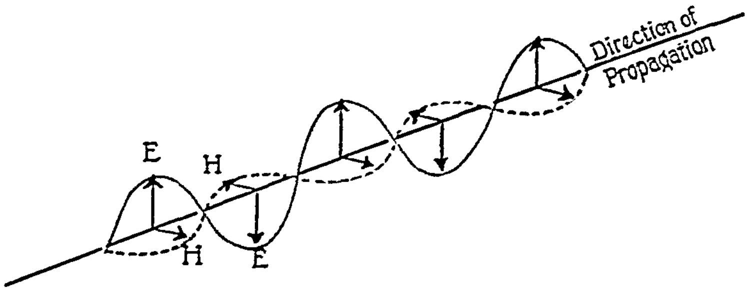


FIG. 264.—Transverse vibrations of an electromagnetic wave travelling from left to right. In this particular example the electrical vibrations are in a vertical plane and the magnetic vibrations in a horizontal plane.

down in a vertical plane and the magnetic vibrations are to and fro in a horizontal plane. In this example, the radiation is due to the coordinated motion of large numbers of electrons in the transmitting antenna. (See Figure 265.)

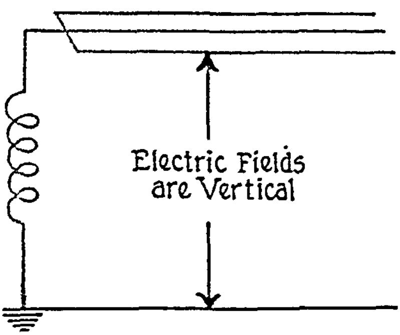


FIG. 265.—In this simple radio transmitter the electric field is vertical—alternately up and then down. The waves radiated will also have their electrical vibrations in a vertical plane.

In the case of ordinary visible light, any quantity of light great enough to be seen must come from a multitude of electrical particles in a great many more or less independent atoms. Consequently some of the electric vibrations may be up and down, some may be horizontal and others may occupy any other possible plane of vibration provided only that the vibration be crosswise to the direction in which the wave is travelling.

Radiation such as the simple wireless waves described above in which the vibrations of the same kind are all in the

same plane is said to be polarized, while radiation with miscellaneous planes of vibration is said to be unpolarized.

It is possible to arrange minute pieces of crystals in such a manner that when ordinary light shines through the arrangement, the waves having vibration directions in one direction with respect to the crystal arrangement will pass through readily while those having vibration directions at right angles to the first group will be almost completely blocked. The emergent light will now be polarized and the device is called a polarizer.

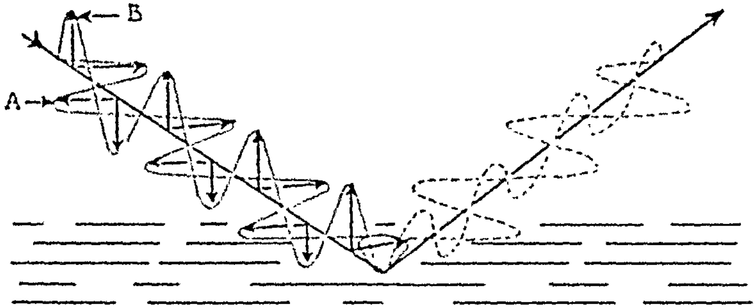


FIG. 266.—Electromagnetic waves having electrical vibrations parallel (*A*) to a reflecting surface are reflected more efficiently than similar waves with electrical vibrations perpendicular (*B*) to the surface. Only the electrical vibrations of each wave are shown.

Some polarization occurs when ordinary light is reflected at angles other than the perpendicular to a surface. Figure 266 shows two wave trains approaching a reflecting surface. The one having electric vibrations parallel to the surface is reflected much more efficiently than the other. It follows that sunlight reflected from water is partially polarized with the electric vibration horizontal. Similarly the light from the head lamps of an approaching automobile is partially polarized after reflection from the street, although the light in the direct beam is unpolarized.

The suggestion has been made that the lenses of automobile headlights and the windshields be made of polarizing material set in opposite directions. For example, the light in the head lamps could be polarized in a vertical plane while the windshields could be set to pass light efficiently only when

it is polarized in a horizontal plane. Such an arrangement would greatly cut down glare and yet not entirely prevent seeing the approaching lights.

Other and more complicated methods for obtaining polarized light have been known for a number of years and many practical applications, including some methods for chemical analysis, have been made.

4.22. Work Done by Waves

Any wave motion may be thought of as a method for getting energy from one place to another without having any direct contact in the ordinary sense of the word.

We know that water waves can be created by moving a block of wood up and down on the surface of a lake. At some distant point the waves can cause another block of wood to move up and down, and this block can be connected to some device so that its motion can do work.

Air waves are created by the motion in the air of a mechanical object, such as the prongs of a tuning fork or the cone of a loudspeaker. Waves of compression and rarefraction travel out through the air, sometimes striking an ear drum or a microphone diaphragm and doing desired work; other times having their energy absorbed in their long travel through air or by material objects that they strike.

Electromagnetic waves are created when electricity is accelerated in an open electrical system (such as an antenna) in such a manner that energy leaves the system and travels through space. The operation of a radio transmitter is a well known example. These waves deliver energy to any device that can absorb it. The effects of electromagnetic waves on ordinary matter depend largely on the wave lengths of the radiation. And since these waves may vary from a millionth of a millionth of a centimeter to millions of millions of centimeters, the variations in effects of absorbed electromagnetic radiation are great.

We have already seen that the principal effect of moderately long electromagnetic waves (wireless waves) is to cause

alternating electric currents to flow in any conductor (such as a radio receiving antenna). Also we are familiar with the effect on human eyes of moderately short radiation which we call visible light. Further well known effects of electromagnetic radiation include the ionizing of atoms and molecules by shorter waves (such as x-rays) and the effect of heat noticed on the absorption of rays just slightly longer than the visible (infra-red).

5.22. Methods for Creating and Absorbing Electromagnetic Radiations

Since the wave lengths of electromagnetic radiations vary so enormously, it is not surprising to find that the methods for producing the radiations are different in detail although they all involve the loss of energy from electrical systems. We have already seen that very long waves are created by the to and fro acceleration of large numbers of conduction electrons in an open system such as the antenna and ground used in a wireless transmitter. On the other hand, we recall that very short waves (x-rays) are produced when individual free electrons are stopped suddenly by the solid target of an x-ray tube. The moderately short rays in the visible region may be caused by the shifting of electrons in their orbits in atoms. From this same source may come infra-red radiation if the energy changes are small enough, or if these changes are large, ultra-violet or even x-rays may be produced.

When electromagnetic radiations are absorbed by objects, we may expect to find that the effects produced are often closely related to the actions involved in producing the radiations. For example, the production of a to and fro current in a receiving antenna is very closely allied to the method of production of the long electromagnetic wireless waves at a transmitting antenna. On the other hand, the removal of electrons from atoms (which is the ionization of atoms) on the absorption of ultra-violet is in the same general field of activity as the production of ultra-violet by a shift of an electron in the orbits of the atom.

6.22. The Detection of Radiation

We can say that the only way to detect any kind of a radiation of energy is to have some kind of absorbing device which can be affected in an observable manner by the absorption of a small amount of the energy of the waves. For example, an easy way to find out if any radio waves are passing is to erect an antenna and to place a detector in the antenna system. Only when enough energy from a train of radio waves is absorbed to operate the detector will we know that such a radiation is present. The fraction of the energy in the train of waves that is absorbed by the antenna is small, so that most of the energy passes on to be picked up by other observers, or to be dissipated in space.

Similarly, if we wish to know whether or not sound waves are passing, we place our heads in the position where the sound is supposed to be, and if our ears pick up enough energy to produce the sensation of sound, we conclude that sound waves are present.

The presence of any kind of radiation at any point can only be discovered when some device responsive to energy in that particular form is at hand.

7.22. Electromagnetic Radiations from the Sun

The electromagnetic radiations most important to man are those that come to the earth from the sun. The most obvious of these are the visible radiations that we call light. But the heat radiation and the ionizing radiations (ultra-violet) are also useful.

The ultra-violet is absorbed to a considerable extent by the earth's atmosphere, so that only a small fraction of what reaches our outer atmosphere gets through to the surface of the solid earth. In the upper atmosphere, the ultra-violet radiation ionizes many molecules, and layers of air are known to exist in which there is great ionization. These layers of partially conducting air are useful, as we have seen, in refracting or reflecting wireless waves. In this way many radio waves, starting off partly skyward, are turned back

to the earth and so the signals are heard at points far away from the sending stations.

In the previous chapter we learned that the ultra-violet which reaches the earth is considered to be the chief health-giving part of the sun's radiations. It is supposed to prevent rickets in growing children and to contribute to a normal person's ability to resist disease. Ultra-violet radiation is also responsible for sunburn. It is well known that more care must be taken against sunburn on top of high mountains than in lower lying regions. This is because the additional layer of air between the level of a high mountain and that of lower regions absorbs a considerable part of the ultra-violet which has penetrated to the level of the top of the mountain.

All of the rays of the sun tend to be absorbed by most objects. In many cases the absorbed energy is converted into heat. These facts are utilized by gardeners who plant seeds under glass in the early spring. The glass is transparent to the visible and the shorter infra-red waves. The infra-red radiation heats the ground directly, while the visible light is absorbed and converted into heat in the soil. Very long waves of infra-red, radiated by the warm soil, are not transmitted too readily by the glass, and so the glass behaves as an energy trap, and the ground beneath it becomes somewhat warmer than it otherwise would.

8.22. A Study of Electromagnetic Radiations Aids Science

In earlier chapters we have seen that a prism can be used to spread out a beam of light into the various colors that it may contain. Pure white light gives a complete rainbow of colors from violet to red. Light from such sources as the gas filled advertising signs so commonly used today show only certain definite colors of light depending on the kind of gas used. For example, the dominant color from the gas neon is a brilliant red, that from helium, a bright yellow, that from mercury vapor, a mixture of yellow and green.

The careful measurement of the exact wave lengths of light from such sources became one of the chief subjects of

research among physicists more than a generation ago. This particular science is called spectroscopy.

Each element in a gas discharge tube was found to produce a number of colors of light in addition to the obvious dominant color. Also there seemed to be some relation between the various exact colors (wave lengths) found for one element and those found for other elements.

This work was first done in the visible region. Later it was extended to the ultra-violet and to the infra-red which these sources of light emit. Experimental formulae relating these wave lengths were built up and studied.

Physicists had long been speculating on the kind of thing that a single atom of an element must be. No one had ever seen an atom and since an atom is smaller than the wave lengths of light to which our eyes respond, there is no reason to suppose that an atom ever will be seen. So physicists had to use their imaginations to picture atoms that could have all the chemical and physical properties that they are known to have.

One of the more important clues to this puzzle was found by Niels Bohr from studying the experimentally found formulae relating the wave lengths of light for different elements. From the fact that only certain exact colors come from one element, he concluded that only certain definite energy changes could take place in the atoms of a single element. This reasoning led him to develop the idea of electrons going about a nucleus; and so his picture of an atom resembled in miniature form our solar system with planets moving about the sun.

Bohr specified that in any given atom there were only certain orbits in which electrons could travel, and different amounts of energy for the electron were required for each orbit. In an electric discharge tube energy is sometimes picked up by an electron in an atom and the electron then goes about in a new orbit. If at some later time it returns either to its original orbit or to any other one which requires less energy than that to which it was moved, it will have to

give up its excess energy. It is this energy that can be radiated in the form of light.

In this development Bohr used a theory of the German physicist, Max Planck, which says that energy comes in little particles which we call quanta. In all of these cases where light is radiated it appears that the *frequency* of the waves of light produced is proportional to the energy that is available. The proportionality factor is always the same and is called Planck's constant. For the emission of light by an energy change in an atom we can write

$$\text{Energy} = (\text{Planck's constant}) \times (\text{frequency of light})$$

Or in symbols

$$\text{Energy} = hf$$

where h is Planck's constant, and f, the frequency of the light.

From a study of the exact wave lengths of radiations from any element, we can determine the energy changes that must have taken place in the electron system of the atom. So our pictures of atoms have become more definite, and this knowledge about atoms has in turn helped in the discovery of other new facts in both physics and chemistry.

9.22. Wave Theories in Science

As a matter of convenience we usually think of an electron as a little particle of electric charge. Of course the electron is too small to be seen and so our ideas of the details of what an electron is like are guesses based on the way electrons behave. For most experiments, (for example: electron emission in a thermionic tube or a photocell) the particle idea seems quite satisfactory.

However, in 1927, G. P. Thomson, a physicist working in Aberdeen, passed beams of electrons through extremely thin layers of crystals; and Davisson and Germer, physicists of the American Telephone and Telegraph Company, reflected beams of electrons from crystals. The beam in each case was split up by the crystal into several beams which came off at

various angles. The results could not be well explained if electrons are particles, but they could be quite nicely accounted for if electrons may be considered as little bundles of waves in space. These experiments do not enable us to say definitely that electrons are waves; but we must say that under some conditions they act as if they were waves.

This discovery came at a time when great efforts were being made to improve our theories concerning the structure of atoms. The Bohr theory had been useful to scientific development, but it was known to have many defects. It had been developed largely on a type of mechanical ideas that are ordinarily applied to the behavior of particles. Beginning in 1923 attempts were made to tackle the problems of atomic structure by using mathematical methods more commonly employed in wave and vibration problems. This procedure came to be called "wave mechanics." It is more abstract than the older method (gives us fewer pictures of the insides of the atom) but does agree with experiment better than the older theories. It is evident that there is still a great deal to be learned about the structure of atoms.

Some Important Facts

1. Electromagnetic wave motions do not require any transmitting medium.

2. In longitudinal waves, such as sound, the oscillation is in the same direction as the wave motion. In transverse waves, such as light, the oscillation is across the direction of wave motion.

3. Since all wave motions involve the transmission of energy, they have the ability to do work.

4. In general, the emission and detection of electromagnetic radiations involve quite similar processes.

5. A device for detecting radiations must intercept a small amount of the radiant energy and transform it so as to affect one of our senses.

6. Radiant energy from the sun includes not only the visible spectrum but the longer infra-red and shorter ultra-violet. Ultra-violet rays, in moderation, are essential to health.

7. Our knowledge of the nature of electromagnetic radiations is useful to research science as in spectroscopy, theories of atomic structure, etc.

Generalization

One of the more important aspects of the physical world is energy transmission. This may be accomplished by means of either transverse or longitudinal waves.

Problems

Group A

1. Compare electromagnetic waves and sound waves as to

   a. The mediums through which they travel.
   b. The direction of the wave motions with respect to the direction of travel.

2. How could you prove that wave motion is a system for transferring energy from one place to another?
3. Make a list of electromagnetic radiations in the order of their wavelengths.
4. Make a list of the principal effects of the various radiations which you have given in answer to problem No. 3.
5. What would you do to detect the presence of the following radiations

   a. Wireless.
   b. X-rays.
   c. Sound waves.

6. Discuss the usefulness to man of the various radiations that reach the earth from the sun.

Group B

1. What experiment shows whether or not sound waves can exist in a vacuum?
2. What simple proof do you have to show that electromagnetic waves do not need a material medium?
3. Use the list of electromagnetic radiations prepared for problem No. A-3 and make out a table showing in successive columns the method of production, a simple method of detection and a practical use for each type of radiation in the list.
4. What is meant by the spectrum of an element?
5. What did Niels Bohr learn from a study of atomic spectra?
6. What is meant by Planck's constant?

Experimental Problems

1. Using a long coiled spring, demonstrate the mechanical difference between longitudinal and transverse waves.

2. The speed with which a radiometer vane spins depends on the degree of absorption of radiant energy. With this in mind, place the radiometer at the same distance from each of several sources of radiation and compare the relative amounts of energy received from each.

How do these results compare with the behavior of the radiometer in direct sunlight?

NATURAL RADIOACTIVITY AND TRANSMUTATION

The transmutation of one element into another is one of the oldest dreams of both scientist and layman. Although its actual discovery was made only in recent times, it is now known to be one of the properties of nature;—a process that has been going on on our earth since its beginning.

Natural transmutation of elements is accompanied by various types of penetrating radiations, and it was search for the sources of these radiations that led to the discovery of radium and other substances able to transmute themselves into new elements.

The study of radiations was stimulated by the discovery of x-rays, the study of the radiations led to the discovery of radioactive substances which transmute themselves, and a study of these substances and their radiations has contributed greatly to our knowledge of the structure of atoms and particularly the structure of the nuclei of the atoms.

1.23. The Discovery of Radioactivity

The term "modern physics" is often used to include chiefly such facts as can best be explained in terms of the structure of atoms. In comparison to many subjects (for example, mechanics, sound, and the more generally known parts of electricity, light and heat) our information on the structure of atoms dates from relatively recent times.

Most scientists agree that it was the discovery of x-rays in 1895 (see Chapter 14) that started off the series of experiments which have put physics and chemistry in possession of their present knowledge about atoms and their behavior. The direct study of x-rays eventually contributed directly to this subject; but perhaps the most vital effect was the immediate interest that was aroused in another group of experiments.

The really startling thing about the x-rays was that they were a type of radiation that could pass right through objects (such as cardboard and black paper) which were opaque to

ordinary light. And, although these x-rays were invisible to the human eye, they could affect a photographic plate similarly to ordinary light.

At once a search started for other sources of radiation that might have similar properties to those of the x-rays. The nature of fluorescence and phosphorence was not well understood, and so these phenomena seemed favorable for examination. Fluorescent objects emit light when they are illuminated; phosphorescent bodies are similar, but they continue to emit light for some time after the external source of illumination has been removed. It seemed possible that

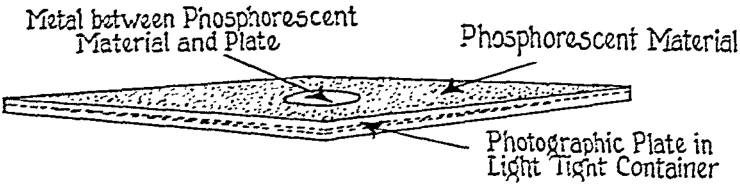


FIG. 267.—Phosphorescent material is tested for more penetrating radiation by placing it on the cover of a box containing a photographic plate. If such radiations exist the plate will appear light struck after it is developed except for the spot protected by the piece of metal.

a part of the light from such substances might be able to pass through cardboard or be otherwise similar to x-rays.

One of the physicists who undertook researches of this type was Professor Henri Becquerel of Paris. He was well qualified for this work, for he had experimented with phosphorescent materials many years earlier. In 1881 he demonstrated that a certain uranium compound (the double sulfate of uranium and potassium) would fluoresce when illuminated with invisible ultra-violet light and that it would phosphoresce after the ultra-violet light had been removed.

He now exposed some of this salt to light and then placed it near a photographic plate protected from light by a container. After a few hours the plate was developed and it gave the appearance of being light struck. (See Figures 267 and 268.)

Becquerel then repeated the experiment without first exposing the uranium salt to light. In this case there was no visible phosphorescence, but the effect on the photographic plate was the same as before. This result led Becquerel to believe that there was something unusual about the uranium salt that gave it this special radiation property which was similar to that of x-rays. Further experiments with other salts showed that it was the uranium in the compound that

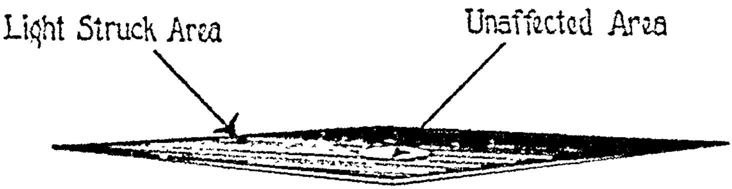


FIG. 268.—The plate of Figure 267 after development.

was responsible for this property of the radiation of penetrating rays.

2.23. The Discovery of Radium

Becquerel published the results of his experiments in 1896, and they attracted the attention of many physicists including Marie Curie. Marie Curie was the wife of Pierre Curie, a professor of physics in the School of Physics and Chemistry of the city of Paris. Marie Curie was also trained as a physicist and was working with her husband in the research laboratories of this school.

In 1897 she began an examination of the known elements with a view to discovering if any others had the ability to produce penetrating radiations such as had been found for uranium.

Becquerel had shown that not only would these rays affect photographic plates, but they would also cause a gold leaf electroscope to lose its charge. Madame Curie chose the electrical method of examination and used a special type of electroscope that had previously been developed by her husband and his brother, Jacques.

Only one other element (thorium) was found at this time to behave in a manner similar to uranium.

On the other hand, a number of mineral ores which Marie Curie examined seemed to have the same property as uranium with regard to emitting penetrating radiations. Of these, pitchblende, an ore from which uranium was obtained, was the most active. In fact, it showed an ability to produce penetrating radiations out of all proportion to what might have been expected from a knowledge of the amount of uranium known to be in the ore.

This discovery led Madame Curie to begin a long and careful chemical analysis of the ore to discover, if possible, to what the additional activity might be due. Finally she isolated two substances which had many million times the radioactivity of uranium. One of these she called polonium after her native land, Poland, and the other was named radium. These discoveries were announced in the year 1898.

The Nobel prize in physics was awarded jointly to Henri Becquerel and Pierre and Marie Curie in 1903 for the discovery of radioactivity, and it was awarded in 1911 to Marie Curie primarily for the discovery of radium.

3.23. The Nature of the Radiations

Not all of the early experiments were made in search of new radioactive substances. Some of them were attempts to determine the nature of the radiations themselves.

In one method of examination, a small piece of radioactive material is placed as shown in Figure 269 so that only rays which pass through the hole, *S*, in the lead plate are under observation. When an intense electric field is applied by means of the plates, *P*, the beam of radiation is broken up into three parts as shown, and three spots are recorded on the photographic plate, *A*.

That part of the beam that is bent towards the negative plate must be positive electrical particles, the part bent towards the positive plate must be negative electrical particles, and the undeviated beam in the original position is now known to be electromagnetic rays.

Further experiment showed that the positive particles are the nuclei of helium atoms; that is, helium atoms that have lost their orbital electrons. They have been named *alpha* particles.

The negative particles are simply high speed electrons. They are called *beta* particles.

The electromagnetic rays are waves that are similar to, but shorter than, x-rays, and are called *gamma* rays.

The alpha particles are very efficient at knocking electrons off any molecules near which they pass. So air is ionized

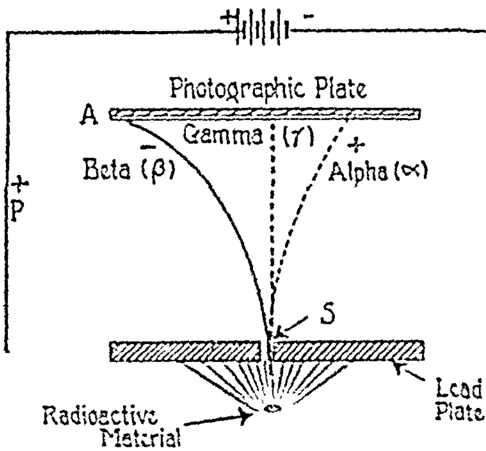


FIG. 269.—Rays from the radioactive material of the radium family can be separated into three kinds by means of an electric field.

extensively in the vicinity of alpha radiation. The kinetic energy of the alpha particle is used up in producing this ionization and even a high speed alpha particle will be stopped in about seven centimeters of air.

Beta particles also ionize many of the molecules near which they pass; but they do not produce so many ions as the alpha particles for equal lengths of path.

Gamma rays produce some ionization in air, but less per centimeter than even the beta particles. On the other hand, they have great penetrating power and can be detected even after passing through a considerable thickness of a heavy substance like lead.

4.23. Observing the Paths of the Rays

The paths of all of these rays can be seen with the aid of a device called a Wilson cloud chamber. A simple form is shown in Figure 270. Water vapor fills the space above the water level in the chamber and a small piece of radioactive material is placed in the side of the chamber.

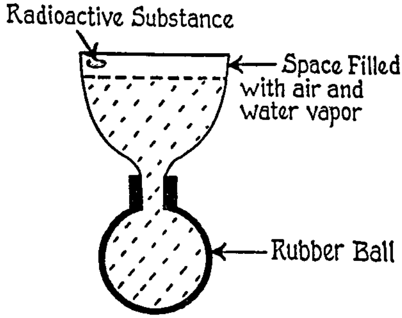


FIG. 270.—A cloud chamber for making visible the paths of alpha and beta particles and gamma rays.

The water level is raised by squeezing the rubber bulb at the bottom of the apparatus. After a moment this bulb is released, the water falls in the chamber, and the air and water vapor are cooled by the sudden expansion.

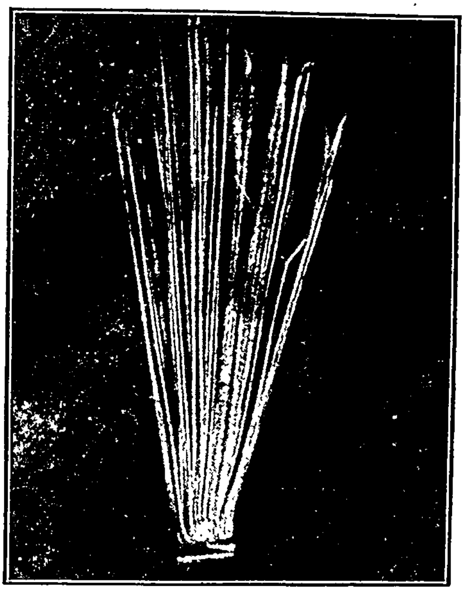


FIG. 271.—The paths of alpha particles are outlined by water vapor condensation on the ions created as the alpha particles pass through the air. (*From Foley's "College Physics," Courtesy Professor W. D. Harkins of Chicago University.*)

If at this time rays of any kind come from the radioactive material and produce a path of ions, water vapor will condense on these ions and so make the paths of the rays visible.

More elaborate forms of this device have been used for taking pictures of the paths of alpha, beta, and gamma rays. Figure 271 shows a photograph of the paths of alpha particles from a small quantity of radioactive material. The water droplets are so close as to give an almost continuous line in the photograph. Figure 272 is a similarly taken photograph of

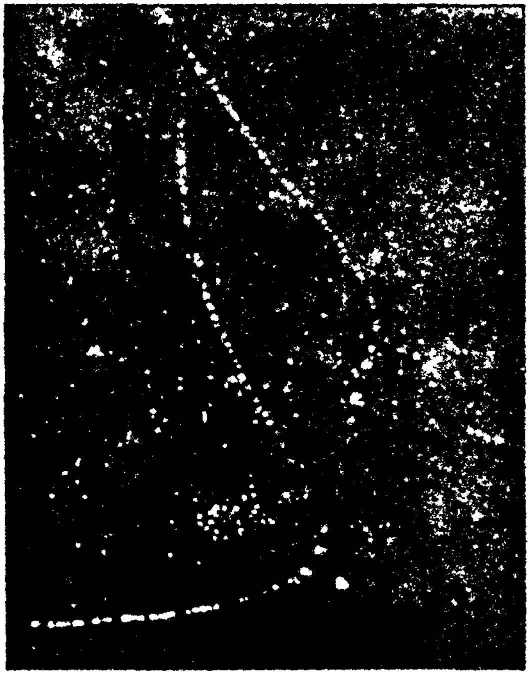


FIG. 272.—A cloud chamber view of a beta ray track. (*Stranathan.*)

beta particle tracks. Water droplets are relatively far apart here indicating that many fewer molecules have been ionized.

If either a magnetic or an electric field is placed across the vapor space in this device, the paths of the alpha and beta particles will be bent, and it is possible to measure the speeds of the particles from such data. (See Figure 273.) The paths of gamma rays are not changed either by electric or magnetic fields.

Another device for making an effect of alpha is called the spinthariscope. It is a simple and

piece of apparatus consisting of an eye glass, A, a bit of radium bromide at B, and a zinc sulfide phosphorescent screen, C, as shown in Figure 274. When an alpha particle from B strikes the screen, the latter phosphoresces with sufficient brightness to be visible to one's eyes if they have first been accustomed to darkness. If the amount of radium bromide is sufficiently small, the individual effects of alpha particles are easily

FIG. 273.—The track of a beta particle deflected by a magnetic field. (*From Rasetti's "Elements of Nuclear Physics," Courtesy of Prentice-Hall, Inc.*)

observed and may be counted. For larger amounts of radium bromide, the screen appears to be continuously luminous.

5.23. Uses for the Radiations

We have already seen that radiations from radioactive materials affect photographic plates and that they knock electrons from atoms and so produce ions. Both of these effects are used in detecting and studying the rays. Useful data on radioactive substances can also be obtained with equipment employing the principle of the spinthariscope.

We have learned that many substances fluoresce or phosphoresce when illuminated with various forms of electromagnetic radiations, such, for example, as visible light, ultra-violet light and x-rays. Gamma rays can be used in the same manner to produce fluorescence or phosphorence. Phosphorescent paints (such as are used on luminous watch and clock dials) may be made by mixing minute quantities of radioactive material with a phosphorescent substance and white paint.

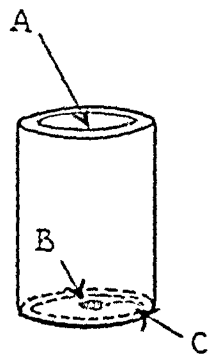


FIG. 271.—A spinthariscope.

Perhaps the most important practical use of radiations from radioactive materials is in the treatment of disease. Neither the alpha nor the beta radiations are used, but only the gamma rays. These rays are, as we have seen, like short x-rays; and they are used in much the same manner as x-rays for the treatment of cancer in its various forms. The advantage of using gamma rays in place of x-rays lies in the fact that the gamma rays are easier to apply. The radioactive material can be arranged in a plaque made to fit the place on the patient which is to be treated, or a capsule containing the material may be inserted directly into the tissue which is to be irradiated.

The alpha and beta particles would not penetrate the tissue deeply enough to have any beneficial effect on the tissue, but would be stopped by the skin. The energy in these rays would be great enough, however, to produce bad local burns. So radioactive materials are shielded from the patient by placing them in metal containers. Even a thin wall of metal will stop the alpha and beta particles, but will not greatly hinder the passage of the gamma rays. An appreciable thickness of a heavy material such as lead is needed to stop the gamma radiation.

6.23. Sources of the Radiations

Up to this point we have been concerned with the discovery of radioactivity and with the nature of the radiations.

Another item of great interest to scientists is the source of the radiations. The early work of Madame Curie showed that radiations were obtained from the elements uranium, thorium, radium and polonium.

Numerous additional experiments were performed to find out what effects or changes took place in the substances when the radiations occurred. Other experiments were carried out to learn whether the amount or type of radiations could be influenced by heat or other physical means; and still other experiments were used to study the effects of chemical combinations on the radiations.

These data showed that the rates and types of radiation were uninfluenced by physical or chemical changes; and the conclusion was drawn that radioactivity is a property of the individual atoms of the radioactive elements, and in particular it is a property of the nuclei of these atoms and not one in which the orbital electrons are concerned.

All of this work on radioactivity indicated that any real understanding of the phenomena hinged on a better knowledge of the structure of atoms themselves. The science of atomic structure had, up to this time, drawn most of its data from spectroscopic experiments, from chemical properties of the elements, and from a knowledge of atomic weights and atomic numbers. Data from the new field of radioactivity was now included in the theories of the structure of atoms and so the new experimental work on radioactivity and the developing of theories of atomic structure became mutually helpful.

7.23. Transmutation of Elements—Radium Family

The present well established belief is that all of the radiations originate in the nuclei of the atoms. For example, a radium atom is believed to have in its nucleus 88 protons and 138 neutrons. ($88 + 138 = 226$, which is the approximate atomic weight of radium.) There are also 88 orbital electrons associated with the normal radium atom. These electrons are not shown in Figure 275 which represents only nuclei of an atom of radium and its products.

When an alpha particle consisting of the nucleus of a helium atom (2 protons and 2 neutrons) is ejected by the radium nucleus, only 86 protons and 136 neutrons are left. So the atom that was once a radium atom now has an atomic number of 86 instead of 88 and an atomic weight of 222 (86 + 136) instead of 226. Since it has a different atomic number it can no longer be an atom of radium and must be an atom of some different element. Experiment agrees with this prediction and shows that the new element is, at ordinary pressures and temperatures, a gas, whereas radium was a

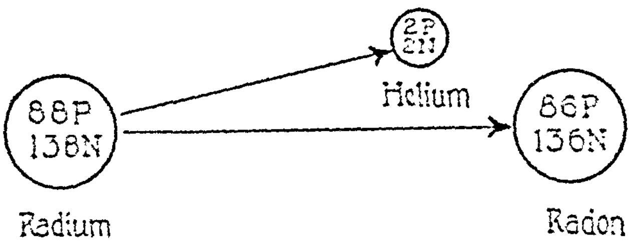


FIG. 275.—The transmutation of an atom of radium into a helium nucleus and an atom of radon.

metallic solid under the same conditions. The name of the new element is radon (formerly called radium emanation).

Here then we have a case of the transmutation of one element into another by natural processes. So nature is discovered performing the trick for which alchemists sought for many ages.

If one has a known number of radium atoms and can count the rate at which alpha particles are given off, he can compute the time at which any given fraction of the radium will remain; or he can compute the probability of any one atom being transmuted in the next second; or he can estimate the probable length of life of any one radium atom. These calculations are not radically different from those employed in census and population statistics with which you may be familiar.

In terms of human life, the life of radium atoms is long. It requires 1,550 years for half of any given quantity of radium to be transmuted.

In the change studied above we found only alpha radiation—no beta particles and no gamma rays. Further study, however, shows that radon atoms are even less stable than radium atoms, and that they also eject alpha particles. Figure 276 shows an atom of radon disintegrating into an alpha particle and a new atom called radium *A*. The latter has 84 protons in the nucleus as compared to 86 for radon atoms. Radium *A* is a white solid substance under ordinary conditions of temperature and pressure. The half period life of any given quantity of radon is only 3.82 days as compared to 1,550 years for radium.

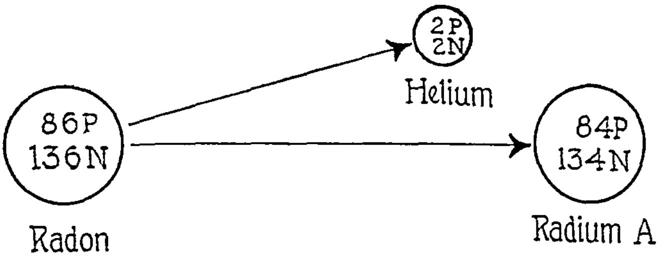


FIG. 276.—The transmutation of an atom of radon into a helium nucleus and an atom of radium A.

A radium *A* atom, like its immediate ancestors, emits an alpha particle and thereby transmutes itself into Radium *B* with an atomic number of 82. (82 protons in the nucleus.)

Radium *B* atoms disintegrate with the emission of a high speed electron which we have called a beta particle. At the same time, gamma rays are emitted. Not many years ago this effect was easily explained by assuming that the electron was emitted from the nucleus and the gamma ray was the result of energy changes in the nucleus. Now, since we believe that the nuclei of atoms contain only protons and neutrons, the explanation is not simple.

A probable explanation is that one neutron in the nucleus of a radium *B* atom changes itself into a proton and an electron. (See Figure 277.) It is this newly created electron which is emitted as the beta particle. The atomic weight of the atom is changed very little by the loss of the electron,

but the number of protons in the nucleus is increased from 82 to 83. Atomic number 83 corresponds to a new element called Radium *C*.

Radium *C* atoms seem to have a choice as to whether they will lose an alpha particle first and then a beta particle, or

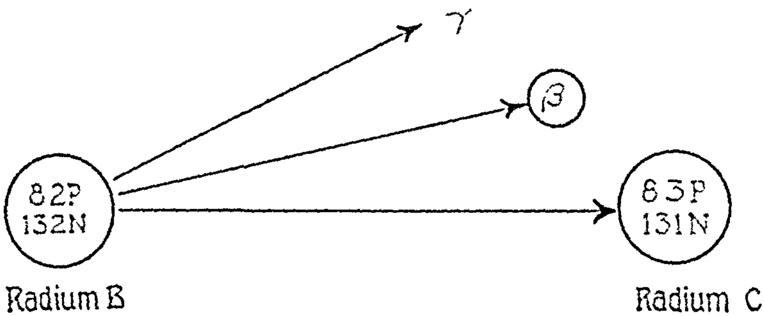


FIG. 277.—The transmutation of an atom of radium *B* into one of radium *C*. The transmutation is accompanied by the emission of a beta particle and gamma rays.

whether they will first lose a beta particle and then an alpha particle. In either case, after the double disintegration, Radium *C* atoms become Radium *D* atoms. The latter element is also called polonium and was discovered by Madame Curie during the same series of experiments in which she discovered radium.

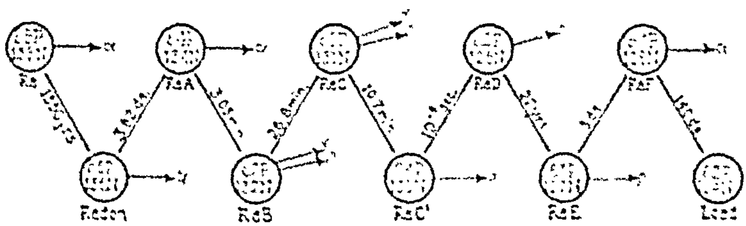


FIG. 278.—Nuclear changes in the radioactive series from radium to lead.

Radium *D* disintegrates into Radium *E* and Radium *E* changes into Radium *F*. Radium *F* also disintegrates and transmutes itself into an atom of lead having an atomic number of 82 and an atomic weight of approximately 206 corresponding to a nucleus with 82 protons and 124 neutrons.

This atom is an isotope of lead. The more common form of lead atoms contains 82 protons and 125 neutrons in each nucleus. The approximate atomic weight is then 207 as compared with the 206 for lead atoms of radium origin.

Figure 278 shows the whole series of transmutations described above. The nuclei of the various elements, their emissions and their half period lives are indicated.

8.23. Radioactive Families

Radium itself comes by a series of several transmutations from the element uranium. It is impossible to keep any one element in a radioactive series in pure form, because of the transmutation into the various products that constantly takes place. So, for example, if one starts with a pure sample of uranium, all of the elements in the chain running all the way to lead gradually form. So all of the radiations, alpha, beta, and gamma, are obtained even though the parent element with which one begins an investigation may emit only alpha rays or only beta and gamma rays.

The element thorium heads another family of radioactive substances which are somewhat similar to the uranium family. The end product of all the various transmutations is also lead, but lead from thorium has 82 protons and 126 neutrons in each atomic nucleus, and so the atomic weight is approximately 208.

Still another radioactive family, the actinium group, is now well known. The end product is again lead, but this time the nucleus contains 82 protons and 125 neutrons, thus giving an atomic weight of approximately 207.

One of the chief interests of modern physics is the structure of the nuclei of atoms, since it is the nucleus that determines what element an atom may be. The first insight into this fascinating problem was gained from the studies outlined above on these transmutations that are to be found in nature.

Some Important Facts

1. The discovery of radioactivity near the end of the 19th century started the modern era of the study of atoms.

2. In searching for radioactive elements, the Curies discovered Polonium and Radium.

3. Radioactive elements, even when chemically combined with non-radioactive elements, emit three types of radiations: (1) positive alpha particles, which are Helium nuclei; (2) negative beta particles, which are high speed electrons; and (3) gamma radiations, which are electromagnetic waves of a frequency even higher than x-rays. These three emanations may be separated from each other by fairly strong magnetic or electrostatic fields.

4. Radioactive emanations are of considerable use in scientific research, due largely to their effect on photographic plates, their ionizing effects, and their ability to produce fluorescence and phosphorescence.

Gamma rays have much the same effect as x-rays in the treatment of cancer and are more penetrating and easier to apply.

5. Since radioactive elements such as Uranium, Radium, Thorium and Polonium emit their radiations quite uninfluenced by any known physical or chemical changes, radioactivity apparently originates in the nuclei of these elements. This conclusion has led to greatly increased interest in, and knowledge of, the structure and nature of atoms.

6. When alpha particles or electrons or both are emitted from the nucleus of an atom of a radioactive element, the atomic number of the element is changed—that is, the radioactive element transmutes itself into another element.

Generalization

Many elements of high atomic number have complex and apparently unstable nuclei, and undergo radioactive disintegration, that is, natural transmutation.

Problems

Group A

1. What is meant by fluorescence and phosphorescence?
2. Describe Becquerel's experiment with a phosphorescent substance and a photographic plate. What did he hope to find?
3. To what substance was the radiation discovered by Becquerel due?
4. What effect did Becquerel discover for radioactivity in addition to the action on a photographic plate?
5. For what did Marie Curie look in her first research in radioactivity?
6. Describe each of the three kinds of radiation found in radioactive substances.
7. Do the three kinds of radiation come from the same identical element? Explain the presence of the three as ordinarily found.
8. How are gamma rays used for the treatment of disease?

9. Why are beta and alpha radiations not used for the treatment of disease and how are they stopped from reaching the patient?

10. What three families of radioactive substances are well known?

Group B

1. Describe the experimental processes by which Marie Curie discovered radium.

2. How can radioactive radiations be shown to consist of three types of rays?

3. How does a Wilson cloud chamber operate, and what can be learned by using one?

4. Describe the operation of a spinthariscope.

5. Describe the changes by which a radium atom is transmuted into an alpha particle and a radon atom.

6. Describe the probable changes by which a radium *B* atom emits a beta particle and is transmuted into a radium *C* atom.

7. What is meant by the half period of a radioactive substance?

Experimental Problems

1. Examine the scintillations that may be seen in an inexpensive spinthariscope.

2. Charge a gold leaf electroscope and note the rate at which the charge escapes by watching the gold leaf fall. Now bring near any feebly radioactive source and observe the increased rate of fall. To what is the effect directly due? Try luminescent material from a watch or clock face, or obtain a small amount of any uranium compound for your radioactive source.

3. Using the same radioactive materials as employed in experiment 2, repeat the original Becquerel experiment.

TRANSMUTING THE ELEMENTS

Transmutation of elements can be produced in laboratories by using the nuclei of small atoms as bullets with which to shoot the nuclei of larger atoms.

Nature provides some high speed bullets,—the most useful being alpha particles from radioactive materials. Other projectiles can be obtained in the laboratory. For example, hydrogen and helium ions can be speeded up in electrical devices so that they also may be used for transmutation work.

Many of the transmutations produce elements in forms not found in nature. These are often unstable and transmute themselves still further just as do natural radioactive atoms.

Chain reacting transmutations can release energy abruptly for use in atomic bombs or in a controlled manner for sources of industrial power.

1.24. Bullets for Transmutation Experiments

In the preceding chapter we learned that the atoms of some elements show a tendency to blow up, and that after an atomic explosion has taken place, the fragments left are the nuclei of atoms of other elements. So nature confirms our theory that it is the nucleus of an atom that determines what element it is; and the way to transmute one element into another is to make changes in the nucleus of each atom.

Ordinary experiments in physics and chemistry are usually concerned with large numbers of atoms grouped together to form a quantity of material. Or, where the behavior of individual atoms is concerned, we are usually interested in the whole atom; that is, its external electrons as well as its nucleus. In nearly all of these cases we can influence the behavior of the things with which we are experimenting by controlling temperature, or pressure, or chemical combination. But in natural radioactivity, which we studied in the last chapter, we learned that none of these changes had any effect. In other words, ordinary physical and chemical changes do not react on the nuclei of atoms.

Finally it occurred to a number of scientists that the way to attack the nucleus of an atom was to shoot at it with high speed bullets having about the same size as the nucleus. Sir Ernest Rutherford (later Lord Rutherford) thought that an easy way to get high speed particles for this shooting would be to use the alpha particles from radium and its products.

2.24. The First Controlled Transmutations

This scheme was followed with some success by Rutherford in 1919. He arranged his radium source so that alpha particles passed into the gas nitrogen. (See Figure 279.) Of course he could not aim the alpha particles, but simply had to

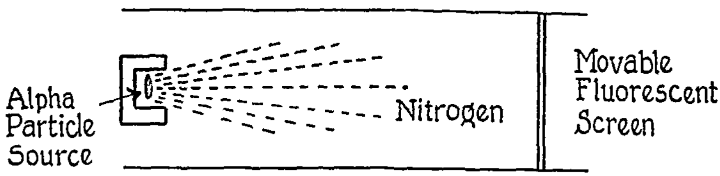


FIG. 279.—The first controlled transmutation performed by man. Alpha particles were used to bombard nitrogen atoms. When a direct hit was made, an oxygen atom and a high speed hydrogen nucleus were produced. The latter could be detected as it struck the fluorescent screen.

depend on the possibility that now and then an alpha particle might travel right through the outer electron region of a nitrogen atom and make a direct hit on the nucleus.

Of course, the kinetic energy of alpha particles is usually used up in removing electrons (ionizing) from the molecules near which they pass, as we have seen in the last chapter. It was reasonable to expect only a few direct hits on nitrogen nuclei in the above experiment and so the question as to how one could tell when a direct hit was made and what result it would have on the nitrogen atom was a serious one. Even if the nitrogen were transmuted into some other element, it was not reasonable to expect to get enough of the new element to detect it by any simple chemical or physical analysis.

Two methods were actually used. One was based on the principle of the spinthariscope which was described in the last chapter. A screen which will fluoresce when struck by alpha

particles can be used to tell how far alpha particles travel in any given gas. The screen is slowly moved away from the radioactive material, and it is fairly easy to determine the distance beyond which no more scintillations occur.

In the case of Rutherford's experiment, it was found that occasionally a scintillation occurred at much greater distances than the known maximum range for alpha particles in nitro-

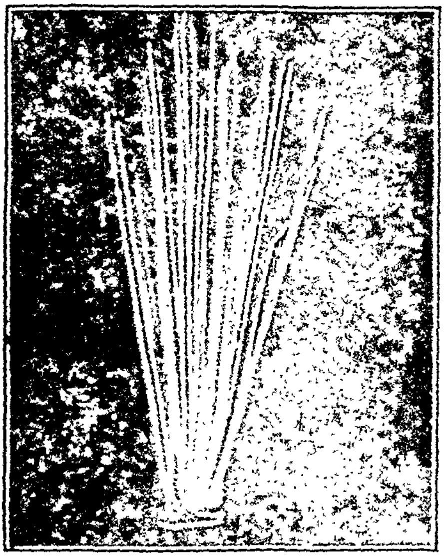


Fig. 280.—Wilson cloud chamber photograph showing the transmutation of a nitrogen atom into oxygen and hydrogen. The radiating lines are the tracks of alpha particles. The one of chief interest is the one in the right hand part of the photograph which breaks into two forked tracks. At the fork the alpha particle has made a direct hit on a nitrogen nucleus. The short heavy track to the right is made by the newly formed heavy oxygen atom while the long light track to the left is made by the newly formed hydrogen nucleus. *(From Foley's "College Physics," Courtesy Professor W. D. Harkins of Chicago University.)*

gen. This discovery led to the belief that some particle might be shot off by a changing nitrogen nucleus. The distance travelled by the particle indicated that it had great velocity, but that it had less mass than the alpha particle. We now believe this high speed particle to be a hydrogen nucleus—that is, one proton.

Rutherford also repeated this experiment with a Wilson cloud chamber and obtained results such as are shown in

Figure 280. Here we see that the path of an alpha particle seems to end abruptly at a fork. From this point a light track goes off in one direction and a short heavy track in another direction. We now believe that the light track is that of the hydrogen nucleus and the heavy track that of the residue of the nitrogen nucleus and the helium nucleus (alpha particle). This combination we believe to be the nucleus of an atom of oxygen.

Figure 281 shows the nuclear changes that seem to have taken place in Rutherford's experiment. The chemical symbols are given beneath the diagrams of the nuclei. The subscript before the symbol gives the atomic number and

$${}_{2}He^{4} + {}_{7}N^{14} = {}_{8}O^{17} + {}_{1}H^{1}$$

FIG. 281.—The nuclear changes involved in transmuting a nitrogen atom into an oxygen atom. The changes are indicated pictorially at the top and in symbols directly below.

the exponent at the right of each symbol gives the atomic weight approximately. We might also say that the subscript before the symbol gives the number of protons in the nucleus and so specifies what element we have. The exponent at the right gives the total number of protons and neutrons in the nucleus.

To make the equations in Figure 281 complete we should include on the left hand side the kinetic energy of the high speed alpha particle (helium nucleus), and on the right, the kinetic energy in the motion of the oxygen and the hydrogen nuclei.

A comparison with a table of atomic weights and atomic numbers will show that the oxygen formed in this transmutation is heavier than ordinary oxygen, for it has an atomic weight of 17 whereas the common form of oxygen has an atomic weight of only 16. (The normal oxygen nucleus con-

tains 8 protons and 8 neutrons, while the oxygen formed here has 8 protons and 9 neutrons.) So the oxygen formed in Rutherford's experiment is a heavy isotope of oxygen. For most physical and chemical purposes, of course, an isotope of an element has all the same properties as the more common form of the element.

This experiment, performed in 1919, is the first recorded case of a transmutation occurring under the control of man. Of course it had no commercial importance and would not have satisfied the dreams of the alchemists who wished to convert lead into gold. But it was a great scientific achievement, for it showed that man had discovered the secret of the differences between atoms of different elements, and that he could use this knowledge to change one element into another.

Rutherford's experiment spurred other workers in his own laboratory and in other laboratories to look for more transmutations and in about a ten year period transmutation of 13 of the lighter elements had been reported. These elements were boron, nitrogen, fluorine, neon, sodium, magnesium, aluminum, silicon, phosphorus, sulfur, chlorine, argon, and potassium. The amount of material transmuted was exceedingly small, and the experiments were important only from the point of view of giving more detailed information along the same lines as that which was obtained from Rutherford's first experiment.

3.24. Discovery of the Neutron

However, in 1930, a startling discovery was made. When transmutation was tried by bombarding the metal beryllium with high speed alpha particles, radiation was observed which did not seem to behave like any known type. When the materials were surrounded by zinc and brass of several millimeters thickness (sufficient to stop any known alpha and beta rays and high speed protons) radiation still came through, although the intensity did decrease a little showing that the metal had some stopping power, but not a great amount.

On the other hand, when the beryllium and high speed alpha source were surrounded by a wall of paraffin, electrical detectors indicated an increase in radiation. A similar result was noticed with water as well and also with cellophane.

However, when the beryllium and alpha particle source was first surrounded by any one of the above three and then by an outer wall of metal, the radiation seemed to be quite completely stopped.

The odd results of this series of experiments were first published in 1930, but it was not until 1932 that the riddle was

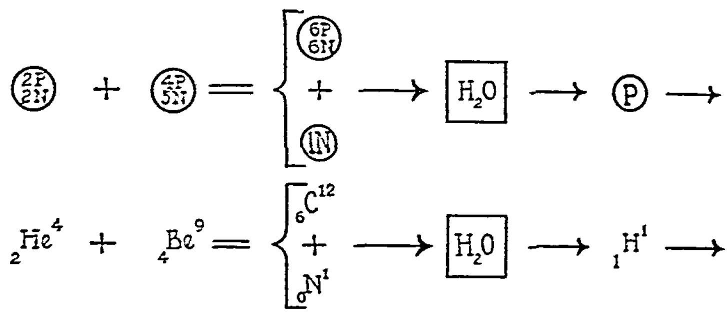


FIG. 282.—The production of neutrons. When an alpha particle strikes beryllium, carbon is produced and a high speed neutron is driven out. The neutron can knock a hydrogen nucleus out of a water molecule and a hydrogen nucleus is easily detected.

solved. In that year, Chadwick, an English physicist, showed that beryllium, when struck with alpha particles, throws out little particles that have about the same weight as protons but do not have any electric charge. They have been named neutrons. This was a striking discovery, because it upset a well established belief that atoms are made up entirely of protons and electrons.

Since neutrons have no electric charge, they do not ionize atoms near which they pass very easily, and so it is more difficult to make observations on them than is the case with alpha and beta particles and high speed protons.

The reason for the apparent increase in radiation when paraffin, water, or cellophane was placed around the source of neutrons seems to be that neutrons strike hydrogen nuclei and

drive them away with great speed but without changing them otherwise. This action is indicated in Figure 282 where we see that a helium nucleus (alpha particle) strikes beryllium, producing a nucleus of carbon and a neutron. When the neutron passes through water molecules it may dislodge a hydrogen nucleus (one proton) and drive it out at great speed. Such a high speed proton is easily detected in a Wilson cloud chamber or any of the electrical devices ordinarily used for detecting beta and alpha radiation.

The above process reminds one of some famous cartoons where very roundabout methods are used for accomplishing some simple thing. First an alpha particle is used as a bullet

$$ {}_{0}N^{1} + {}_{7}N^{14} = {}_{5}B^{11} + {}_{2}He^{4} $$

FIG. 283.—A neutron is a good bullet with which to perform transmutations. Here a nitrogen atom is changing into the elements boron and helium after being bombarded with a neutron.

to shoot a beryllium nucleus. A neutron comes out at high speed and hits a hydrogen nucleus. The latter ionizes some atoms through which it passes. If these atoms are in a Wilson cloud chamber, water molecules condense around them, and we finally observe a fog track. It requires a good bit of experimenting and cross checking to become convinced that somewhere between the original alpha particle and final fog track, there was a particle with mass like a proton but with no electric charge. But Chadwick was able to prove the truth of this explanation to the satisfaction of the scientific world.

The neutron turns out to be a useful bullet with which to perform other transmutations even though it has no transmuting effect on the simple nucleus of the hydrogen atom. The transmutation of nitrogen into boron and helium by shooting the nitrogen with a neutron is illustrated in Figure 283.

4.24. Discovery of the Positron—Cosmic Rays

The year 1932 also saw another unusual discovery and one of a type similar to the discovery of the neutron. A new particle, called the positron, was discovered by Anderson at California Institute of Technology. It has very low mass, like the electron, but it has a positive charge of electricity like a proton. It might very well be called a positive electron.

The neutron was discovered as a result of work on the transmutation of elements, but the discovery of the positron came out of work on cosmic rays.

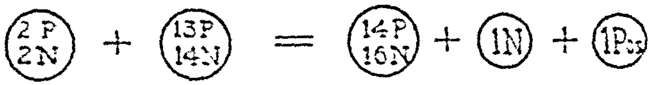
For many years it had been suspected that some kind of radiation enters the earth's atmosphere from outer space. One reason for this belief was that it is impossible to keep a gas, such as air, from having a small percentage of its atoms always ionized. Therefore we conclude that some ionizing radiation must constantly be present. This effect was studied by a number of scientists in this country and abroad, and finally Millikan, the famous physicist of California Institute of Technology, in a long series of brilliantly carried out experiments—data taken hundreds of feet beneath the surfaces of lakes, data taken in airplanes, data taken by recording meters in free balloons—demonstrated that some kind of ionizing radiation actually does enter the earth's atmosphere from the great open places of space.

This work was first published in the early 1920's and work by Millikan's laboratory and by a number of other laboratories has continued ever since in an endeavor to learn more about the nature of this radiation and the place from which it comes. Something of a controversy has existed; for Millikan believes that the radiation, before it reaches the earth's atmosphere, is a wave radiation like ultra-ultra penetrating x-rays or gamma rays. Many of the other experimenters think the data proves the cosmic rays to consist of very high speed particles instead of waves.

Regardless of what the true nature of these cosmic radiations may be, it is quite well established that sometimes, when a cosmic ray makes a direct hit on the nucleus of an

atom, a particle like an electron, but with a positive charge, is given off at very high speed. This particle is the positron.

Since positrons were discovered by Anderson in 1932, other experimenters have been able to produce them by using the gamma radiation from natural radioactive substances such as radium *B* and radium *C*. Positrons have also been produced by letting very high speed alpha particles strike some elements, such as beryllium and aluminum. The probable action with aluminum is illustrated in Figure 284. (Later, see Figure 288, we shall see that some steps in the



$$ {}_{2}He^{4} + {}_{13}Al^{27} = {}_{14}Si^{30} + {}_{0}N^{1} + {}_{1}Pos^{0} $$

FIG. 284.—The production of positrons when aluminum is bombarded with alpha particles.

process have been omitted in Figure 284.) Here an alpha particle strikes an aluminum nucleus, transmuting it into the nucleus of an atom of silicon and emitting both a neutron and a positron.

5.24. The Restlessness of Nature

Hence we see that positrons are involved in some transmutations. In this particular case, when the helium nucleus enters the aluminum nucleus, it appears that one proton must be thrown out. But this proton promptly converts itself into a neutron and a positron.

This action suggests a certain restlessness on the part of nature. It is as if a proton, when it gets tired of being a proton, can say to itself, "It is time that I split my personality. Hereafter I will be a neutron and a positron." (See Figure 285.)

There are other times when it appears that a neutron becomes tired of being a neutron and changes itself into a

proton and a negative electron. This is what we believe happens when a beta particle (high speed electron) is emitted in natural radioactivity as we learned in the preceding chapter.

A few years ago we thought we knew a great deal about the nature of the elements and the parts of which they are made. Now we have learned enough about them to know that there is still much more to learn.

P = N + Pos

N = P + -e

FIG. 285.—A proton may change into a neutron and a positron; and a neutron may change into a proton and an electron.

6.24. More Bullets for Transmutations—Heavy Hydrogen

So far we have considered transmutations resulting from bombardment with alpha particles or neutrons. The success of these experiments suggests that other light nuclei, for example the nucleus of a hydrogen atom, could be used if we had any method for getting them up to sufficiently high speeds. The nucleus of a simple hydrogen atom consists, as we know, simply of a proton. A heavy isotope of hydrogen, consisting of a proton and a neutron, also exists in nature, although its percentage is small in comparison to the simple hydrogen.

The heavy isotope of hydrogen has an atomic weight approximately twice that of simple hydrogen. It was discovered by Urey, a chemist of Columbia University, in 1932—this being the same year that saw the discoveries of the neutron and the positron. The differences between the heavy isotope of hydrogen and the normal atoms of the element are relatively greater than in the cases of the isotopes of elements with higher atomic weights. Heavy hydrogen has been named deuterium, and the nucleus is called a deuteron. This is the only isotope that has been given a name differing from that of the more common form of the element.

Since it is easy to get simple hydrogen, deuterium and helium in an ionized condition, any of these gases could have their ions brought up to a high speed by placing them in a

tube and applying sufficient electric potential differences between the electrodes. Such an arrangement is shown in simple form in Figure 286. A practical difficulty is due to the fact that electric potentials of the order of several million volts are needed to bring these ions up to speeds that compare with those of natural alpha particles.

Some attempts have been made to develop electric generators of several million volts, and one of these (the Van

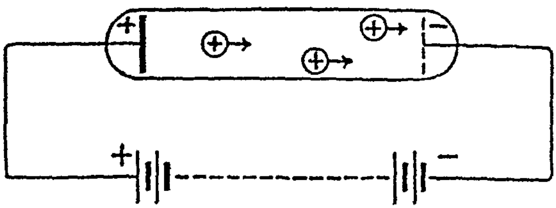


FIG. 286.—Positive ions may be given a high velocity as they travel through a large electrical potential difference.

de Graaf generator) was described on page 388 as an illustration of a modern electrostatic machine.

7.24. The Cyclotron

A different sort of arrangement has been built up on the idea that if a relatively small voltage (say 20,000 volts) could be applied to the same ions a number of times, they could be brought up to the necessary high speeds. The most successful device working on this principle has been named the cyclotron. It was described in a paper published in 1932 by Lawrence of the University of California and was put into immediate use to obtain high speed protons for transmutation experiments.

Figure 287 shows a photograph of a cyclotron. The most obvious part of the apparatus is an extremely large electromagnet. Between the poles, and insulated from them, is a flat metal box cut into two sections. These two sections are connected to a high frequency electrical generator of a type similar to that used in radio stations. Ions in the box go around in circles due to the action of the magnetic field on any ion that moves in it. Every time an ion passes from one half of the

box to the other it receives an impulse due to the voltage difference between the two halves of the box.

As the ions speed up they go round in paths of larger radius and finally they escape at a slot in the edge of the box.

The development of the cyclotron has speeded up the work of experimenters in transmutation; for it provides a large source of high speed particles in comparison with what most

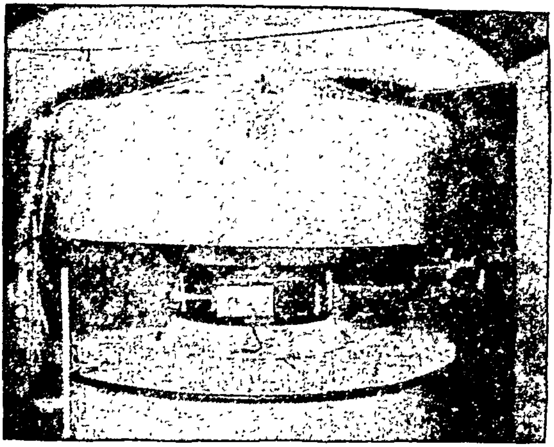


FIG. 287.—A cyclotron. (*Stranathan.*)

laboratories would be able to obtain from the amount of radium that they might be able to purchase.

8.24. Synthetic Radioactivity

One more discovery of great importance is to be described here. In 1934, F. Joliot and Madame Joliot (Irene Curie, daughter of the famous Madame Curie) discovered that when they attempted to transmute some elements, such as aluminum, boron, and magnesium with alpha particles, radiation of some sort seemed to take place from these elements after the alpha particle source had been taken away. In other words, there was a delayed activity.

This activity fell off with time in much the same manner that the activity of natural radioactivity decreases with time. For example, when alpha particles bombard aluminum, phosphorus is first created and neutrons are given off. This

reaction is shown in Figure 288(*a*). But this phosphorus is not stable. After some time it changes according to the reaction shown in Figure 288(*b*). The phosphorus changes into silicon and emits a high speed positron, and some gamma radiation. For any one atom the emission of the positron and gamma radiation may happen immediately after the formation of the phosphorus, or the phosphorus atom may exist for a considerable time before it blows up.

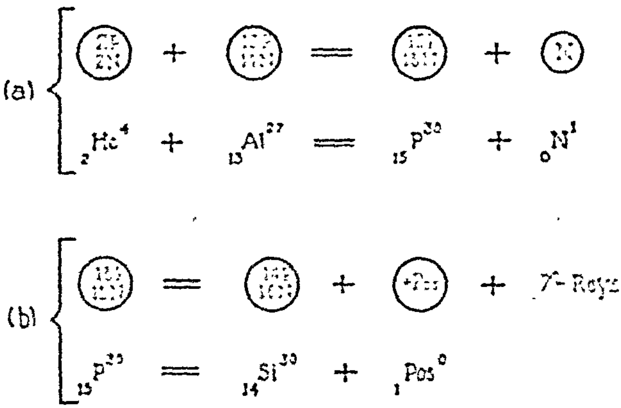


FIG. 288.—(*a*) When an alpha particle strikes an aluminum nucleus, an unstable isotope of phosphorus is produced. (*b*) At some later time the phosphorus blows up, emitting a positron and gamma rays and leaving silicon behind.

These experiments marked the discovery of man's ability not merely to transmute one element into another, but to create unstable elements which would disintegrate in a manner similar to that of the natural radioactive elements.

For this discovery the Joliots were awarded the Nobel prize in 1935. It is a fitting coincidence that the daughter of the woman who was twice awarded the Nobel prize for work in natural radioactivity should receive the same prize for the discovery of artificially produced radioactivity.

A great number of radioactive transmutations are already known. For example, when ordinary sodium such as is in table salt is irradiated with high speed heavy ions (deuterons), a heavy, but unstable

created and a proton is emitted. This reaction is shown in Figure 289(*a*).

At some later time, the heavy isotope of sodium spontaneously disintegrates and transmutes itself into an atom of magnesium, a high speed electron (beta particle) and gamma rays. This reaction is shown in Figure 289(*b*). So, if one wanted gamma radiation in his stomach, he could eat its source by sprinkling on his food ordinary salt which had first been exposed to high speed deuterons.

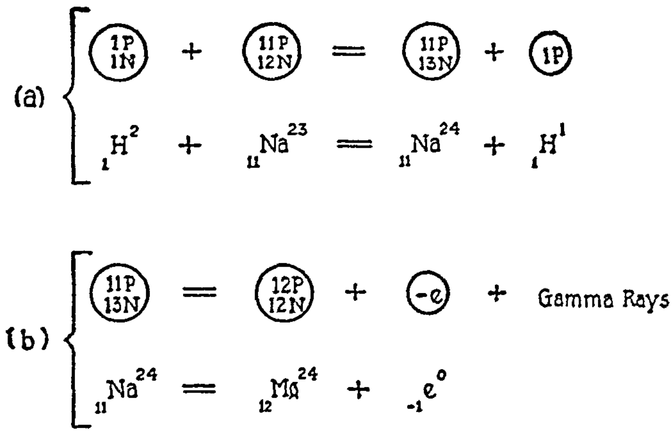


FIG. 289.—(*a*) Ordinary sodium may be converted into an unstable form of sodium by bombardment with heavy hydrogen. The byproduct is ordinary hydrogen. (*b*) The new sodium atoms will later blow up with the emission of beta and gamma rays similar to those obtained in natural radioactivity. The end product is magnesium.

9.24. More Examples of Transmutations

Nearly all of the transmutations first performed were with atoms at the lighter end of the periodic table. Also in these reactions, one of the products of the transmutation was a high speed particle. In later experiments, transmutations were accomplished with atoms in the heavier groups. In general, neutrons were most successful as bullets in producing these transmutations. Usually the neutron was captured by the heavy nucleus and no high speed particle was produced. A

typical example is the bombardment of silver with neutrons. The reaction is shown in Figure 290(*a*).

In this transmutation an ordinary silver atom captures an impacting neutron and becomes a heavy isotope of silver. This heavy form of silver is not found in nature, for it is unstable. It disintegrates by emitting a high speed electron. The half life for $_{47}Ag^{108}$ is only 2.3 minutes.

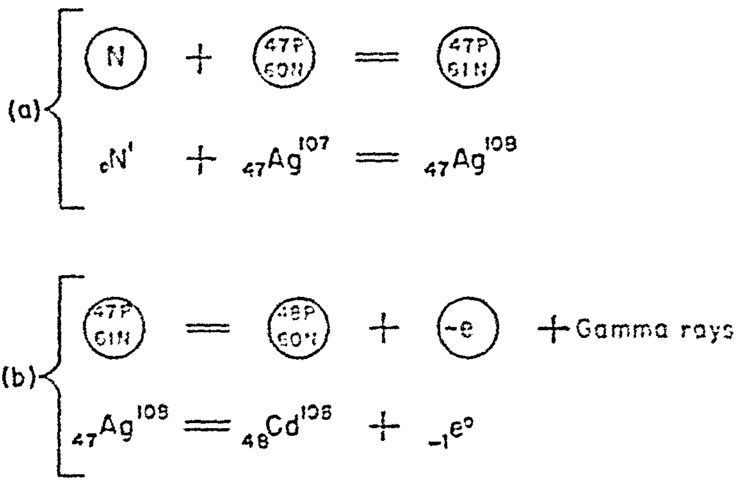


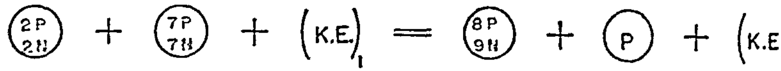
FIG. 290.—(a) A silver atom can capture an impacting neutron and so become a heavy isotope of silver. (b) This heavy isotope of silver is unstable and disintegrates with the emission of a beta particle and gamma rays. The end product is cadmium.

Emission of an electron in this case can occur if one neutron in the heavy silver nucleus is converted into a proton and an electron. The atomic number of the remaining nucleus will now be 48 (since the number of protons has been increased by 1), which corresponds to cadmium, and the atomic weight will remain at 108. This form of cadmium is stable and is found in nature, although heavier forms of cadmium are more common.

Numerous radioactive isotopes of many common elements have now been produced by a process like that described above

for silver. The radioactive isotopes disintegrate in one or more steps until they become stable isotopes of some element.

The discovery of artificially produced radioactivity has caused great interest among laymen and in the medical profession, as well as among physical scientists; for both industrial and medical applications may be developed. For example, some article of food or some drug may be made radioactive and then fed to a person. If the proper substance is chosen it will go to the part of the body that needs radiation treatment. Also, radioactive elements can be fed to a person or injected into a human body, and the rate at which the substance is carried to various parts of the body can be checked by a radiation indicator. Such information is daily adding to our medical knowledge.



$$_2He^4 + {}_7N^{14} + (K.E.)_1 = {}_8O^{17} + {}_1H^1 + (K.E$$

$$\text{Mass numbers}\ \underbrace{4.00389 + 14.00750}_{18.01139} \qquad \underbrace{17.00450 + 1.00812}_{18.01262}$$

FIG. 291.—The transmutation of Fig. 281 is repeated here with allowance for the kinetic energy of the particles on each side of the equation. There is little change in total nuclear mass during the transmutation and little change in kinetic energy.

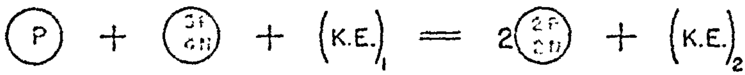
Gamma radiations both from artificially produced radioactive substances and also from natural radioactive materials (particularly the radium series) are used for the treatment of cancer and other diseases in a manner similar to that described on page 569 for x-rays.

10.24. Energy in Nuclear Changes

The first transmutation discovered by Rutherford is indicated in Figure 281 (page 694). The equation given in this figure does not completely account for the transmutation, since it does not show the energy of the particles either before or after the transmutation. Figure 291 shows a more complete form of the reaction given in Figure 281. In the new

figure $(K.E.)_1$ is the kinetic energy of the alpha particle, ${}_2He^4$ (we may assume that the nitrogen atom was at rest), and $(K.E.)_2$ is the combined kinetic energies of the heavy oxygen atom, ${}_8O^{17}$, and the proton, ${}_1H^1$. Experiment indicates that $(K.E.)_1$ is slightly greater than $(K.E.)_2$, but hardly enough greater to make it certain that the difference is beyond experimental inaccuracies.

As a check on this result, we may examine a second one of the early transmutations. This time we will bombard lithium with hydrogen. The resulting product is helium and the reaction is suggested in Figure 292. This time, experiment shows that $(K.E.)_2$ is much greater than $(K.E.)_1$. In other words, the kinetic energy of the two helium nuclei is



$$ {}_1H^1 + {}_3Li^7 + (K.E.)_1 = 2\,{}_2He^4 + (K.E.)_2 $$

Mass numbers $\underbrace{1.00813 + 7.01818}_{8.02631}$ $\qquad \underbrace{2 \times 4.00389}_{8.00778}$

FIG. 292.—When lithium is bombarded by hydrogen, helium is produced. There is a pronounced loss in nuclear mass and a corresponding increase in kinetic energy.

much greater than that of the hydrogen nucleus which was used to bombard the stationary lithium atom.

The next item of interest is that the sum of the masses of the lithium nucleus and the hydrogen nucleus is greater than the sum of the masses of the two helium nuclei. So on the left hand side of the equation we have more mass and less kinetic energy than on the right hand side of the equation where the mass is less and the kinetic energy is greater.

Long before this experiment was performed it had been predicted from Einstein's relativity theory that mass and energy can be interchanged and that when one is converted into the other the relation is

$$ \text{Energy} = \text{mass} \times (\text{velocity of light})^2 $$
$$ E = mc^2 $$

When the loss in mass of the particles on the left hand side of the equation in Figure 292 is converted to kinetic energy units by this formula and is added to the kinetic energy of the impacting hydrogen nucleus, we obtain the amount of kinetic energy $(K.E.)_2$ observed to be present in the two helium nuclei.

This experiment offers elegant proof for the theory that mass and energy are interchangeable; and at the same time it suggests the possibility of obtaining energy by transmutation processes. Whenever a transmutation results in a loss of mass, we may expect energy to appear.

If we examine the reaction shown in Figure 291 where only a small change in kinetic energy is observed, we find, also, that there is very little change in nuclear mass on the two sides of the equation. The small change that does exist gives greater mass on the right hand side of the equation than on the left. In other words, this particular transmutation results in increased mass and this increase in mass must come about by absorbing some of the kinetic energy put into the transmutation. This mass change, although small, accounts for the observation that the kinetic energy on the right hand side of the equation is slightly less than that on the left. On the other hand, in Figure 292 we have a transmutation where there is an appreciable loss of mass and consequently an appreciable gain in kinetic energy.

In both of the above reactions, the number of protons and neutrons on each side of each equation is equal to that on the other side. Since the masses of the nuclei are different while the number of particles remains the same, we must conclude that these particles have different masses depending on how they are packed together. This concept was mentioned briefly in Part II, Chapter 1, Section 12.1*d*. Precise measurements have now been made on the masses of many atoms so that the information used in discussing the masses of atoms in the above transmutations has been accurately determined by experiment.

11.24. Smashing Heavy Atoms

By the early part of the year 1939, experiments were reported in which very heavy atoms (in particular atoms of uranium) were bombarded by neutrons. In some cases the neutron was captured and a heavy radioactive isotope was produced as has been described above for the case of silver. In other cases it was observed that two relatively heavy particles were produced at once and that they were emitted at great velocity. In one of the early experiments barium was identified as one of the particles.

Later experiments have confirmed the early observation that a uranium nucleus can split into two medium heavy atoms corresponding to atoms in the middle of the atomic

$$ {}_0N^1 + {}_{92}U^{235} + (K.E.)_1 = X + Y + s\,{}_0N^1 + (K.E.)_2 $$

FIG. 293.—When uranium 235 captures a slow speed neutron it breaks into two medium heavy atoms and a number of neutrons. There is a large loss of mass and a corresponding gain in kinetic energy.

weight table; and these experiments also have shown that several high speed neutrons are produced. The two medium heavy atoms are in the form of unstable isotopes and they decay in the customary manner of radioactive elements until they reach some stable form. The splitting of an atom in the manner described here is called *fission*. There are at least three interesting items to be noticed at once: (1) The mass of all the fragments is much less than that of the original neutron and the bombarded uranium atom; hence there is a great amount of energy available. (2) In the smashing process, additional neutrons are produced which may possibly be used to smash other uranium atoms. In this manner the process can be self-sustaining, or as a chemist would say, a chain reaction. (3) Extensive radioactivity is present in the products of the transmutation.

Natural uranium consists chiefly of two isotopes—slightly more than 99 per cent of atomic weight 238 and less than 1 per cent of atomic weight 235. It is the uranium 235 which splits in the manner described above. Moreover, the chances of capture of a neutron by uranium 235 are greater

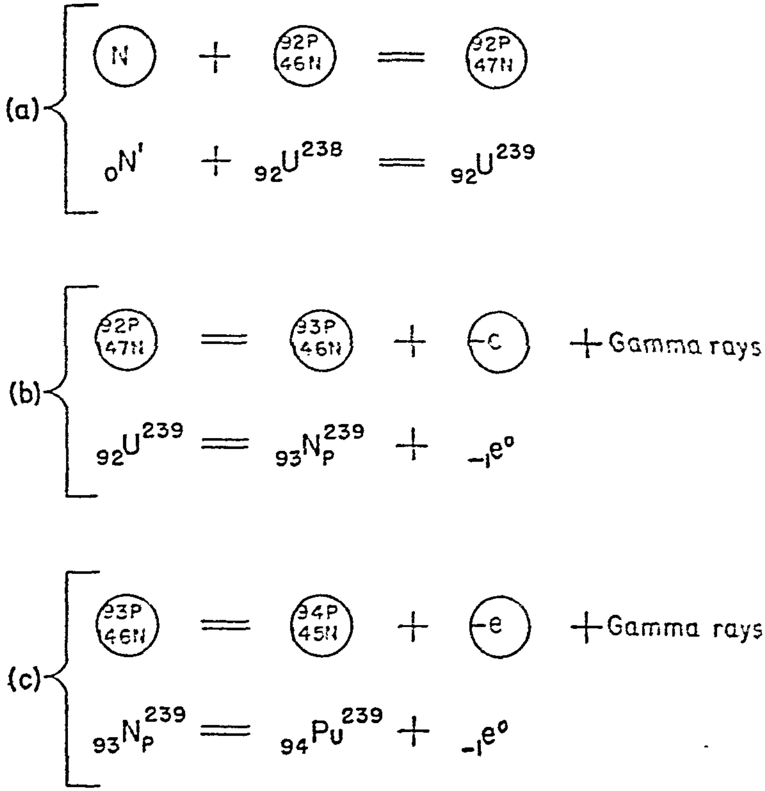


FIG. 294.—(a) Uranium 238 may capture a medium high speed neutron and become a heavy and unstable isotope of uranium. (b) The heavy uranium (239) emits a beta particle and becomes neptunium. (c) Neptunium is also unstable. It emits another beta particle and becomes plutonium. Plutonium, like uranium 235, can absorb a slow speed neutron and split into two heavy fragments with release of great energy.

for a slow speed neutron than for a high speed one. On the other hand, uranium 238 will capture medium high speed neutrons more readily.

When uranium 238 captures a neutron, it does not split as

does uranium 235, but becomes uranium 239. This heavy isotope of uranium is unstable. (See Figure 294.) It disintegrates by the process of having one of its neutrons changed into a proton and an electron. The electron is emitted at high speed and the atomic number of the remaining nucleus is 93 as compared to 92 for uranium. The new substance is called neptunium. Neptunium is also unstable and disintegrates by a repetition of the above process producing an atom with atomic number 94. This element is known as plutonium. Plutonium is important because it behaves like uranium 235 in that it can capture a slow speed neutron and promptly split into two heavy fragments and several neutrons and at the same time release large amounts of energy.

We now have two substances, uranium 235 and plutonium 239, which can be used as atomic energy sources. Uranium 235 is found in nature. Plutonium we can produce from uranium 238.

Some feeling for the amount of energy involved in the smashing of a uranium or plutonium atom can be gained from data which shows that this energy is about 100 million times that obtained from a simple chemical reaction such as the burning of a molecule of gasoline.

12.24. Using Energy from Nuclear Sources

Since the smashing of a uranium or plutonium atom by bombardment with a neutron produces several new neutrons, it is to be expected that the process will be chain reacting, provided the newly produced neutrons can be made to enter the nuclei of neighboring uranium or plutonium atoms. If this process can be handled in a controlled manner, a source of continuous power should result. If the process goes out of control there will be an explosion.

Obtaining a chain reaction will be more likely if the new high speed neutrons can be slowed down to the speed where their chances of capture are greater. Neutrons can be slowed down by allowing them to hit atoms of light elements with which they do not combine. For this purpose, carbon

has been used very successfully. If a small piece of uranium or plutonium is surrounded by pure carbon, the neutrons are slowed down to the point where the percentage captured is greatly increased. A material used to slow down high speed neutrons is called a *moderator.* (See Figure 295.)

It is also obvious that the chances of capture for a neutron by a nucleus of uranium or plutonium will also be increased if there is a large quantity of this material in the immediate neighborhood. We can therefore conclude that a small quantity of uranium 235 or plutonium might not be capable

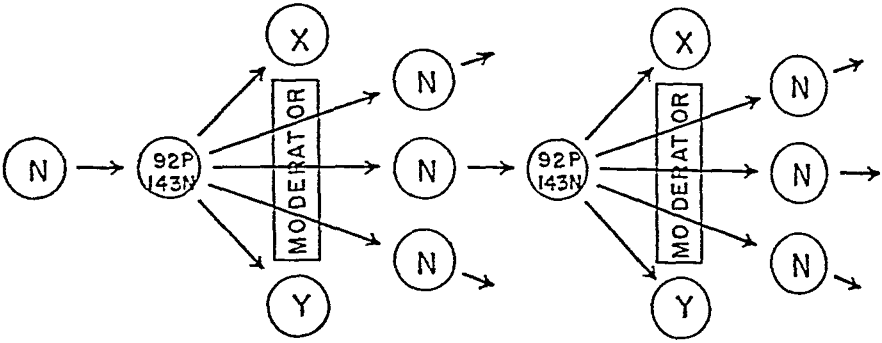


FIG. 295.—If the neutrons from the explosion of a uranium 235 (or plutonium 239) nucleus are slowed down, the chance for entering the nucleus of another similar atom is increased. Probability of a chain reaction is improved by this technique. Presence of a large supply of uranium 235 (or plutonium 239) nuclei also increases the likelihood of a chain reaction.

of sustaining a chain reaction, whereas a larger quantity might readily do so. This conclusion leads to the suggestion that a considerable quantity of uranium or plutonium might safely be kept in small quantities well separated, and that a violent explosion might take place if the small quantities were suddenly shoved close together. The same result might be accomplished by placing neutron barriers between the pieces of uranium and plutonium and then withdrawing the barriers. Also a lesser degree of concentration of the material, or a half-way position of the barriers might permit chain reaction to proceed at slower than an explosion rate. From these con-

siderations one may proceed to design either a controlled source of power or an atomic bomb.

If the reaction proceeds with explosive violence, it is to be expected that: (1) A great concussion wave will take place similar to that from any other form of violent explosion. (2) Tremendous heat will be developed corresponding to the very high kinetic energies of the atoms produced in the fission. (3) Powerful radiations will take place as the unstable isotopes disintegrate. The concussion wave will knock down structures; the heat will burn, melt or vaporize material; the radiations may kill or injure any animal life not destroyed by other effects.

If the reaction takes place under controlled conditions, a source of power for industrial purposes may be obtained. Also, the unstable products of the atom smashing might be used for medical or industrial purposes.

If nuclear fission is used as a source of power, the material used must be surrounded by barriers that will protect the area from all forms of radiation. Such protection is both bulky and massive and at present leads to the belief that nuclear power is not likely to be developed for such uses as driving an automobile. On the other hand, the use of nuclear power plants for driving large electric generators or for the propulsion of battleships looks practicable. The discovery of unlocking energy from the nuclei of atoms ushers in a new age of scientific discovery and application.

Some Important Facts

1. Unlike ordinary physical and chemical changes, transmutation involves changes in atomic nuclei. Transmutation is accomplished artificially by shooting the nucleus with high speed bullets, such as alpha particles, neutrons, etc.

2. In 1919, Rutherford bombarded nitrogen with alpha particles (helium nuclei) and obtained oxygen and protons (hydrogen nuclei). This is the first recorded case of controlled transmutation.

3. Using alpha particles as projectiles, Chadwick in 1932 transmuted beryllium into carbon and neutrons. The neutron has the weight of the proton but no electric charge.

4. While investigating cosmic rays, Anderson in 1932, discovered the positron—a particle with the mass of an electron but with a positive electric charge.

5. Recent research indicates nature to be in a highly complicated state of dynamic equilibrium.

6. In 1932, Urey discovered deuterium (heavy hydrogen) whose nucleus is called the deuteron. The deuteron consists of a proton and a neutron.

7. The cyclotron is a device for combining a powerful electromagnetic field with a high voltage, high frequency, electric field, to the end of producing high velocity particles for transmutation. For this purpose, the cyclotron excels radium in economy and ultra high voltage static machines (such as the Van de Graaf generator) in convenience and safety.

8. In 1934, the Joliots produced elements of unstable, and therefore radioactive, nuclei. For this synthetic radioactivity, they were awarded the Nobel prize in 1935.

9. Mass and energy can be interchanged. The relation is

$$\text{Energy} = \text{mass} \times (\text{velocity of light})^2$$

10. The exact mass of particles in nuclei varies from one element to another.

11. If a transmutation results in the loss of mass, a corresponding increase in energy takes place.

12. Controlled release of nuclear energy can be used as a source of power. Uncontrolled release of nuclear energy can take place with explosive violence.

13. Radiations from nuclear energy sources may be exceedingly dangerous to life. Under controlled conditions they may be used for medical or industrial purposes.

Generalization

Transmutation (or radioactive disintegration), whether occurring spontaneously or artificially produced, involves fundamental changes in atomic nuclei—thereby differing from ordinary chemical and physical changes.

Problems

Group A

1. Contrast modern transmutation with that of the alchemists as to:

   a. Desired result.
   b. Method of achievement.

2. Tabulate several artificial transmutations to date, listing in each case: (1) scientist; (2) year; (3) elements involved; (4) method used.

3. Tabulate known sub-atomic particles giving: (1) name; (2) discoverer; (3) year; (4) method of discovery.

Group B

1. Trace the so-called cosmic ray controversy from its inception to date.

2. Illustrate by labelled diagram, the principle of the cyclotron.

3. Compare the cyclotron with other devices designed for the same general purpose.

4. What items appear to be especially important in the transmutation resulting from the capture of a neutron by a silver nucleus?

5. Why is the equivalence of mass and energy important? What proof for the theory now exists? In what way is this fact disconcerting to older beliefs?

6. List the differences between the absorption of a neutron by uranium 238 and by uranium 235.

7. Make a diagram to show how a chain reaction can be obtained with uranium 235.

8. Suggest a detailed mechanism for (*a*) producing a controlled chain reaction; (*b*) producing an explosive chain reaction.

Experimental Problem

1. Conceive, if you can, an experiment relevant to the general problem of transmutation.

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